No - envy Arrow's conditions

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Abstract

This paper studies the relationship between the theory of distributive justice based on the concept of envy-freeness and Arrovian social choise theory. We define two conditions of No-envy and study their relationship with Arrow' scondition of independence of irrelevant alternatives, a weakening of this condition called Personal States Independence and the condition of Minimal Equity, that says that each individual must have the power to veto (in a limited sense) at least one alternative (presumably, one alternative which is particularly unfair to him).

1. Introduction

This paper analyses the concept of envy-freeness (see Foley, 1967) in the framework of Arrovian social choice theory. Two fairness conditions are introduced. Strong No-envy says that only envy-free alternatives can be chosen. Under unrestricted domain and Strong Pareto Optimality, this condition conflicts with the requirement that the choice set always be non-empty.

The (weaker) No-envy conditions just asks that only envy-free alternatives be chosen when they exist. It is argued that in an Arrovian framework the No-envy condition cannot be satisfied either because it violates Arrow's independence condition. However, No-envy is consistent with a weakening of the independence condition, that we shall call Personal States Independence.

We then introduce a weak ethical principle, Minimal Equity. Minimal Equity says that any individual must have the power to veto (in a weak sense) at least one alternative – presumably one which is particularly unfair to him[1]. In the presence of Personal States Independence, Minimal Equity implies Strong No-envy and therefore cannot possibly be satisfied. Thus we still have an impossibility result if Arrow's independence is weakened into Personal States Independence but Nondictatorship is strengthened into Minimal Equity.

2. Framework

Let X be the universal set of alternatives and $N = \{1, 2, ..., n\}$ the finite set of individuals. Alternatives are vectors $\alpha = (z, x_1, ..., x_i, ...x_n)$ where z is a vector that

[1]For instance, it could be argued that in a pure exchange economy any individual should be allowed to reject allocations that give him the zero bundle. Even this very weak condition implies in fact that every individual can veto many alternatives and is therefore stronger than Minimal Equity.

describes those feature of the particular situation under consideration that are common to all individuals, and x_i denotes the personal state of individual i. For instance, in an economy with public and private goods z may be interpreted as the vector of public goods and x_i as the vector of private goods pertaining to individual i.

We say that two alternatives $\alpha = (z, x_1, ..., x_n)$ and β belong to the same redistributive class if there is a permutation σ of N such that $\beta = (z, x_{\sigma(1)}, ..., x_{\sigma(n)})$. Thus α and β are characterised by the same public state z and contain the same list of personal states, so that β can be obtained from α through a redistribution of personal states across individuals. It is clear that redistributive classes are equivalence classes and hence partition X. Let $G(\alpha)$ denote the set of all logically conceivable alternatives that can be obtained from α through a redistribution of personal states across individuals. We assume that X is complete in the sense that $\alpha \in X$ implies $G(\alpha) \subset X$. This condition is actually stronger than necessary but is maintained for simplicity.

We assume that X contains at least two alternatives α and β that do not belong to the same redistributive class ($\beta \notin G(\alpha)$) and are such not all the personal states involved in each alternative are the same (that is, $\mid G(\alpha) \mid \geq 2$ and $\mid G(\beta) \mid \geq 2$). The assumption that X is complete then implies that X contains at least 4 distinct alternatives.

Each individual i is endowed with a weak ordering R_i over X. (A weak ordering is a complete, reflexive and transitive binary relation.) Let P_i denote strict preference and I_i denote indifference. We assume that individual preferences are selfish in the sense that each individual i is interested in the public state z and his personal state x_i only. That is,

$$(z, x_1, ..., x_i, ...x_n) I_i (z, y_1, ..., x_i, ..., y_n)$$

holds for all individuals i and all feasible alternatives. This means that individual preferences are essentially defined on pairs (z, x_i). To simplify notation we shall at times write $(z, x_i) R_i (z', x_i')$ etc.

A preference profile $p = (R_1, ..., R_n)$ is a list of preference relations, one for each individual. Let $\mathcal{P}$ be the set of all logically conceivable profiles. We assume unrestricted domain, so that any $p \in \mathcal{P}$ is an admissible profile.

Given a preference relation R over X, we denote by $R \mid_S$ the restriction of R to $S \subset X$. Similarly, we denote $p \mid_S = (R_1 \mid_S, ..., R_n \mid_S)$.

In this paper we deal with a particular type of collective choice rules, namely, Social Choice Correspondences.

Definition 1. *A Social Choice Correspondence assigns to each admissible profile* $p \in \mathcal{P}$ *a non empty set of alternatives* $C(p) \subset X$.

$C(p)$ is to be understood as the set of the choosable alternatives at profile p.

A Social Choice Correspondence may be viewed as a Social Choice Function the agenda domain of which is restricted to the universal set X only. We choose to work in a fixed agenda framework because it could be argued that this is the context where Foley's fairness criterion is most naturally defined (see Sen, 1986, pp. 1106-7). However, we believe our results could be translated into the more standard variable agenda framework. Obviously, in a fixed agenda framework it would be pointless to invoke Arrow's condition of independence of irrelevant alternatives because it is always vacuously satisfied. Instead of Arrow's independence we therefore use an independence condition due to Hansson (1969).

Hansson's independence condition (henceforth, H-independence) says that if an alternative α is chosen and another alternative β is rejected at a certain profile p, and if a profile p' has the same restriction to the pair $\{\alpha, \beta\}$ as profile p, then β should also be rejected at profile p'. Formally:

Definition 2. *A Social Choice Correspondence satisfies H-independence if for all* $\alpha, \beta \in X$ *and all* $p \in \mathcal{P}$, *if* $\alpha \in C(p)$ *and* $\beta \notin C(p)$ *and* $p\ |_{\{\alpha,\beta\}} = p'\ |_{\{\alpha,\beta\}}$ *then* $\beta \notin C(p')$.

Denicolò (1993) has shown that this condition is exactly the fixed agenda counterpart of Arrow's condition of independence of irrelevant alternatives.

We next define two conditions of Pareto optimality.

Definition 3. *A Social Choice Correspondence satisfies Pareto Optimality if at any profile* p *with* $\alpha P_i \beta$ *for all individuals* i *we have* $\beta \notin C(p)$.

In the framework used in this paper it may happen that two alternatives coincide up to a permutation of the personal states of a subset M of the set N of individuals. Individuals in $N - M$ will then be indifferent between such two alternatives by the assumption that preferences are selfish and in these cases the condition of Pareto Optimality would be silent. That is why we at times need the stronger condition of Strong Pareto Optimality.

Definition 4. *A Social Choice Correspondence satisfies Strong Pareto Optimality if at any profile* p *with* $\alpha R_i \beta$ *for all individuals* i *and* $\alpha P_i \beta$ *for at least one individual* i *we have* $\beta \notin C(p)$.

3. NO-ENVY

Now we come to the notion of envy-freeness and the no-envy conditions. First of all we formally define the concept of envy.

Definition 5. *Individual i envies individual j at $\alpha = (z, x_1, ..., x_n)$ if $(z, x_j)P_i(z, x_i)$.*

In other words, i envies j if he prefers j's personal state over his own.

Definition 6. *An alternative α is envy-free at profile p if no individual envies anybody else at α.*

It would appear natural to ask that only envy-free alternatives be chosen. Formally, let $F(p)$ denote the set of all envy-free alternatives at profile p.

Definition 7. *A Social Choice Correspondence satisfies Strong No-envy if $C(p) \subseteq F(p)$ for all $p \in \mathcal{P}$.*

Under unrestricted domain, the condition of Strong No-envy is very demanding because the only way to guarantee that envy-free alternatives exist is to consider alternatives where the personal states of all individuals are identical. But such alternatives can easily be Pareto dominated and therefore it is possible to make examples where no envy-free and Pareto optimal alternatives exist. Thus under unrestricted domain and the Strong Pareto Principle, the Strong No-envy condition cannot possibly be satisfied[2].

Proposition 1. *There is no Social Choice Correspondence satisfying Strong Pareto Optimality and Strong No-envy.*

Proof. By assumption, there are at least two distinct alternatives α and $\beta \in G(\alpha)$. Let i and j denote two individuals whose personal states at α and β do not coincide. Consider a profile p such that all other alternatives are weakly Pareto dominated by α and β, and i envies j at α whereas j envies i at β. Then Strong Pareto Optimality and Strong No-envy together would imply that $C(p)$ be empty, which is impossible. ■

Thus under unrestricted domain one cannot insist that only envy-free alternatives be chosen. A weaker no-envy condition is that only envy-free alternatives be chosen if there are any. We call this condition, No-envy.

Definition 8. *A Social Choice Correspondence satisfies No-envy if $C(p) \subseteq F(p)$ for all $p \in \mathcal{P}$ such that $F(p) \neq \emptyset$.*

This condition is clearly consistent with Strong Pareto Optimality. However, in the next section we shall show that it conflicts with H-independence.

[2]There is a large literature that studies conditions for the existence of envy-free alternatives in economic contexts; in most of this literature the condition of Pareto Optimality is also imposed. Though there are problems in production economies, we know that envy-free and Pareto efficient allocations exist in a large class of economies. By way of contrast, in this paper we mantain the unrestricted domain assumption and weaken the Strong No-envy condition. See the concluding section for further remarks on this point.

4. UTILITY INFORMATION AND ENVY

Even if each individual evaluates alternative social states according to his own personal state only, an external ethical observer may wish to base his judgement on the way individuals evaluate (according to their own preferences) the positions of others. This is implicit in the No-envy condition. In this section we argue that Hansson's independence condition rules this information out and therefore must be violated if one insists that only envy-free allocations be choosable.

Consider the following two alternatives:

$$\alpha = (x_1, x_2) \text{ and } \beta = (x_1, y_2)$$

where x_1 is the (constant) personal state of individual 1 and x_2 and y_2 are two possible personal states of individual 2 (there are no public issues). Now consider two preferences profiles, p and p'. Assume $R_2 = R_2'$ and, in particular, at both profiles, $x_2 I_2 y_2$.

Thus, the two alternatives are indistinguishable in terms of utility levels. Any Bergson-Samuelson social welfare function must declare they are socially indifferent. To put things in a slightly different way, H-independence implies that α and β should be treated in the same way in social choices made at profiles p and p'.

But suppose that at profile p, $x_2 P_1 x_1 P_1 y_2$, whereas at profile p', $y_2 P_1' x_1 P_1' x_2$. This means that individual 1 envies individual 2 at α at profile p, and envies him at β at profile p'. (For simplicity, suppose that at neither profile individual 2 envies individual 1.) Thus only β is envy-free at profile p, and only α is envy-free at profile p'.

This example neatly shows that the independence condition is not consistent with the condition of No-Envy. The intuition is as follows. Arrow's independence and its fixed agenda relative, H-independence, not only imply a weak form of neutrality thus forbidding the use of non utility information; they also severely limit the use of utility information as they allow social choice to depend only on the utilities actually enjoyed by individuals. This rules out the envy comparisons which are the starting point of the No-envy condition.

Since the notion of envy-freeness does not require any non utility information, and also economizes on the use of utility information (for instance, it does not rest on any kind of interpersonal comparisons), however, it may be consistent with some weakening of the independence condition. But which weakening exactly? We tackle this question in the following section.

5. PERSONAL STATES INDEPENDENCE

In the light of the above discussion one is led to weaken the Hansson independence condition in order to make room for the envy comparisons.

Loosely speaking, independence conditions say that the treatment of any two alternatives in the social choice should not change (in some sense) if the relative ranking of the two alternatives in individuals' preferences has not changed (in some sense). Our weakening of H-independence is based on a stricter interpretation of what is meant by saying that the relative ranking of two alternatives has not changed.

It should be clear that in order to use envy comparisons, one cannot say that the individual preference orderings on $\{\alpha, \beta\}$ have not changed unless the preference orderings of each individual over *all the relevant personal states,* even those pertaining to other individuals, have not changed. Formally, in order to claim that individual preferences over two alternatives α and β have not changed it does not suffice that $p\mid_{\{\alpha,\beta\}} = p'\mid_{\{\alpha,\beta\}}$; it is necessary that the relative ranking of all alternatives in $G(\alpha)$ and $G(\beta)$ has not changed either, that is $p\mid_{G(\alpha)\cup G(\beta)} = p'\mid_{G(\alpha)\cup G(\beta)}$. Otherwise, the envy information is lost.

This leads to the following weakening of H-independence.

Definition 9. *A Social Choice Correspondence satisfies Personal States Independence if $\alpha \in C(p)$ and $\beta \notin C(p)$ and $p\mid_{G(\alpha)\cup G(\beta)} = p'\mid_{G(\alpha)\cup G(\beta)}$ imply $\beta \notin C(p')$.*

We have:

Proposition 2. *If X is finite*[3]*, there exist Social Choice Correspondences satisfying Personal States Independence and No-envy.*

Proof. Consider the Social Choice Correspondence that at any profile $p \in \mathcal{P}$ selects all envy-free alternatives at p if there are any; otherwise, it selects the alternatives at which only individual 1 envies somebody else if there are any; otherwise, it selects the alternatives at which only individuals 1 and 2 envy somebody and so on.

First of all, note that this Social Choice Correspondence is well defined. Indeed, if an alternative is Pareto Optimal it cannot be the case that everybody envies somebody else (see Varian, 1974; his proof holds also with unrestricted domain); since when X is finite there always exist Pareto optimal alternatives, this implies that the choice set cannot be empty.

We next show that this correspondence satisfies Personal States Independence.

Suppose $\alpha \in C(p)$ and $\beta \notin C(p)$. Then there is an $s \geq 0$ such that no $i > s$ envies anybody at α whereas $s+1$ envies somebody else at β. Now consider a profile p' such that $p\mid_{G(\alpha)\cup G(\beta)} = p'\mid_{G(\alpha)\cup G(\beta)}$. Then all the envy relationships concerning α and β are preserved at profile p'. That is, there is an $s \geq 0$ such that no $i > s$ envies anybody at α whereas $s + 1$ envies somebody else at β and therefore β cannot be chosen at p'. ■

[3]Here and in Proposition 4 below the assumption that X is finite could be relaxed; all what matters is that at any profile there is at least one Pareto optimal alternative.

The set of envy-free alternatives is often very large and may contain alternatives which are Pareto dominated. The condition of Strong Pareto Optimality can be used to overcome this problem. However, while the condition of No-envy alone is consistent with Personal States Independence, we now demonstrate that if the condition of Strong Pareto Optimality is added one gets an inconsistency.

Proposition 3. *There is no Social Choice Correspondence that satisfies No-envy, Strong Pareto Optimality and Personal States Independence.*

Proof. An example will suffice. Assume there are no public issues (the vector z has dimension 0) and there are only two individuals. Consider three alternatives $\alpha = (x_1, x_2)$, $\beta = (y_1, y_2)$ and $\gamma = (w_1, w_2)$. Construct a profile p such that α and β are envy-free, there is envy at γ and γ Pareto dominates α. All other alternatives are Pareto dominated. Then construct another profile p' such that $p \mid_{G(\alpha)\cup G(\beta)} = p' \mid_{G(\alpha)\cup G(\beta)}$, so that α and β are still envy-free, there is still envy at γ but now γ Pareto dominates β. Again, all other alternatives are Pareto dominated. (We leave it to the reader to check that these profiles can be constructed under unrestricted domain.) Then only β can be chosen at profile p, and only α can be chosen at profile p'. This violates the condition of Personal States Independence. ■

This result motivates the following weakening of the condition of Personal States Independence.

Definition 10. *A Social Choice Correspondence satisfies Weak Personal States Independence if* $\{\alpha\} = C(p)$ *and* $\beta \in G(\alpha)$ *and* $p \mid_{G(\alpha)} = p' \mid_{G(\alpha)}$ *imply* $\beta \notin C(p')$.

Personal States Independence has thus been weakened in two ways. First, in the premise we no longer assume that α be chosen and β be rejected but we require that α is uniquely chosen. Second, the condition of Weak Personal States Independence is operative only when α and β belong to the same redistributive class; otherwise, it is silent. It is therefore clear that Weak Personal States Independence is implied by Personal States Independence.

We now demonstrate that the inconsistency shown in Proposition 3 disappears if Personal States Independence is replaced by Weak Personal States Independence.

Proposition 4. *If X is finite, there exist Social Choice Correspondences satisfying Weak Personal States Independence, No-envy and Strong Pareto Optimality.*

Proof. Consider the Social Choice Correspondence that at any profile $p \in \mathcal{P}$ selects all envy-free and Pareto optimal alternatives at p if there are any; otherwise, it selects all Pareto optimal alternatives. Clearly, the choice set is never empty and therefore this Social Choice Correspondence is well defined.

We now show that this correspondence satisfies Weak Personal States Independence.

Suppose that $\{\alpha\} = C(p)$ with $\beta \in G(\alpha)$. We must show that if $p' |_{G(\alpha)}= p |_{G(\alpha)}$then $\beta \notin C(p')$. Suppose first that α is envy free; it follows that β cannot be envy-free at p. The reason is that if an alternative is envy-free it weakly Pareto dominates any other alternative in its redistributive class. Thus if β was envy-free, it must be Pareto indifferent to α and since α is Pareto optimal, β must also be Pareto optimal and therefore should have been chosen. Thus β is not envy free at profile p. Since the envy relationships are preserved within $G(\alpha)$, β cannot be envy-free at profile p' and therefore $\beta \notin C(p')$ if there are envy-free and Pareto optimal alternatives. If α is no longer Pareto optimal at p', β (which is Pareto dominated by α) must also be Pareto dominated at p' and therefore cannot be chosen.

To complete the proof, we show that if α is uniquely chosen it must indeed be envy-free. Suppose to the contrary that α is not envy-free at p, then since it is chosen it must be the case that there are no envy-free alternatives at p. Since α is uniquely chosen and X is finite, it follows that α Pareto dominates all other alternatives, including all alternatives in $G(\alpha)$. But this means that α is envy-free. ■

It is an open question whether No-envy is consistent with some independence condition stronger than Weak Personal States Independence.

6. NO-ENVY AND MINIMAL EQUITY

There is a long tradition in economics, dating back at least to Wicksell, that argues that certain decisions should be taken in accordance to the unanimity principle. But it is difficult to find an unanimous agreement on redistributive issues. For instance, in the case where individuals are egoistic and non satiated it is obvious that a conflict of interest will emerge and the unanimity principle cannot be applied. Nonetheless, it would seem appropriate to give individuals some veto power, to avoid that an aggressive majority can expropriate those who are left out.

However, it is well known that the independence condition (together with Pareto Optimality and unrestricted domain) implies that if one gives an individual veto power on a single issue, one must give him veto power on all issues. If all individuals are treated symmetrically, we are back to the unanimity principle.

When the H-independence condition is weakened into Weak Personal States Independence the veto epidemic is more limited but still an impossibility result emerges. More precisely, we show that under unrestricted domain, Weak Personal States Independence and Strong Pareto Optimality, then giving everybody a veto power implies Strong No-envy. By Proposition 1, one then obtains an impossibility result.

In keeping with the Wicksellian tradition, we assume that all individuals are given at least a very limited veto power. More precisely, we postulate that for any individual there is at least one alternative (one can think of a distribution of resources which is particularly unfavorable to him) that he can veto in a limited sense. We shall call this condition Minimal Equity.

Veto power as defined in this paper is limited in the following sense. In order to exclude from the choice set the alternative which has been assigned to him, say α, an individual must find another alternative in the same redistributive class as α that he strictly prefers to α. Thus it does not suffice that an individual can do better than α; he can use the veto power only if he can do better choosing a different personal state in the list of personal states contained in α. Formally:

Definition 11. *Individual i has a veto on an alternative α if $\beta P_i \alpha$ for some $\beta \in G(\alpha)$ implies $\alpha \notin C(p)$.*

To clarify, imagine that an alternative is defined in three steps: in the first step, the public state is chosen; in the second step, the unordered list of personal states is chosen; in the third step, personal states are assigned to individuals. Our veto power is limited to the third step of such a procedure.

Definition 12. *A Social Choice Correspondence satisfies Minimal Equity if for all individuals $i \in N$ there is at least one alternative $\alpha_i \in X$ such that $G(\alpha_i) \geq 2$ and i has a veto on α_i.*

The condition of Minimal Equity says that each individual must have veto power, in the sense we have just clarified, on at least one alternative[4]. This alternative could be the same for all individuals, but must be such that there are at least two distinct alternatives in its redistributive class (otherwise the veto power would be vacuous because it could never be implemented).

Minimal Equity is a very weak condition. However, we now demonstrate that, in the presence of Strong Pareto Optimality and Weak Personal States Independence, Minimal Equity implies the (much stronger) condition of Strong No-Envy.

Proposition 5. *If a Social Choice Correspondence satisfies Strong Pareto Optimality, Weak Personal States Independence and Minimal Equity, then it must satisfy Strong No-envy.*

Proof. First we show that if an individual i has veto power on one alternative, it must have veto power on all alternatives.

[4]For instance, the vetoed alternative α might be the proposal to tax away all the resources of individual i and redistribute them among the other $(n-1)$ individuals. It can be easily shown that there exist Social Choice Correspondences that satisfy Minimal Equity and Strong Pareto Optimality.

Suppose that i has veto power on α and $G(\alpha) \geq 2$. We show that it must have veto power on any other alternative $\gamma \notin G(\alpha)$.

If $G(\gamma) = 1$ the claim is trivially true, thus we assume $G(\gamma) \geq 2$. Consider then four alternatives, $\alpha, \beta \in G(\alpha), \gamma \notin G(\alpha)$ and $\delta \in G(\gamma)$. Consider a profile p such that $\delta P_i \beta P_i \alpha P_i \gamma$ and, for all $j \in N - \{i\}$, $\delta R_j \beta$ and $\alpha R_j \gamma$; no further restriction is placed on j's preferences. All other alternatives are Pareto dominated.

Since i has veto power on α and $\beta P_i \alpha$, we have $\alpha \notin C(p)$. By Strong Pareto Optimality, $\beta \notin C(p)$ and $\gamma \notin C(p)$. Hence $\{\delta\} = C(p)$. By Weak Personal States Independence, this means that γ cannot be chosen whenever there is a $\delta \in G(\gamma)$ such that $\delta P_i \gamma$; that is, i has veto power on γ.

Repeated applications of this argument show that if an individual has veto power on one alternative, it must have veto power on all alternatives.

We next show that Strong No-envy must hold. By Minimal Equity and our previuos claim, all individuals must have veto power on all alternatives. Suppose α is not envy free at profile p. Then there is at least one individual i who envies somebody else at α. Since i has veto power on α, it follows $\alpha \notin C(p)$. But this is just what the Strong No-envy condition says. ■

Since we know that under unrestricted domain the Strong No-envy condition cannot be satisfied, it follows:

Corollary 1. *No Social Choice Correspondence satisfies Strong Pareto Optimality, Weak Personal States Independence and Minimal Equity.*

This result resembles the (choice theoretical formulation of) the impossibility of a Paretian liberal of Sen (1970). Minimal Equity, however, is different from Minimal Liberty. First, Minimal Equity assigns a more limited veto power than that attributed to individuals by Minimal Liberty. Second, Minimal Equity could be satisfied even if all individuals had the power to veto the *same* alternative. Suppose for instance there are two individuals and let $\alpha = (x_1, x_2)$. If x_1 is the very worst personal state of individual 1 and x_2 is very bad to individual 2, it is not unreasonable that both 1 and 2 are given the power to veto α: in any event, this would be enough to respect the Minimal Equity condition. These differences are the reason why our impossibility theorem requires a (weak) form of independence whereas Sen's theorem does not use independence at all.

7. Concluding Remarks

We have shown that the condition of No-envy is inconsistent with Arrow's independence of irrelevant alternatives or with its fixed agenda counterpart, Hansson' independence. However, it is consistent with a weaker independence condition, Personal

States Independence. Personal States Independence allows for the use of a larger set of utility information in making social choice than H-independence.

We have also shown that Strong No-envy can be derived from a more basic ethical principle, Minimal Equity, that says that any individual must have the power to veto at least one alternative if he dislikes it on purely redistributive grounds. Since under unrestricted domain Strong No-envy cannot be satisfied, our result implies that there is no Paretian Social Choice Correspondence that obeys both Weak Personal States Independence and Minimal Equity.

A task for future research is to analye whether our results can be extended to economic domains. In particular, since on economic domains Strong No-envy and Pareto Optimality can be satisfied, if Proposition 5 could be extended to economic domains this would provide a new justification of the No-envy condition.

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