CARTEL STABILITY AND THE CURVATURE OF MARKET DEMAND

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Abstract

The stability of collusion is analysed for a family of demand functions whose curvature is determined by a parameter varying between zero and infinity. If demand is sufficiently convex, firms may prefer to act as quantity setters in order to increase cartel stability. Otherwise, price-setting behaviour enhances their ability to collude.

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1. Introduction

The issue of cartel stability has been widely and deeply analysed under many perspectives (see D'Aspremont and Gabszewicz (1986) for an exhaustive survey). Particularly noteworthy efforts have been devoted to the investigation of the bearings of heterogeneity among firms on cartel stability (D'Aspremont et al. (1983), Donsimoni (1985), Donsimoni et al. (1986)). Another relevant stream of literature deals with the relationship between product differentiation and the stability of collusion (see, *inter alia*, Deneckere (1983), Chang (1991), Ross (1992) Rothschild (1992) and Häckner (1994)). The role of imperfect information in affecting firms' ability to collude has also received attention (Green and Porter (1984), Rees (1985), Rotemberg and Saloner (1986)).

Here I want to investigate what kind of linkages there exist between the curvature, i.e., the convexity/concavity of the market demand function, and the stability of a cartel made up by symmetric firms supplying a homogeneous good, under both quantity-setting and price-setting behaviour. It turns out that under Cournot behaviour the critical discount factor is increasing and concave in the parameter describing demand curvature, so that cartel stability is a decreasing function of the same parameter, while under Bertrand behaviour the critical value of the discount factor, and consequently cartel stability, are indipendent of the curvature of the demand function. These results revise and encompass those already available in the literature (see Majerus, 1988) as special cases.

2. The model

I adopt the simple model described by Anderson and Engers (1992, p.129), assuming, in order to simplify calculations, that only two firms operate in the market. They sell a homogeneous product. The market demand function is defined by

$$Q = 1 - p^{\alpha}, \quad \alpha > 0; \qquad (1$$

this demand function is always downward sloping, and can be either convex (when $\alpha \leq 1$) or concave (when $\alpha \geq 1$). Fixed costs can be assumed away without loss of generality, while it must be assumed that marginal costs are nil in order to obtain explicit solutions. Thus, firm *i*'s profit function coincides with her revenue. The collusive setting is analysed under the standard hypothesis that after deviation agents revert to the Nash equilibrium strategies forever. The next subsection is devoted to the analysis of Cournot behaviour. Subsection 2.2 then briefly describes Bertrand behaviour.

2.1. Quantity-setting duopoly

Consider first what happens if firms noncooperatively maximize single-period profits by simultaneously choosing quantitites. Firm *i*'s profit is given by

$$\pi_i = q_i(1-q_i-q_j)^{\frac{1}{\alpha}}, \quad i,j=1,2, \quad i \neq j. \tag{2}$$

The first oder condition for firm *i* is then

$$\frac{\partial \pi_i}{\partial q_i} = (1-q_i-q_j)^{\frac{1}{\alpha}}\left(1-\frac{q_i}{\alpha(1-q_i-q_j)}\right) = 0, \tag{3}$$

yielding

$$q_i^N = \frac{\alpha}{1+2\alpha} \tag{4}$$

as the Nash equilibrium quantity, and

$$\pi_i^N = \frac{\alpha}{(2\alpha+1)^{\frac{\alpha+1}{\alpha}}} \tag{5}$$

as the noncooperative equilibrium profit (see Anderson and Engers, 1992, p.131).

If instead firms collude, they cooperatively set quantities in order to maximize joint profits:

$$\Pi^C = \pi_1 + \pi_2; \tag{6}$$

the generic first order condition is:

$$\frac{\partial \Pi^C}{\partial q_i} = \frac{(1-q_i-q_j)^{\frac{1}{\alpha}}(\alpha-q_i-q_j-\alpha q_i-\alpha q_j)}{\alpha(1-q_i-q_j)} = 0, \tag{7}$$

yielding as a solution

$$q_i^C = \frac{\alpha}{2(\alpha+1)}, \tag{8}$$

while each firm's collusive profit amounts to

$$\pi_i^C = \frac{\alpha}{2(\alpha+1)^{\frac{\alpha+1}{\alpha}}}. \tag{9}$$

A quick comparison between (4-5) and (8-9) shows that the cartel resticts both individual and total output, consequently enhancing profits. Let's now consider unilateral deviation from the cartel agreement. Provided firm *j* sticks to her collusive output q_j^C, the deviation output by firm *i* is obtained from condition (4):

$$q_i^D = \frac{\alpha(\alpha+2)}{2(\alpha+1)^{\frac{2(\alpha+1)}{\alpha}}} \tag{10}$$

yielding

$$\pi_i^D = \frac{\alpha(\alpha+2)^{\frac{\alpha+1}{\alpha}}}{2^{\frac{\alpha+1}{\alpha}}(\alpha+1)^{\frac{2(\alpha+1)}{\alpha}}} \tag{11}$$

as the single-period deviation payoff. Deviation by firm *i* entails the following profit for firm *j*:

$$\pi_j(q_i^D, q_j^C) = \frac{\alpha(\alpha+2)^{\frac{1}{\alpha}}}{2(\alpha+1)\,(2\alpha^2+4\alpha+2)^{\frac{1}{\alpha}}} \tag{12}$$

Since, as it is easily checked, $\pi_j(q_i^D, q_j^C) < \pi_j^N$ for all $\alpha > 0$, the punishment strategy is always credible. Thus, for collusion to be sustainable each firm's discount factor must sutisfy the following condition:

$$\beta_i \geq \beta(\alpha) = \frac{\pi_i^D - \pi_i^C}{\pi_i^D - \pi_i^N} = \frac{(1+2\alpha)^{\frac{\alpha+1}{\alpha}}\left(2^{\frac{1}{\alpha}}(\alpha+1)^{\frac{\alpha+1}{\alpha}} - (\alpha+2)^{\frac{\alpha+1}{\alpha}}\right)}{2^{\frac{\alpha+1}{\alpha}}(\alpha+1)^{\frac{2\alpha+2}{\alpha}} - (\alpha+2)^{\frac{\alpha+1}{\alpha}}(2\alpha+1)^{\frac{\alpha+1}{\alpha}}} \qquad (13$$

After some simple albeit tedious calculations it can be verified that $\partial\beta/\partial\alpha > 0$ and $\partial^2\beta/\partial\alpha^2 < 0$, implying that the critical discount factor is increasing and concave in α. The same obviously holds for π_i^N, π_i^C and π_i^D. Furthermore,

$$\lim_{\alpha\to 0} \pi_i^J = 0, \quad \lim_{\alpha\to\infty} \pi_i^J = \frac{1}{2}, \quad J = N, C, D, \qquad (14$$

so that as α approaches either zero or infinity, the critical discount factor β is indeterminate. Nevertheless,[1]

$$\lim_{\alpha\to 0} \beta(\alpha) = 0, \quad \lim_{\alpha\to\infty} \beta(\alpha) = 1. \qquad (15$$

This results can be given an intuitive explanation. First, as α decreases, $\pi_i^D - \pi_i^C$ shrinks more rapidly than $\pi_i^D - \pi_i^N$. Second, as α approaches infinity, $\pi_i^C - \pi_i^N < \pi_i^D - \pi_i^C$, i.e., as α increases, π_i^N catches up with π_i^C faster than the latter does with π_i^D.

2.2. Price-setting duopoly

The case of Bertrand behaviour can be quickly dealt with. Single-period noncooperative profit $\pi_i^N = 0$ whatever the value of α, while cooperation yields half the monopoly profit,$\pi_i^C = \pi^M/2$, and finally $\pi_i^D = \pi^M$. Consequently, the critical discount factor is independent of α and each firm's discount factor must satisfy the following condition (see Tirole, 1988, p.246):

$$\beta_i \geq \beta = \frac{\pi^M - \pi^M/2}{\pi^M} = \frac{1}{2} \qquad (16$$

1. The limit of β as α approaches zero has been computed through a binomial expansion, which has not been included in the text due to its length.

Numerical calculation shows that $\beta_C = \beta_B$ when α~0.4753, where subscripts *C* and *B* stand for Cournot and Bertrand behaviour, respectively. This means that a quantity-setting cartel is more stable than a price-setting cartel for $\alpha \in]0, 0.4753[$.

3. Conclusions

The influence of the curvature of market demand on cartel stability has been analysed. The results point to the conclusion that the degree of convexity/concavity of the demand function can affect firms' ability to collude only if they act as quantity-setters. In the Cournot framework, the critical discount factor increases at a decreasing rate as the parameter determining the curvature of demand increases. When firms play *à la* Bertrand, the nature of both competition and equilibrium payoffs is such that the critical discount factor is indipendent of the curvature of the demand function. Furthermore, if the latter is sufficiently convex, the critical discount factor associated to quantity collusion is lower than that associated to price competition, so that collusion in quantities is more stable than collusion in prices, while the opposite holds if the demand function is only slightly convex, linear, or concave. This implies that, should firms be able to coordinate the choice of the strategic variable in order to enhance cartel stability, they would choose to set quantities only when facing a rather limited class of convex demand functions. The result reached by Majerus (1988), who takes into consideration a linear demand function and shows that when products are close or perfect substitutes price collusion is more easily sustainable than quantity collusion, appears as a special case of the results drawn from the model presented here.

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