SHUT DOWN OPTION AND PROFIT SHARING

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No 190

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February 1994

JEL classification L20, D92

Abstract

Aoki's profit sharing firm organization is associated with the option evaluation model of investment. The firm is endowed with a shut down option it can exercise when the market price, assumed uncertain, falls below a certain trigger level. The distributive parameter is the result of a bargaining process and it is influenced by the shut down option. Workers can delay the firm's shut down by sharing not only profits but also losses. In that case the workers' policy changes both the optimal distributive parameter and the trigger price in a non trivial way. The overall result implies an increase of the profit share going to shareholders as compared to the original Aoki's finding.

We acknowledge comments and suggestions by Sergio Pastorello with the usual disclaimer.
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1. Introduction

The decision to give birth to a firm or to invest has usually been modelled as a project evaluation problem, mainly based on present value criteria. More recent literature - surveyed in Pindyck (1991), Dixit (1992) - has emphasized the role of uncertainty on two aspects of investment decisions, which are not properly taken into account by the present value approach. The first concerns the irreversibility of most investment expenditure because of firm specificity. The second deals with the possibility of delaying the investment while keeping the opportunity of doing so at the most favourable time. Sunk costs and the possibility of delaying entry or exit decisions are able to explain many hysteresis phenomena, i.e. the persistence of an effect well beyond its cause. In our case hysteresis means that some firms do not exit when the price goes just below the average variable cost and do not enter when the price goes just above the total average cost (variable plus fixed average cost). Firms wait and do not exit because they think that the price could revert to a higher level which is considered normal or simply that the price goes up and down with the same probability. There are optimal trigger prices, which make the firm enter or exit: below the average variable cost, for exit, and above the average total cost, for entry. As a consequence, entry and exit are to be considered options which can be exercised or kept alive by delaying them. Then a firm is defined by its know-how and by the option to invest and to produce.

The innovation of this paper is to introduce, within this framework, a different firm organization as compared to the traditional neoclassical one adopted so far. We define the firm as a joint organization run by a manager who acts on behalf of shareholders and mediates between workers and shareholders as to the distribution of profits. But before mediating with workers on the firm's organization the manager has to take decisions as to the firm investment, i.e. the dimension of the firm. Any investment in fixed capital is simultaneously accompanied by the decision to hire workers. Therefore the manager of the firm, in addition to the investment project, has to design the industrial relations and discusses them with workers' representatives. This second aspect of the investment decision has not yet been coupled with the option model. In this paper we try to model both aspects of the investment decision, i.e. the optimal trigger prices and the optimal profit sharing policy.

Workers receive part of the profits as an extra wage above the market wage. The share of profit they get is the result of bargaining and is aimed to avoid quits causing a loss of human capital that the firm accumulates on workers, even though there might be a certain kind of specificity of human capital hindering marketability. Workers are paid above the market wage also because the firm has an option to shut down, if the market price goes below a certain trigger level. To model this kind of organization we resort to Aoki's (1980, 1984) model of the firm. Aoki's firm is a specific organisation of stockholders and employees endowed with "skill and knowledge specific to the firm as a result of quasi permanent association with it". The firm's specificity of human capital and the capital supplied by stockholders make it possible to attain results which depart from the standard neoclassical model where the mere combination of capital and labour is sufficient to set up a firm. These results can be reached because of cooperation between employees and stockholders, which can be modeled as a bargaining over the distribution of profit, called by Aoki, "organizational rent". In this case a market for human capital appears, but it is internal to the firm and its reward depends on the firm's performance.

A firm investing in physical assets has a perpetual option to exit the market by shutting down the firm. By paying a certain exercise price equal to the unitary cost of production the firm gets an uncertain revenue. We see how the exit decision is influenced by profit sharing and how profit sharing influences the trigger price which lets the firm change regime. We assess the value of a firm with an Aoki organization, endowed with a shut-down option, i.e. with a flexible but costly organization. When the firm shuts down, workers quit, simply getting a bonus which represents the entire sunk cost of shutting down. Workers enter the bargain over the share of organizational rent but do not control the shut down option, making the periods at the firm stochastic. The amount of profit workers receive depends upon their bargaining power and price dynamics. The bargaining power of workers represents their attitude to risk of open conflict when there is an opportunity of gaining from a shift in the opponent's position.

Workers' participation and the shut down option take us away from the original neoclassical model of the present value and also from the original Aoki model. We see whether Aoki's weighting rule is liable to undergo substantial changes, when the manager has the possibility of shutting down and when workers adopt the policy of sharing not only profits but also losses.

As in Aoki (1980) we assume that there are no layoffs and that workers are homogeneous in terms of skill.

The paper is organized as follows. In the next section we present the basic model and notation. In section 3 we turn to the efficient bargaining set using an option valuation approach. In section 4 and 5 we explore some comparative statics and the general features of bargaining equilibrium. Finally section 6 provides conclusions.

2. The model

We consider a firm endowed with a certain capital stock. The life of the physical assets is infinite from the current period (period 0) to infinity. In each period the firm produces one unit of output. Marginal and average costs are constant and equal to c. The number of workers employed is, for simplicity, normalized to 1.

2.1. The market price

The firm faces uncertain revenue, equal to the market price since the firm produces only one unit of output. The price is driven by a geometric Brownian - motion or random walk in continuous time,

$$dp_t = \alpha p_t dt + \sigma p_t dz_t \tag{1}$$

$$\text{with} \quad p_0 = p$$

where dz_t is the increment of a standard Wiener process, uncorrelated over time and satisfying the conditions that $E(dz_t) = 0$ and $E(dz_t^2) = dt$.[1]
This means that the firm will never gain by waiting for information, but may gain by waiting for a more favourable realization. The price assumption is a first departure from the original Aoki (1980) model where the firm is assumed to face no market uncertainty but just an internal organizational uncertainty. In Aoki the firm sets the price and the amount of investment to devote to growth, within the bargaining, simultaneously with the workers' profit sharing scheme.

The firm we model is quite flexible since we assume that it has, in the spirit of Dixit (1989), an option to shut down in the future and later to reopen according to market conditions. A hurdle to flexibility is the existence of a sunk cost which might prevent the firm from shutting down and/or reopening just after one deviation of price. To do this the firm has to pay a total statutory allowance for firing workers equal to K, whenever it decides to stop production. The investment the firm needs, to start production, is equal to I and will be completely lost when it stops. Then, if I goes to infinity the reentry option becomes worthless and no more investment is allowed after the shut down has occurred. We also assume that K and I are constant and nonstochastic.

For a firm that already exists, the operating profits, termed organizational rent of the firm, at time t are

$$\pi(p_t) = p_t - c \tag{2}$$

when the firm is working and zero if the firm decides not to produce.
Since there is a sunk exit cost, the firm stays in the market even when there are operating losses. Since the firm believes that the market price has the same probability of going up and down, it stays in the market even if $p_t < c$, because, by doing so, it might be able to cover the sunk costs rather than exiting soon. It will become optimal to exit only if the price falls below a trigger level p_L which

1 By standard Brownian motion theory, the variable $\log p_t$ is normally distributed with mean equal to $\log p_0 + \left(\alpha + \frac{1}{2}\sigma^2\right)t$ and variance $\sigma^2 t$. Using the properties of lognormal distributions we get $E(p_t \mid p_0) = p_0 e^{\alpha t}$.
The term α represents the positive or negative trend of the market price.

has to be endogenously identified. This is the main crux of the paper. The same intuition applies if a (re)entering policy is allowed: the market price must rise somewhat above c to induce (re)entering. We call this second trigger level p_H, and it must be the case that $p_L < c < p_H$.

2.2. Profit sharing

We assume that profits are distributed partly to shareholders and partly to employees. Payments to employees are composed of two parts: a wage component $\overline{w}$, constant over time, which is equal to the competitive wage set in the external labour market; a premium earning Δw_t which represents the employees' share of the economic rents accruing to the firm. Therefore:

$$w_t = \overline{w} + \Delta w_t. \tag{3}$$

Following the literature on dually controlled (shareholders-employees) firms, let θ be the shareholders' share of profits. The premium per employee is then given by:

$$\Delta w(p_t;\theta, m) = (1-\theta)\max[-m, \pi(p_t)] \tag{4}$$

$$0 \leq \theta \leq 1 \quad \text{and} \quad 0 \leq m \leq c - p_L$$

where p_L is the critical low value of revenue that triggers the shutting down of the firm, when the option of doing so is taken into account.

Equation (4) is also crucial to interpret the corporate policy for employees. The parameter m represents the workers' willingness to share firm's losses. When $m = 0$ workers refuse to share the losses of the firm and accept only positive average premiums. On the other hand, if $m = c - p_L$ the average premium becomes $\Delta w(p_t) = (1-\theta)(p_t - c)$, which means that workers and shareholders completely share the firm's total losses before exit. We assume that the parameter m is not subject to bargaining.

2.3. The workers' objective

We, further, assume that hired employees, at time 0, are interested in the amount of lifetime earnings they can get by taking part in the firm's production. Under the simplifying assumptions a) that employees are remunerated equally, b) that the market wage is constant c) that the employees' relative share $1-\theta$ remains unchanged over periods, d) that workers are fired only when the firm exercises the shut down option, the levels of lifetime well-being of an individual employee up to the shut down may be ordered according to the expected discounted sum of the premium earnings at the firm. That is:

$$LE(p;\theta,m) = E_0\left\{\int_0^T \Delta w(p_t;\theta,m)e^{-\beta t}dt + Ke^{-\beta T} \mid p_0 = p\right\} \geq 0, \quad \text{for} \quad p \in [p_L,\infty), \tag{5}$$

where E_0 denotes the expectation at time zero, T represents the unknown future time when the firm is shut down, i.e. the stopping time $T(p_L) = \inf\{t \geq 0 \mid p_t < p_L\}$, K is the transfer to the employees if the firm is shut down, and $\beta(> \alpha)$ is the workers' constant discount rate[2]. If $I < \infty$ the firm may decide to reenter in the future and the same workers may or may not be hired again. If rehiring is allowed and warranted we get a series of periods during which the workers are alternatively working or laid off. Each period is expressed by a premium earning function as (5) and the lifetime well-being of the employees is given by the sum of (5) over these periods. If $I = \infty$ the firm never reenters and the shutting down leads to the termination of the contract. Then (5) is the well-being at the firm. In both cases, however, the analysis does not change qualitatively, therefore, for sake of simplicity, we shall confine to the latter. Finally, $LE(p;\theta, m)$ must be non-negative to induce participation[3].

2.4. The shareholders' objective

Now we consider the shareholders, assumed to be homogeneous in all respects as workers. An opportunity to make a real investment can be viewed as a *call option* the firm possesses because of its access to a particular production technology (Dixit, 1989). The firm has an option to produce a unit of output: paying c (the marginal cost that is certain and can be considered the fixed exercise price) the firm gets p_t which is uncertain (and could be thought of as the market value of the asset). Then a firm can be thought of as a series of options which are exercised every time it produces and not exercised if it does not produce. The value of a firm can then be assessed by the standard techniques of finance. Unfortunately there is a great difference due to the sunk cost the firm has to pay if it stops, i.e. if it doesn't exercise the option to produce. Production can be suspended, as in Dixit (1989), incurring a fixed cost K. Such a cost could be considered the certain cost to pay to exercise another option to get zero profits rather than expected losses. The exercise price of this option is K and the firm avoids uncertain future losses. Flexibility has a cost, which means that reversing the investment decision is going to change the valuation of the firm.

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2 We must assume that the firm, once it has decided the investment level, will not be able to vary the number of workers, i.e. either it produces with that number of workers or it doesn't produce at all. For possibility of layoffs see Aoki (1982).

3 If rehiring is allowed and not warranted the employees might receive K plus an option to reenter the firm when it reopens. Since the option changes the employees' outside opportunity it would imply a modeling of the labour market which is beyond the scope of our analysis.

In addition, if $I < \infty$, the firm may decide to reenter the market, which gives the firm the possibility again to produce and to shut down.

Conditionally upon the workers' corporate policy, the shareholders' decision problem can be described by two state variables, the current revenue p_t and the discrete variable j which is equal to *1* if the firm is active and *0* if the firm is not active. In state $(p_t, 1)$ the firm has to decide whether to continue in production or to exit. If the firm is in state $(p_t, 0)$ it has to decide whether to continue stay out of the market or become active.

Letting $\rho(> \alpha)$ be the cost of capital or the shareholders' discount rate, the shareholders' problem is one of an optimal operation policy, i.e. when to enter and when to exit ($j=0$ or 1) in order to maximize the expected sum of discounted profits. That is:

$$\max_{j=0,1} \quad E_0\left\{\int_0^\infty jR(p_t;\theta,m)e^{-\rho t}dt + \sum_i (K[\Delta j_i]_- - I[\Delta j_i]_+)e^{-\rho i}\right\} \tag{6}$$

where $R(p_t;\theta, m)$ represents the amount of organizational rent accruing to shareholders, given the policy of loss sharing decided by workers. Using J as the indicator function, R can be written as,

$$R(p_t;\theta,m) = \theta\pi(p_t)J_{1\{\pi(p)\geq -m\}} + \pi(p_t)(1 - J_{1\{\pi(p)\geq -m\}})$$

and

$$J_{1\{\pi(p)\geq -m\}} = \begin{cases} 1 & \text{if} \quad \pi(p_t) \geq -m \\ 0 & \text{if} \quad \pi(p_t) < -m \end{cases}$$

Moreover, $[\Delta j_i]_- = -1$ denotes at time $t = i$ the move from the active state ($j = 1$) to the idle state ($j = 0$), while $[\Delta j_i]_+ = 1$ denotes at time $t = i$ the move from the idle state ($j = 0$) to the active state ($j = 1$).

Define $S_1(p;\theta, m)$ the competitive valuation of the firm's share (i.e. the expected net present value) of starting with a price p at the initial time *0* in the active state and following optimal policies. Then, making use of dynamic programming, the problem (6) can be rewritten using the Bellman value function

$$S_1(p;\theta,m) = E_0\left\{\int_0^T R(p_t;\theta,m)e^{-\rho t}dt + (S_0(p_L;\theta,m) - K)e^{-\rho T} \mid p_0 = p\right\}, \quad \text{for} \quad p \in [p_L,\infty) \tag{6'}$$

where $S_0(p_L;\theta,m)$ represents the firm's valuation in the idle state at time $T(p_L) = \inf\{t \geq 0 \mid p_t < p_L\}$ when the firm is shut down. We get a similar result if we start in the idle state and pursue the optimal policy of (re)starting whenever the price p_t rises above the critical high level p_H. The Bellman value function becomes:

$$S_0(p;\theta,m) = E_0\{(S_1(p_H;\theta,m) - I)e^{-\rho T'} \mid p_0 = p\}, \quad \text{for} \quad p \in (0,p_H] \tag{6''}$$

where $S_1(p_H;\theta,m)$ represents the firm's valuation in the active state at time

$T'(p_H) = \inf\{t \geq 0 \mid p_t \geq p_H\}$ when the firm (re)starts production.

2.5. The bargaining

Employees and shareholders have different objective functions, as can be seen by comparing (5) and (6). However they share a common interest in reaching an agreement as to the distribution of the "organizational rent". Such a binding agreement has to be reached before the firm starts producing. The bargaining process is carried out between a representative employee and the manager of the firm. The essential function of the manager is to mediate between the body of shareholders and the employees' representative to find a cooperative game solution. The manager-mediator chooses a distributive share θ at the beginning of the planning period, to maximize the joint interests of shareholders and employees. This is equivalent to finding a Nash bargaining solution (NBS) of the game, as formulated by Harsanyi (1956, 1977) and Aoki (1980).

Both players share the same information about the future evolution of p_t. They have the same attitude towards the risk of internal conflicts in the bargaining, which can be represented by concave Von Neumann Morgenstern utility functions: $u(LE)$ for the representative employee, defined on the domain of possible total expected premium earnings and $v(S_1)$ for the manager, defined on the domain of firm's share value. The utility $v(S_1)$ is supposed to represent the manager's view as to shareholders' interest.

In the bargaining both players are averse to the risk of open conflict. If cooperation fails, the bargaining has no solution and the players get the worst result, i.e. the utility levels $\hat{u} \geq 0$ and $\hat{v} \geq 0$, which are given and represent the threat points of the bargaining set. $\hat{u}$ represents the workers' wage utility they can get in alternative jobs available in the labour market. $\hat{v}$ is the utility shareholders can get elsewhere. Even though shareholders can diversify their portfolio we assume that not only shareholders but also workers have acquired some specific knowledge which would be wasted if a binding agreement was not reached.

As in Aoki (1980, 1984) we assume that the utilities coming from the cooperative bargaining exceed the reservation values $\hat{u}$ and $\hat{v}$. This implies that both players have appropriate threat potentials so that the utility of cooperation exceeds the utility of open conflict[4].

The organizational equilibrium can be characterized as the result of the maximization of the joint objective function:

$$\nabla = \log[u(LE) - \hat{u}] + \log[v(S_1) - \hat{v}], \tag{7}$$

subject to the relevant constraints, (2), (4), (5), (6).

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4 We assume that the threat points are constant even though outside opportunities may be subject to fluctuations.

3. The efficient bargaining set: an option valuation approach

We now have to specify the efficient bargaining set of the two players (shareholders and employees), i.e. we find the set of efficient combinations of the firm's share value and total earnings for the incumbent employees. Then the bargaining will let them choose, within the efficient set, the mutually preferred point.

3.1 The shareholders

We begin with the shareholder's problem, that is depicted in (6). This is a stochastic dynamic programming problem that can be solved by using the option pricing analogy.

The opportunity to invest can be considered as an asset that is held for a series of small intervals of time *dt*. We first consider this opportunity over the range of prices p where it is optimal for the operating firm to continue its state ($p \geq p_L$). The investment return is given by dividends plus capital gains. Dividends accruing to shareholders are the residue of the "organizational rent" over the payments to employees. The capital gain (or loss) will be given by the change of the value of $S_1(p;\theta,m)$ over time as price changes, i.e. $E(dS_1(p;\theta,m)/dt)$.

In equilibrium[5], to avoid arbitrage, the sum of the dividends plus capital gain has to be equal to the normal return, or the market cost of capital: $\rho S_1(p;\theta,m)$.

Investors holding shares of the firm at time t are interested in maximization of the firm's value $S_1(p;\theta,m)$.[6]

Since p_t is a stochastic variable we apply Ito's Lemma to the value of the firm $S_1(p;\theta,m)$ and obtain:

$$dS_1 = S'_1 dp + \frac{1}{2} S''_1 \sigma^2 p^2 dt, \tag{8}$$

and, therefore, using (1), the expected capital gain is

$$E(dS_1) = \left[S'_1 \alpha p + \frac{1}{2} S''_1 \sigma^2 p^2 \right] dt. \tag{9}$$

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5 See Pindyck (1991) and Dixit (1992) for a survey on contingent claims methods applied to real investment opportunities.

6 To be precise we must assume that shares are treated on the stock exchange only after an agreement concerning employees' earnings is reached, but before current production takes place. But there is a bargaining process between the management (on behalf of shareholders) and the employee representative, which presupposes the existence of shareholders. To solve this conundrum Aoki (1980, 1984) calls those who hold shares of the firm before going to the stock exchange ex-ante shareholders, and those who become shareholders after all corporate decisions are taken, ex post shareholders.

The asset equilibrium condition yields the following differential equation

$$\frac{1}{2}\sigma^2 p^2 S''_1 + \alpha p S'_1 - \rho S_1 = -R(p;\theta,m) \quad \text{for} \quad p \in [p_L,\infty) \tag{10}$$

The asset return, when the firm is idle, $S_0(p;\theta,m)$, can be calculated similarly.
The only difference is that there are no operating profits. That is

$$\frac{1}{2}\sigma^2 p^2 S''_0 + \alpha p S'_0 - \rho S_0 = 0 \quad \text{for} \quad p \in (0,p_H] \tag{11}$$

These equations, (10) and (11), must be solved subject to the following boundary conditions:

$$S_0(0) = 0; \tag{12a}$$

$$S_1(p_L;\theta,m) = S_0(p_L;\theta,m) - K; \tag{12b}$$

$$S_1(p_H;\theta,m) - I = S_0(p_H;\theta,m); \tag{12c}$$

$$S'_1(p_L;\theta,m) = S'_0(p_L;\theta,m); \tag{12d}$$

$$S'_1(p_H;\theta,m) = S'_0(p_H;\theta,m); \tag{12e}$$

where K and I represent the sunk costs respectively of shutting down the firm and of (re)starting. Condition (12a) states that, in the idle state, if p is zero it will remain zero and the firm has no value since the price may only change in the distant future when heavy discount makes it irrelevant; in other words *0* is an *absorbing state* for the process $\{p_t\}$.
The *value-matching conditions* (12b) and (12c) state that at the trigger price there is indifference. At p_L the value of the active firm equals the value of the idle firm minus the sunk cost of exit K. At p_H the active firm has the same value of the idle firm plus reentry cost. The *smooth-pasting conditions* (12d) and (12e) say that the firm's value is a continuous and smooth function of p at the trigger points p_L and p_H. This means that at the trigger price entering and exit has the same marginal effect on the value of the firm.

Equations (10) and (11) are linear differential equations in S_1 and S_0. The general solution will be composed by two parts, a complementary function and a particular solution. Since in both states the homogeneous part is the same, we can find the complementary function simultaneously. We try a functional form of the type p^γ and check by substitution if it works. We can show that

$$S_1(p;\theta,m) = A(p)^{\gamma_2} + V(p;\theta,m) \qquad \text{for} \quad p \in [p_L,\infty]; \tag{13a}$$

$$S_0(p;\theta,m) = B(p)^{\gamma_1} \qquad \text{for} \quad p \in (0,p_H). \tag{13b}$$

where γ_1 and γ_2 are respectively the positive and the negative roots of the quadratic equation

$$\frac{1}{2}\sigma^2\gamma^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\gamma - \rho = 0, \tag{14}$$

and $V(p;\theta,m)$ stands for a particular solution of (10). A convenient particular solution is given by the expected discounted flow of the firm's operating profits ignoring the absorption at p_L. That is

$$V(p;\theta,m) = E_0\left\{\int_0^\infty R(p_t;\theta,m)e^{-\rho t}dt\right\}$$

$$= E_0\left\{\int_0^\infty \theta(p_t - c)J_{1\{\}}e^{-\rho t}dt\right\} + E_0\left\{\int_0^\infty (p_t - c)(1 - J_{1\{\}})e^{-\rho t}dt\right\} \quad (15)$$

If $g(p_t;p)$ represents the probability density function of p_t conditional on p, then (15) becomes

$$V(p;\theta,m) = \theta\int_0^\infty\left[\int_{c-m}^\infty (p_t - c)g(p_t;p)dp_t\right]e^{-\rho t}dt + \int_0^\infty\left[\int_0^{c-m}(p_t - c)g(p_t;p)dp_t\right]e^{-\rho t}dt$$

$$= \int_0^\infty\left[\int_0^\infty (p_t - c)g(p_t;p)dp_t\right]e^{-\rho t}dt - (1-\theta)\int_0^\infty\left[\int_{c-m}^\infty (p_t - c)g(p_t;p)dp_t\right]e^{-\rho t}dt$$

$$= \left(\frac{p}{\rho - \alpha} - \frac{c}{\rho}\right) - (1-\theta)\int_0^\infty H(p;c-m,t)e^{-\rho t}dt. \quad (15')$$

Equation (15') says that, as p becomes very large and the firm is active, the probability that, over any finite time span, it will fall below cost and production will cease, becomes very small. The first part of equation (15') represents the discounted value of profits when the firm is active for ever and there is no profit sharing. This value is decreased by the "cost of employees' corporate policy" which accrues to the workers in terms of future operating profits.
Since the expected value of p rises at the trend rate α, the value of the firm $V(p;\theta,m)$ is just the expected present value that can be obtained by keeping it in operation forever, starting from an initial price p.

Using the option pricing analogy we can see that

$$H(p;c-m,t) = h(p;c-m,t) - mN(d_2),$$

where, $h(p;c-m,t)e^{-\rho t}$ is the value of a European call option with expiration date t and exercise price c - m written on an asset (share) that pays continuous dividends at rate $\rho - \alpha$ and with a current or spot price of p (see Merton, 1973; McDonald - Siegel, 1985; Ingersoll, 1987). The value of this option is decreased by $m\ N(d_2)$[7]. As m tends to 0 workers refuse to share losses and the entire risk is born by shareholders. In that case the option to produce is less valuable because of the expected losses and the value of the firm has to decrease. As m increases the option value is decreased

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7 H represents a transformation of a call option with two strike prices c and c - m.

further, since workers bear some losses and the production option becomes less valuable. The price of this option is given by:

$$h(p;c-m,t)e^{-\rho t} = pe^{-(\rho-\alpha)t}N(d_1)-(c-m)e^{-\rho t}N(d_2), \tag{16}$$

$$d_1 = \frac{\ln(p/(c-m))+\left(\alpha+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t},$$

and $N(.)$ is the unit normal distribution function. It is worth noting that, when $m = c - p_L$, i.e. the workers wholly share firm losses with shareholders, the option $h(p;c-m,t)e^{-\rho t}$ becomes a perpetual (non-expiring) call option whose price is asymptotically equal to[8]:

$$h(p;p_L,\infty)e^{-\rho t} = pe^{-(\rho-\alpha)t} - p_L e^{-\rho t}, \tag{16'}$$

and, integrating over t, the particular solution (15') reduces to[9]

$$V(p;\theta) = \theta\left(\frac{p}{\rho-\alpha}-\frac{c}{\rho}\right). \tag{16"}$$

Moreover, since the price of a call option is a nonincreasing function of the exercise price[10] c-m ,we get $\frac{\partial V(p;\theta,m)}{\partial m} > 0$ and in particular $V(p;\theta,m) \le V(p;\theta)$.(See Appendix A).

This means that the value of the firm to shareholders is higher when there is complete sharing of losses rather than incomplete sharing ($m > 0$).

The constants A and B, and the trigger points p_L and p_H are determined by using the boundary conditions (12b)-(12e). Since they are nonlinear in p_L and p_H we cannot get any closed solution for (13a) and (13b)[11].

Nevertheless, solutions (13a) and (13b) have some interesting interpretations. Consider the active firm. Regardless of the value of p, if the firm has no choice but the feasible strategy of never shutting down, the present value of future flows of operating profits would be given by $V(p;\theta, m)$. However, if p falls below p_L, the firm considers the possibility of stopping production to avoid

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8 This is nearly equal to the value of the asset if $\alpha = \rho$, (Ingersoll, 1987; p.300-312).

9 The same result is obtained directly noting that, if

$$m = c - p_L \quad \text{then} \quad R(p_t;\theta) = \theta(p_t - c) \quad \text{and} \quad E_0\int_0^\infty \theta(p_t - c)e^{-\rho t}dt = (16")$$

10 See Ingersoll (1987) ch.14 and 17.

11 Numerical simulations should be undertaken to evaluate the properties of the solution. See, for a similar procedure, Dixit (1989).

losses. The value of this option to stop production is given by $A(p)^{\gamma_2}$. Since the option strategy should do better, the constant A must be positive.

Similarly, since the idle firm is not producing, the "organizational" rent is zero and $S_0(p;\theta,m)$ represents the value of the option to become active in the future when the price rises above p_H.

3.2. The workers

As for shareholders, when employees work in the firm they set up a sort of system of workers shares. The employees are interested in higher operation profits because the dividends from their "firm membership" are higher the more profitable the firm is. In other words, workers get their extra wage in the form of new shares, coming from the dividends which are not distributed to shareholders and which are granted to workers for their participation in the firm's activity. Either the incumbent workers have a right to be hired again by the firm when it reopens or the contract terminates with the shut down: in both cases their lifetime well-being may be ordered according to $LE(p;\theta,m)$.

The employees do not have to pay for the shares and, on the other hand, they have no voting rights in the corporate board. When the firm is shut down, workers get the bonus K.

Again, if we assume that the uncertainty over p is spanned by existing assets, we can value the employee shares owned by a single worker by using contingent claim methods. Suppose that X is an asset which pays instantaneous dividends $\Delta w(p_t;\theta,m)$ and the lump sum K the first time p falls below the trigger value p_L. In a risk neutral world, if the investors have access to an asset with an expected rate of return $\beta(>\alpha)$, to avoid arbitrage the value of X must be the present value of expected dividends discounted at rate β as expressed in (5).

To find the workers' efficient set, let us define $LE(p;\theta, m)$ as the value of the shares owned by an employee. The asset of "being employed at the firm" is willingly held by the workers if the average premium of the employee (operating earnings) $\Delta w(p;\theta,m)$ plus the expected capital gains $E(dLE(p_t;\theta,m)/dt)$ is equal to the workers' normal return $\beta LE(p;\theta,m)$.

Applying Ito's Lemma to $LE(p;\theta, m)$ the asset equilibrium condition becomes the differential equation

$$\frac{1}{2}\sigma^2 p^2 LE'' + \alpha p LE' - \beta LE = -\Delta w(p;\theta,m) \quad \text{for} \quad p \in [p_L,\infty). \tag{17}$$

This equation must be solved subject to the following boundary conditions

$$LE(0) = 0; \tag{18a}$$

$$LE(p_L;\theta,m) = K; \tag{18b}$$

Again the first condition follows from the fact that zero is an absorbing state for the process $\{p_t\}$. The value-matching condition (18b) says that, if $p = p_L$ employee's total extra-earning is equal to the lump sum transfer K.

The general solution of (17) may be expressed as the sum of a complementary function and a particular solution. Moreover, since the homogeneous part is the same as in (10), we can find the complementary function applying the functional form p^{λ} and checking by substitution if it works. That is,

$$LE(p;\theta,m) = C(p)^{\lambda_2} + F(p;\theta,m), \quad \text{for} \quad p \in [p_L,\infty) \tag{19}$$

where λ_2 is the negative root of the quadratic equation

$$\frac{1}{2}\sigma^2\lambda^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\lambda - \beta = 0, \tag{20}$$

and $F(p;\theta, m)$ stands for a particular solution of (17). The constant C accounts for the difference between the lost earnings the workers face if the firm suspends its production and the per-capita transfer. When it is positive workers agree on the shut down. It can be found using the boundary condition (18b)

$$C = [K - F(p_L;\theta,m)]\,(p_L)^{-\lambda_2} \tag{21}$$

As a convenient particular solution of (17) we consider the expected discounted flow of the employee's average premium ignoring the absorption at p_L. That is

$$F(p;\theta,m) = E_0\left\{\int_0^{\infty}(1-\theta)\max[-m,\pi(p_t)]e^{-\beta t}dt\right\} \tag{22}$$

$$= (1-\theta)\left(\int_0^{\infty} E_0\max[0,p_t-(c-m)]e^{-\beta t}dt - \frac{m}{\beta}\right)$$

Using the density function $g(p_t; p)$

$$= (1-\theta)\left[\int_0^{\infty}\left(\int_{c-m}^{\infty}(p_t - c + m)g(p_t;p)dp_t\right)e^{-\beta t}dt - \frac{m}{\beta}\right]$$

$$= 1-\theta\left(\int_0^{\infty} h(p;c-m,t)e^{-\beta t}dt - \frac{m}{\beta}\right)$$

Again $h(p;c-m,t)e^{-\beta t}$ is the value of a European call option with expiration date t and exercise price c-m written on an asset which pays continuous dividends discounted at rate $\beta - \alpha$ and with a current price p. The same way we used for the shareholders, it can be noted that, when $m = c - p_L$ it follows that $\max[-m, \pi(p_t)] = p_t - c$ and integrating (16') over t (using β instead of ρ), yields

$$F(p;\theta) = (1-\theta)\left(\frac{p}{\beta-\alpha} - \frac{c}{\beta}\right).$$

Applying the comparative statics properties of a call option we get $\partial F(p;\theta,m)/\partial m < 0$ and in particular $F(p;\theta,m) \geq F(p;\theta)$ (see appendix A).
As with (15"), when p becomes very large, the option of shutting down the firm becomes nearly worthless and the value of the employee's lifetime earning is just the expected present value that can be obtained by receiving the average premium for ever.

However, if p falls below p_L the firm stops producing and the worker is laid off. The loss attributable to the firm's option to stop producing is given by the term $C(p)^{\lambda_2}$.

4. Some comparative statics

Before we analyse the equilibrium distribution, i.e. the profit sharing parameter θ, we wish to assess the comparative statics properties of $S_1(p;\theta, m)$ and $LE(p;\theta, m)$. To avoid complications we assume that both actors have the same opportunities in the stock market, i.e. shareholders and employees have the same discount rate: $\rho = \beta$. This also implies that $\gamma_2 = \lambda_2$.

Moreover, to obtain a closed form solution for $S_1(p;\theta,m)$, we assume that the firm cannot restart production once it has shut down. As it was stated, this is just a marginal limit of the analysis that does not greatly modify the conclusions as far as θ is concerned. This can be done assuming $I = \infty$, that is, the entry option becomes worthless and the constant B goes to zero. The shut down moment becomes also the termination of the employees' contract with the firm.

Then it is possible to solve (12b) and (12d) for $A(\theta,m)$ and $p_L(\theta,m)$, that is:

$$S_1(p_L;\theta,m) = A(p_L)^{\gamma_2} + V(p_L;\theta,m) = -K \tag{24}$$

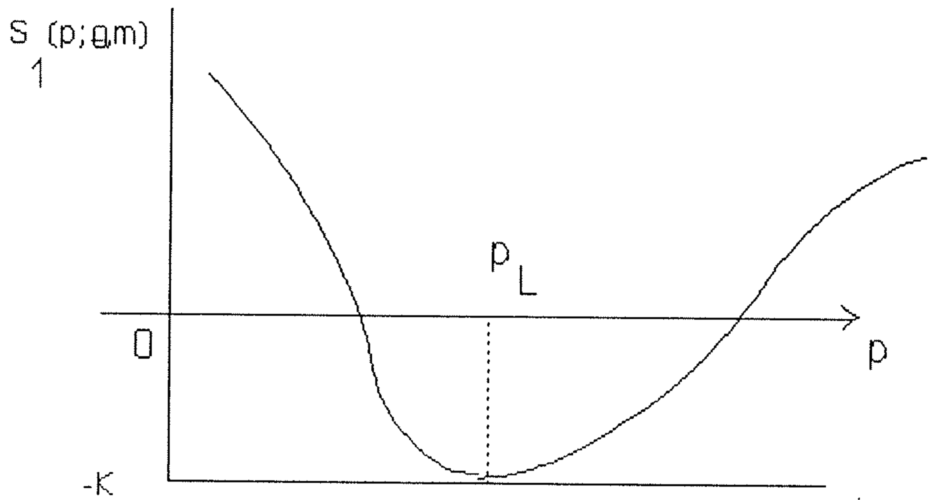
$$S'_1(p_L;\theta,m) = A\gamma_2(p_L)^{\gamma_2-1} + V'(p_L;\theta,m) = 0 \tag{25}$$

Since we are looking for a positive lower price level p_L, the SOC for a minimum is:

$$S''_1(p_L;m) = A\gamma_2(\gamma_2-1)(p_L)^{\gamma_2-2} + V''(p_L;\theta,m) > 0 \tag{26}$$

The first term on the r.h.s. of (26) is always negative while the sign of the second term is ambigous (see Appendix A). Therafter we assume that (26) always holds.
A simple diagram adapted from Dixit (1989) provides a clear illustration of the solution. As consequences of the convexity / concavity of $S_1(p;\theta, m)$, the general appearance is as shown in fig. 1.

Figure 1



Then, to solve the problem, we must adjust the constant A until S_1 becomes tangent to the horizontal line at $-K$ and the respective point of tangency defines p_L. Thereafter we assume that (26) always holds.

To get a better idea of the range of admissible p_L, let us evaluate (10) at p_L and take account of (24), (25) and (26). We then have:

$$-R(p_L;\theta,m) = \frac{1}{2}\sigma^2 p^2 S''_1(p_L;\theta,m) + \rho K > \rho K$$

or, by the definition of m and R

$$p_L < c - \rho K. \tag{27}$$

As shown in Dixit (1989) the r.h.s. of (27) differs from the variable cost c because the firm faces a lump sum exit cost. If $\rho K/c > 1$ production is never abandoned and in equation (13a) A goes to zero as the option of exiting becomes worthless.

Hysteresis is explained by (27) and by the above expression in our case. As it can be seen the firm has an exit policy which departs from the one recommended by the present value criterion. The firm is going to stay in the market longer and exit at a lower price.[12]

Substituting (24) into (25) and substituting the expression for $V(p_L;\theta,m)$ and $V'(p_L;\theta,m)$

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12 In general our criterion differs from the simple option criterion of Dixit (1989); however, for $\theta = 1$, that is analogous to the expression for p_L obtained by Dixit (1989) equation (24), p.630.

p_L is determined implicitly by the expression:

$$p_L - (1-\theta)(\rho-\alpha)\frac{\gamma_2}{\gamma_2-1}\left[\int_0^{\infty}\left[H(p_L;c-m,t)-\frac{p_L}{\gamma_2}\frac{\partial H}{\partial p_L}(p_L;c-m,t)\right]e^{-\rho t}dt\right] =$$

$$= \frac{\rho-\alpha}{\rho}\frac{\gamma_2}{\gamma_2-1}(c-\rho K). \qquad (28)$$

where

$$\frac{\rho-\alpha}{\rho}\frac{\gamma_2}{\gamma_2-1} < 1.$$

From (28) it is immediate to verify that, as $\theta = 1$ the optimal level of p_L is given by

$$\frac{\rho-\alpha}{\rho}\frac{\gamma_2}{\gamma_2-1}(c-\rho K) > 0,$$

that satisfies inequality (27).

On the contrary if θ tends to zero it may be the case that p_L becomes negative. However, since p_L must be positive to induce the firm to keep the shut down option alive there may exist a *reservation distributive parameter*

$\hat{\theta} = \inf\{\theta \geq 0 : p_L > 0\}$.

To have a better idea of the relationship between p_L and θ we focus the attention on the two special cases $m = c - p_L$ and $m = 0$.

a) m = c - p_L

If the workers wholly share firm losses, equation (28) reduces to:

$$p_L = \frac{\rho-\alpha}{\rho}\frac{\gamma_2}{\gamma_2-1}\left(c - \frac{\rho}{\theta}K\right) \qquad (28')$$

which is positive only if

$$\hat{\theta} \equiv \frac{\rho K}{c} < \theta < 1.$$

Moreover $dp_L/d\theta$ is always positive.

The sub-case of $K = 0$ yields $p_L < c$ and $dp_L/d\theta = 0$.

b) m = 0

If the workers refuse completely to share the firm's losses equation (28) becomes

$$p_L - (1-\theta)(\rho-\alpha)\frac{\gamma_2}{\gamma_2 - 1}\int_0^{\infty}\left[h(p_L;c,t) - \frac{p_L}{\gamma_2}\frac{\partial h}{\partial p_L}(p_L;c,t)\right]e^{-\rho t}dt =$$

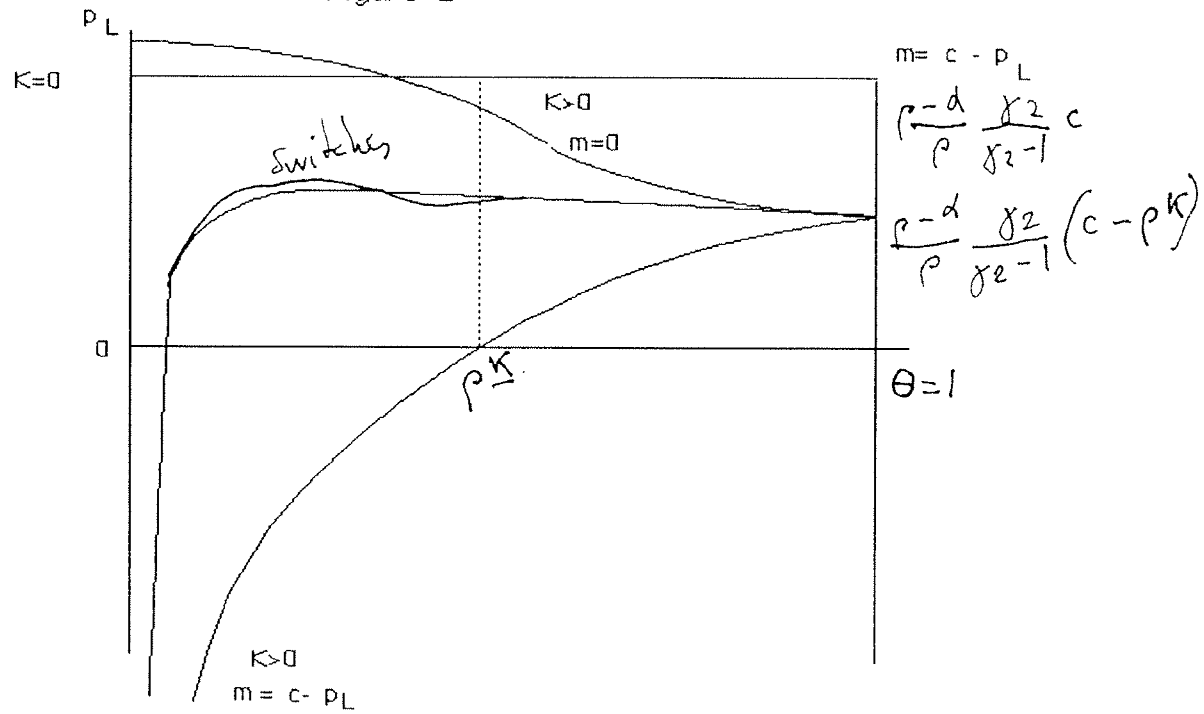
$$= \frac{\rho-\alpha}{\rho}\frac{\gamma_2}{\gamma_2 - 1}(c - \rho K). \qquad (28")$$

When $\theta = 1$, again

$$p_L = \frac{\rho-\alpha}{\rho}\frac{\gamma_2}{\gamma_2 - 1}(c - \rho K).$$

On the contrary, when $\theta < 1$, since the integral in (28") is always positive, in order for (28") to be satisfied it is necessary that p_L increases. By continuity the same applies for all $0 < \theta < 1$, which yields $dp_L/d\theta < 0$,
(see Appendix A).
Summing up these results in figure 2, we conclude that the intermediate case of $0 < m < c - p_L$ lies between the (28') and the (28") even though there are switches in the sign of $dp_L/d\theta$ in the range $\theta \in [0, 1]$. In other words the extent of labour partecipation to losses leads to opposite effects of the share parameter on the trigger price.

Figure 2



For any value of the profit sharing parameter we get that the hysteresis effect is going to become more important when workers share part of the losses. Because of the switches in $dp_L/d\theta$ the opposite is not always true.

Let us now turn to the comparative statics of $S_1(p;\theta, m)$. Differentiating (13a) with respect to θ for given m, and taking account of (24) and (25) we obtain:

$$\frac{dS_1(p;\theta,m)}{d\theta}=\frac{dA(\theta,m)}{d\theta}p^{\gamma_2}+V_\theta(p;\theta,m) \tag{29}$$

$$=V_\theta(p;\theta,m)-V_\theta(p_L;\theta,m)\left(\frac{p}{p_L}\right)^{\gamma_2}=$$

$$=E_0\int_0^T H(p;c-m,t)e^{-\rho t}dt>0$$

where the expected value is taken only with respect to the stopping time T (see Appendix A).

Going through the same steps for $LE(p;\theta, m)$ we obtain:

$$\frac{dLE(p;\theta,m)}{d\theta}=\frac{dC(\theta,m)}{d\theta}p^{\gamma_2}+F_\theta(p;\theta,m) \tag{30}$$

$$=F_\theta(p;\theta,m)-F_\theta(p_L;\theta,m)\left(\frac{p}{p_L}\right)^{\gamma_2}-[\gamma_2 p_L^{-1}(K-F(p_L;\theta,m))+F'(p_L;\theta,m)]\frac{dp_L}{d\theta}\left(\frac{p}{p_L}\right)^{\gamma_2}=$$

$$=-\left(E_0\int_0^T h(p;c-m,t)e^{-\beta t}-\frac{m}{\beta}\right)-[\gamma_2 p_L^{-1}(K-F(p_L;m))+F'(p_L;m)]\frac{dp_L}{d\theta}\left(\frac{p}{p_L}\right)^{\gamma_2}$$

whose sign depends also upon $\frac{dp_L}{d\theta}$.

Although the first term on the right hand side of (30) is negative, the sign of the second term and then of $dLE/d\theta$ remains indeterminate.

However, strong worker participation in the firm losses and a low value of the bonus K is accompanied by a negative effect of an increase in the distributional parameter θ on the workers' expected earnings.

We conclude this section by analysing the comparative statics of $S_1(p;\theta, m)$ and $LE(p;\theta, m)$ with respect to m. Totally differentiating (24) and (25) with respect to m, for given θ, we get:

$$\frac{dp_L}{dm}=\frac{\gamma_2 p_L^{-1}V_m(p_L;\theta,m)-V'_m(p_L;\theta,m)}{S''_1(p_L;\theta,m)}<0 \tag{31}$$

By the fact that $V_m(p_L;\theta,m) > 0$ and $V'_m(p_L;\theta,m) < 0$ we can conclude that $dp_L/dm < 0$. (See Appendix A).

If there is a greater worker participation in the firm's losses (through an increase in m) there should be a decrease in the trigger level p_L below which the firm decides to stop producing and exits the market. In other words an increase in m implies an increase in the expected length of the firm's operative life and therefore an increase in the expected discounted flow of operating profits which balance out the negative effect on the value associated with the option of shutting down (See Appendix A).

$$\frac{dS_1(p;\theta,m)}{dm} = \frac{dA(\theta,m)}{dm}(p)^{\gamma_2} + V_m(p;\theta,m) =$$

$$= V_m(p;\theta,m) - V_m(p_L;\theta,m)\left(\frac{p}{p_L}\right)^{\gamma_2} =$$

$$= -(1-\theta)E_0\int_0^T \frac{\partial H}{\partial m}(p;c-m,t)e^{-\rho t}dt > 0 \tag{32}$$

The expected value is taken with respect to the stopping time.

The same cannot be said in general regarding the worker's expected sum of lifetime premium earning $LE(p;\theta, m)$. Going through the same steps as in (32) we get:

$$\frac{dLE(p;\theta,m)}{dm} = \frac{dC(\theta,m)}{dm}(p)^{\gamma_2} + F_m(p;\theta,m)$$

$$= F_m(p;\theta,m) - F_m(p_L;\theta,m)\left(\frac{p}{p_L}\right)^{\gamma_2} - [\gamma_2 p_L^{-1}(K - F(p_L;\theta,m)) + F'(p_L;\theta,m)]\frac{dp_L}{dm}\left(\frac{p}{p_L}\right)^{\gamma_2} =$$

$$= (1-\theta)\left(E_0\int_0^T \frac{\partial h}{\partial m}(p;c-m,t)e^{-\beta t}dt - \frac{1}{\beta}\right) - [\gamma_2 p_L^{-1}(K - F(p_L;\theta,m)) + F'(p_L;\theta,m)]\frac{dp_L}{dm}\left(\frac{p}{p_L}\right)^{\gamma_2} \tag{33}$$

The first term on the r.h.s. is negative while the second term is ambiguous. However, a high value of K makes loss sharing less attractive for workers. As m increases the life expectancy of the firm becomes longer reducing the possibility of getting K.

5. Equilibrium distribution

This section is devoted to deriving the NBS between a representative employee and the manager-mediator at time zero. Throughout this section we maintain the assumptions introduced in section 4 $(\rho = \beta; \gamma_2 = \lambda_2)$.

Substituting (13a) and (19) into (7) and differentiating with respect to θ, we get the following optimality condition (see appendix B).

$$L_\theta = -\frac{B_u}{1-\theta} LE(p;\theta,m) + \frac{B_v}{\theta} S_1(p;\theta,m) -$$

$$-\frac{B_V}{\theta}\left\{[V(p;\theta,m) - \theta V_\theta(p;\theta,m)] - [V(p_L;\theta,m) - \theta V_\theta(p_L;\theta,m)]\left(\frac{p}{p_L}\right)^{\gamma_2} - K\left(\frac{p}{p_L}\right)^{\gamma_2}\right\} +$$

$$+\frac{B_u}{1-\theta}\left\{\frac{(1-\theta)\gamma_2}{p_L} F(p_L;\theta,m) + K\left(1 - \frac{(1-\theta)\gamma_2}{p_L}\right) - (1-\theta)\frac{dp_L}{d\theta} F'(p_L;\theta,m)\right\}\left(\frac{p}{p_L}\right)^{\gamma_2} = 0 \qquad (34)$$

for $\theta \in (\hat{\theta}, 1)$ and where, $B_u = \frac{u'}{u - \hat{u}}$ and $B_v = \frac{v'}{v - \hat{v}}$ represent the reciprocal of the marginal risk premium needed to compensate the employees and the shareholders, respectively, for risking an infinitesimally small probability of open conflict. The larger the value of B_u (B_v) the bolder the employees (shareholders) will be in demanding higher earnings (dividends).

Solving (34) for θ^* we can obtain the equilibrium solution of the bargaining. We focus only on distributive decisions; therefore the share parameter defining the division of profits between dividends and wage premia is subject to bilateral agreement. Other firm's corporate decisions are prerogative of the management.[13] The decision about m is a prerogative of the worker's body.

The level of θ^* would in general reflect the relative bargaining power between the shareholders and the employees. Obviously, if the option to shut down the firm is worthless the bargaining power is represented by Aoki's weighting rule. That is, the weighted average lifetime earnings LE and share value S_1, each multiplied by the bargaining power of their respective representatives (B_u and B_v), cannot be increased by any marginal change in θ (see appendix B). This is the case when the sharing parameter θ lies within the interval $(0, \hat{\theta}]$

In order to highlight, in an uncertain environment, the importance of the cost of reversing the investment decision, we further assume that the two parties' utility functions exhibit constant relative risk aversion. In particular let $u(LE) = LE^{1-R}$ be the employee's utility function where R is the degree of relative risk aversion and $v(S_1) = S_1^{\,q}$ the manager's utility function. Moreover, for

13 In the models proposed by Aoki (1980, 1984) the bargaining is more comprehensive and also corporate decisions as to growth and price policy enter the bargaining. Focusing only on the distributive share makes the model somehow a compromise between Aoki (1980) and Weitzman (1984).

simplicity let us also assume that the failure of cooperation between the two parties results in $\hat{u} = \hat{v} = 0$.[14]

The results are summarized in the following proposition:

Proposition 1

If $K > 0$ and $0 \le m \le c - p_L$ the optimal relative shares of the shareholders and employees in the "organizational" rent are determined by the following necessary condition:

$$\frac{\theta^*}{1-\theta^*} - \phi_1(\theta^*;K,m) - \phi_2(\theta^*;K,m) = \frac{q}{1-R} \qquad \text{if} \quad \hat{\theta} < \theta^* < 1;$$

$$\frac{\theta^A}{1-\theta^A} = \frac{q}{1-R} \qquad \text{if} \quad 0 < \theta^A \le \hat{\theta}. \quad (pure\,Aoki)$$

where

$$\phi_1(\theta^*;K,m) = -\frac{q}{1-R}\frac{1}{S_1(p;\theta,m)}\left\{[V(p;\theta,m) - \theta V_\theta(p;\theta,m)] - [V(p_L;\theta,m) - \theta V_\theta(p_L;\theta,m)]\left(\frac{p}{p_L}\right)^{\gamma_2} - K\left(\frac{p}{p_L}\right)^{\gamma_2}\right\} > 0$$

$$\phi_2(\theta^*;K,m) = \frac{\theta}{1-\theta}\frac{1}{LE(p;\theta,m)}\left[\frac{(1-\theta)\gamma_2}{p_L}F(p_L;\theta,m) + K\left(1 - \frac{(1-\theta)\gamma_2}{p_L}\right) - (1-\theta)\frac{dp_L}{d\theta}F'(p_L;\theta,m)\right]\left(\frac{p}{p_L}\right)^{\gamma_2}$$

Proof: See Appendix B.

Because of the non-linearity of ϕ_1 and ϕ_2 we have found it hard to reach an overall specification of a SOC for a local maximum. However appendix B presents enough results pointing out how the maximum depends on the various characteristics of the model.

From Proposition 1, if the option to shut down the firm is worthless ($p_L \le 0$; i.e. $\theta^A \in (0,\hat{\theta}]$, the optimal relative shares of the shareholders and employees in the "organizational" rent is given by the simple constant sharing rule $\theta^A = q/(1-R+q)$. That is, the relative share of the employees

14 If we allowed for a reentering option the threat point could not be deemed equal to pure outside opportunities.

in the organizational rent is determined only by the relative bargaining power of the incumbent employees vis-à-vis the shareholders body (Aoki 1980, 1984). However, if the option is worth keeping alive ($p_L > 0$ i.e. $\theta^* \in (\hat{\theta}, 1)$), we have:

Corollary 1

$$\left[\frac{\theta^*}{1-\theta^*}\right] > \left[\frac{\theta^A}{1-\theta^A}\right] = \frac{q}{1-R}$$

Proof: see Appendix B.

The result of Corollary 1 is independent from the workers corporate policy (m) and from the bonus. In fact, by Proposition 1 it is immediate to verify that, even if the exercise price of the option to shut down is zero and workers wholly share the firm's losses, the bargaining power is still not represented by Aoki's weighting rule, as corollary 2 shows:

Corollary 2

If $K = 0$ and $m = c - p_L$ then

$$\frac{\theta^*}{1-\theta^*} - \phi_2(\theta^*) = \frac{q}{(1-R)} \quad \text{for} \quad 0 < \theta^* < 1$$

$$\phi_1 = 0$$

$$\phi_2 = \theta^* \frac{\frac{1}{\rho-\alpha}\left(\frac{p}{p_L}\right)^{\gamma_2}}{\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) - \left(\frac{p_L}{\rho-\alpha} - \frac{c}{\rho}\right)\left(\frac{p}{p_L}\right)^{\gamma_2}}$$

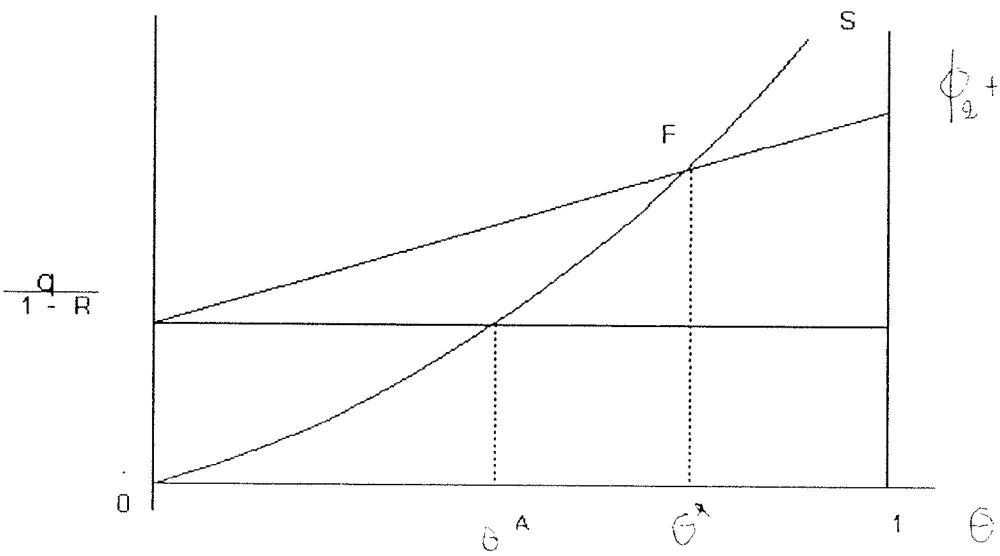
$$p_L = \frac{\rho-\alpha}{\rho}\frac{\gamma_2}{\gamma_2 - 1}c < c$$

$$\frac{dp_L}{d\theta} = 0$$

Proof: see Appendix B

The equilibrium described in Corollary 2 is represented by point F in figure 3, where the line representing $\phi_2 + \frac{q}{1-R}$ crosses OS representing $\frac{\theta}{1-\theta}$.

Figure 3



6.Conclusions

We have tried to model the organization of a firm in which the employees and management decide cooperatively the share of profits to distribute respectively to shareholders and to workers, when the firm has an option of shutting down by paying workers a bonus. The focus has been on the distributive parameter and on the trigger price that prompts the firm to quit the market. The participation parameter indicating the share of profits going to dividends is always higher when the firm is endowed with a shut down option. In other words workers do not get any further premium from accepting the shut down, but the bonus. We have gone a step further towards labour participation considering the workers'willingness to share losses. Some outcomes may appear trivial, like the decrease in the trigger exit price when workers share losses. Others are more intriguing like the effect of the parameter representing the willingness to share losses on the workers' lifetime earning. Since workers keep the firm alive longer, sharing losses can lead to an increase or to a decrease of their lifetime earning, without a definite result.
Even when the shut down option is costless the solution of the bargaining as to the distributive share departs from Aoki's one, represented by relative bargaining power of employees vis à vis the manager. This is no surprise since Aoki's model is not applied to the case of a firm facing irreversible investment in an uncertain environment as in our case. Uncertainty, the possibility of delaying investment and sunk costs are at the basis of hysteresis, which becomes more important when workers'loss sharing increases.

There seem to be many lines of research open to deeper examination of the issue of flexibility and labour partecipation. To get a closed solution we have assumed that there is no reentry opportunity for the firm, while it would quite relevant to know whether a reentry option could change the distributive parameter. Finally the willingness to share losses could be endogenously determined within the bargaining.

Appendix A

In this appendix we analyse the comparative statics properties of $S_1(p;\theta,m)$ and $LE(p;\theta,m)$ with respect to p, θ, and m.
Let us start by totally differentiating (24) and (25) with respect to θ, for given m:

$$\frac{dA(p_L;\theta,m)}{d\theta}=-\frac{V_\theta(p_L;\theta,m)}{p_L^{\gamma_2}}<0 \qquad (A1)$$

$$\frac{dp_L}{d\theta}=\frac{\gamma_2 p_L^{-1}V_\theta(p_L;\theta,m)-V'_\theta(p_L;\theta,m)}{S''_1(p_L;\theta,m)} \qquad (A2)$$

and from (21)

$$\frac{dC}{d\theta} = -\frac{[\gamma_2 p_L^{-1}(K - F(p_L;\theta,m)) + F'(p_L;\theta,m)]\frac{dp_L}{d\theta} + F_\theta(p_L;\theta,m)}{p_L^{\gamma_2}} \qquad (A3)$$

From (15') we obtain, differentiating with respect to θ:

$$V_\theta(p;\theta,m) = \int_0^\infty H(p;c-m,t)e^{-\rho t}dt > 0 \qquad (A4)$$

while differentiating with respect to p yields:

$$V'(p;\theta,m) = \frac{1}{\rho-\alpha} - (1-\theta)\int_0^\infty \frac{\partial H}{\partial p}(p;c-m,t)e^{-\rho t}dt \qquad (A6)$$

where

$$\frac{\partial H}{\partial p} = e^{\alpha t}N(d_1) + pe^{\alpha t}N'(d_1)\frac{dd_1}{dp} - cN'(d_2)\frac{dd_2}{dp} = \qquad (A7)$$

$$= e^{\alpha t}N(d_1) + [pe^{\alpha t}N'(d_1) - cN'(d_2)]\frac{dd_1}{dp} =$$

$$= e^{\alpha t}N(d_1) - e^{\alpha t}\frac{pm}{c-m}N'(d_1)\frac{dd_1}{dm} =$$

$$= e^{\alpha t}\left[N(d_1) - \frac{1}{\sigma\sqrt{t}}\;\frac{m}{c-m}N'(d_1)\right]$$

(A7) is obtained applying

$$\frac{dd_1}{dp} = \frac{dd_2}{dp} = \frac{1}{\sigma\sqrt{t}}p^{-1} > 0 \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{t}.$$

Differentiating again (A6) we get:

$$V''(p;\theta,m) = -(1-\theta)\int_0^\infty \frac{\partial^2 H}{\partial p^2}(p;c-m,t)e^{-\rho t}dt > 0 \qquad (A8)$$

where

$$\frac{\partial^2 H}{\partial p^2} = e^{\alpha t}\left[N'(d_1)\frac{dd_1}{dp} - \frac{1}{\sigma\sqrt{t}}\;\frac{m}{c-m}N''(d_1)\frac{dd_1}{dp}\right] =$$

$$= e^{\alpha t}\frac{p^{-1}}{\sigma\sqrt{t}}\left[1 + \frac{m}{c-m}\frac{1}{\sigma\sqrt{t}}d_1\right]N'(d_1) > 0 \qquad (A9)$$

A by-product of (A4) and (A6) is the cross derivative

$$V'_{\theta}(p;\theta,m) = \int_0^{\infty} \frac{\partial H}{\partial p}(p;c-m,t)e^{-\rho t}dt$$

whose sign depends on $\partial H/\partial p$.

Finally let us calculate the difference:

$$\gamma_2 p^{-1} V_{\theta}(p;\theta,m) - V'_{\theta}(p;\theta,m) =$$

$$= \gamma_2 p^{-1} \int_0^{\infty} \left[e^{-\alpha t} p\left(\frac{\gamma_2 - 1}{\gamma_2}\right) N(d_1) - cN(d_2) + e^{\alpha t} \frac{1}{\sigma\sqrt{t}} \frac{m}{c-m} N'(d_1) \right] e^{-\rho t}dt \qquad (A10)$$

Substituting (A10) evaluated at p_L into (A2) and considering the boundary cases in the range of m, we get:

$$\frac{dp_L}{d\theta} > 0 \quad \text{if} \quad m = c - p_L \qquad (A11a)$$

$$\frac{dp_L}{d\theta} < 0 \quad \text{if} \quad m = 0 \qquad (A11b)$$

By continuity of the above derivative in the range of m there is a switching point of the derivative $dp_L/d\theta$.

Finally, by (A1) and (A4) the total effect of a change in θ on $S_I(p;\theta, m)$ is given by:

$$\frac{dS_1(p;\theta,m)}{d\theta} = \frac{dA(\theta,m)}{d\theta}(p)^{\gamma_2} + V_{\theta}(p;\theta,m) =$$

$$= V_{\theta}(p;\theta,m) - V_{\theta}(p_L;\theta,m)\left(\frac{p}{p_L}\right)^{\gamma_2} = \qquad (A12)$$

$$= E_0 \int_0^{T} H(p;c-m,t)e^{-\rho t}dt > 0$$

where the third equality follows from the strong Markov property of the process $\{ p_t \}$ and the expected value is taken with respect to the stopping time T. (See Harrison (1985) proposition 11, p.47).

Let us now consider $F(p;\theta, m)$. Taking the derivative with respect to θ yields:

$$F_{\theta}(p;\theta,m) = -\int_0^{\infty} h(p;c-m,t)e^{-\beta t} + \frac{m}{\beta} < 0 \qquad (A13)$$

and, by the call option property

$$pe^{\alpha t}N'(d_1)-(c-m)N'(d_2)=0$$

and the fact that the value of an option is an increasing function of the spot price, we get

$$F'(p;\theta,m)=(1-\theta)\int_0^\infty \frac{\partial h}{\partial p}(p;c-m,t)e^{-\beta t}>0 \qquad (A14)$$

where

$$\frac{\partial h}{\partial p}=pe^{\alpha t}N'(d_1)\frac{dd_1}{dp}-(c-m)N'(d_2)\frac{dd_2}{dp}+e^{\alpha t}N(d_1)=$$

$$=e^{\alpha t}N(d_1)>0 \qquad (A15)$$

Again, a byproduct of (A13) and (A14) is :

$$F'_\theta(p;\theta,m)=-\int_0^\infty \frac{\partial h}{\partial p}(p;c-m,t)e^{-\beta t}dt<0 \qquad (A16)$$

Although the signs of (A14) (A15) (A16) are well defined , the sign of (A3), and hence of $dLE/d\theta$ remains indeterminate. However if K is small and m goes to c - p_L the sign becomes definitely negative.

To say something about the effect of m on $S_1(p;\theta, m)$ and $LE(p;\theta, m)$, let us totally differentiate (24) and (25) for a fixed value of θ

$$\frac{dA}{dm}=-\frac{V_m(p_L;\theta,m)}{p_L^{\gamma_2}}<0 \qquad (A17)$$

$$\frac{dp_L}{dm}=\frac{\gamma_2 p_L^{-1}V_m(p_L;\theta,m)-V'_m(p_L;\theta,m)}{S''_1(p_L;\theta,m)}<0 \qquad (A18)$$

From (15') we obtain

$$V_m(p;\theta,m)=-(1-\theta)\int_0^\infty \frac{\partial H(p;c-m,t)}{\partial m}e^{-\rho t}>0 \qquad (A19)$$

where

$$\frac{\partial H}{\partial m} = p e^{\alpha t} N'(d_1) \frac{dd_1}{dm} - c N'(d_2) \frac{dd_2}{dm} = \qquad (A20)$$

$$= [p e^{\alpha t} N'(d_1) - c N'(d_2)] \frac{dd_1}{dm} =$$

$$= -\frac{pm}{c-m} e^{\alpha t} N'(d_1) \frac{dd_1}{dm} =$$

$$= -\frac{pm}{(c-m)^2} \frac{1}{\sigma\sqrt{t}} N'(d_1) < 0$$

(A20) is obtained applying

$$\frac{dd_1}{dm} = \frac{dd_2}{dm} = \frac{1}{\sigma\sqrt{t}}(c-m)^{-1} > 0 \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{t}.$$

A byproduct of (A19) and (A6) is the cross derivative:

$$V'_m(p;\theta,m) = -(1-\theta)\int_0^\infty \frac{\partial^2 H(p;c-m,t)}{\partial p \partial m} e^{-\rho t} dt \qquad (A21)$$

where

$$\frac{\partial^2 H}{\partial p \partial m} = e^{\alpha t}\left[N'(d_1)\frac{dd_1}{dm} - \frac{1}{\sigma\sqrt{t}}\frac{m}{c-m} N''(d_2)\frac{dd_1}{dm} - \frac{1}{\sigma\sqrt{t}}\frac{c}{(c-m)^2} N'(d_1)\right] = \qquad (A22)$$

$$= e^{\alpha t}\left[N'(d_1)\left(\frac{dd_1}{dm} - \frac{1}{\sigma\sqrt{t}}\frac{c}{(c-m)^2}\right) - \frac{1}{\sigma\sqrt{t}}\frac{m}{c-m} N''(d_1)\frac{dd_1}{dm}\right] =$$

$$= e^{\alpha t}\left[-\frac{1}{\sigma\sqrt{t}}\frac{m}{(c-m)^2} N'(d_1) - \frac{1}{\sigma^2 t}\frac{m}{(c-m)^2} N''(d_1)\right] =$$

$$= -e^{\alpha t}\frac{m}{(c-m)^2}\frac{1}{\sigma\sqrt{t}}\left[N'(d_1) + \frac{1}{\sigma\sqrt{t}} N''(d_2)\right] =$$

$$= e^{\alpha t}\frac{m}{(c-m)^2}\frac{1}{\sigma\sqrt{t}}\left[N'(d_1)\frac{d_1 - \sigma\sqrt{t}}{\sigma\sqrt{t}}\right] =$$

$$= e^{\alpha t}\frac{m}{(c-m)^2}\frac{1}{\sigma^2 t} d_2 N'(d_2) =$$

$$= \begin{Bmatrix} >0 & if & d_2 > 0 \\ <0 & if & d_2 < 0 \end{Bmatrix}$$

(A22) is obtained by $N''(d_1) = -d_1 N'(d_1) < 0$ and $d_2 = d_1 - \sigma\sqrt{t}$.

Evaluating d_2 at p_L and recalling that
$p_L/c \leq p_L/(c-m) \leq 1$
we obtain:

$$\lim_{t \to 0} \frac{1}{\sigma^2 t} d_2 \,|_{p=p_L} = -\infty$$

$$\lim_{t \to \infty} \frac{1}{\sigma^2 t} d_2 \,|_{p=p_L} = \begin{cases} 0^+ & \text{if} \quad \alpha - \frac{1}{2}\sigma^2 > 0 \\ 0^- & \text{if} \quad \alpha - \frac{1}{2}\sigma^2 < 0 \end{cases}$$

Taking account of these limits, the normal distribution property $N'(-\infty) = N(+\infty) = 0$ and the convergence assumption $(\rho - \alpha) > 0$ we can conclude that, even though

$$\frac{\partial^2 H}{\partial p \partial m}_{|p=p_L}$$

is not always negative in $(0, \infty]$, it turns out that the integral of it is negative. This definitely happens if $\alpha < \frac{1}{2}\sigma^2$, that is, price variability has to be large vis à vis the trend value. Therefore

$V'_m(p_L;\theta, m) > 0,$

which assures, togheter with (A19), that $dp_L/dm < 0$.
As the willingness to share losses increases hysteresis becomes larger since the trigger price decreases.
Summing up, the total effect on the firm's value is positive. That is

$$\frac{dS_1(p;\theta,m)}{dm} = \frac{dA(\theta,m)}{dm}(p)^{\gamma_2} + V_m(p;\theta,m) =$$

$$= V_m(p;\theta,m) - V_m(p_L;\theta,m)\left(\frac{p}{p_L}\right)^{\gamma_2} =$$

$$= -(1-\theta)E_0 \int_0^T \frac{\partial H}{\partial m}(p;c-m,t)e^{-\rho t}dt > 0 \qquad (A23)$$

Moreover, as m goes to zero $V_m(p_L;\theta, m)$ and $V'_m(p_L;\theta, m)$ go to zero and then even dp_L/dm goes to *0* as well.
The same happens to the worker's expected sum of the lifetime premium earning $LE(p;\theta, m)$. By (21) we get (assuming $\lambda_2 = \gamma_2$):

$$\frac{dC}{dm} = -\frac{[\gamma_2 p_L^{-1}(K - F(p_L;\theta,m)) + F'(p_L;\theta,m)]\frac{dp_L}{dm} + F_m(p_L;\theta,m)}{p_L^{\gamma_2}} \qquad (A24)$$

and differentiating (22) with respect to m we get

$$F_m(p;\theta,m) = (1-\theta)\left(\int_0^{\infty}\frac{\partial h}{\partial m}(p;c-m,t)e^{-\beta t}dt - \frac{1}{\beta}\right) < 0 \qquad (A25)$$

where

$$\frac{\partial h}{\partial m} = p e^{\alpha t}N'(d_1)\frac{dd_1}{dm} - (c-m)N'(d_2)\frac{dd_2}{dm} + N(d_2) = \qquad (A26)$$

$$= N(d_2) > 0$$

(A26) is obtained applying the call option property

$$p e^{\alpha t}N'(d_1) - (c-m)N'(d_2) = 0,$$

the fact that the value of a call option is a non increasing function of the exercise price and

$$\frac{dd_1}{dm} = \frac{dd_2}{dm}.$$

Since

$$\int_0^{\infty} N(d_2)e^{-\beta t}dt \le \int_0^{\infty} e^{-\beta t}dt = \frac{1}{\beta}$$

(A25) is always negative.

Although the signs of (A14), (A25) and dp_L/dm are well defined, the total effect of an increase of m on (A24) and hence on $LE(p;m)$ is indeterminate.

However if K is large enough dC/dm tends to be negative and then also dLE/dm becomes definitely negative.

Appendix B

The optimization requires the maximization of the objective function (7) with respect to θ given the constraints (13a), (15'), (16), (19), (21), (22), (24), (25), $\rho = \beta(>\alpha), \gamma = \lambda, 0 \le \theta \le 1$. The objective function after substitutions is

$$L = \log\left\{u\left[C(\theta,m)(p)^{\gamma_2} + (1-\theta)\left(\int_0^\infty h(p;c-m,t)e^{-\rho t}dt - \frac{m}{\rho}\right)\right] - \hat{u}\right\} +$$

$$+\log\left\{v\left[A(\theta,m)(p)^{\gamma_2} + \left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) - (1-\theta)\int_0^\infty H(p;c-m,t)e^{-\rho t}dt\right] - \hat{v}\right\} -$$

$$-\xi_1(1-\theta) - \xi_2(\hat{\theta}-\theta) \qquad (B1)$$

Where $\xi_{1,2}$ are the Lagrange multipliers. ξ_2 is associated with the subset $[\hat{\theta} < \theta]$, where $\hat{\theta}$ is the minimum level corresponding to a trigger price equal to zero. The first order conditions (FOC) for a maximum are

$$L_\theta \leq 0, L_\theta \theta = 0; 0 \leq \theta \leq 1, \hat{\theta} - \theta \geq 0;$$

$$\xi_1, \xi_2 \leq 0, \xi_1(1-\theta) = 0, \xi_2(\hat{\theta}-\theta) = 0.$$

The differentiated Lagreangian is:

$$L_\theta = -\frac{B_u}{1-\theta}LE(p;\theta,m) + \frac{B_v}{\theta}S_1(p;\theta,m) - \frac{B_V}{\theta}\left[\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) - \int_0^\infty (H(p;c-m))e^{-\rho t}dt\right] +$$

$$+\left[\frac{B_u}{1-\theta}\left(C(\theta,m) + (1-\theta)\frac{dC(\theta,m)}{d\theta}\right) - \frac{B_v}{\theta}\left(A(\theta,m) - \theta\frac{dA(\theta,m)}{d\theta}\right)\right](p)^{\gamma_2} + \xi_1 + \xi_2 = 0 \qquad (B2)$$

If we take into account the results obtained in the comparative statics section for $dC/d\theta$ and $dA/d\theta$ (see Appendix A) and we substitute them in (B2) we obtain the expression

$$L_\theta = -\frac{B_u}{1-\theta}LE(p;\theta,m) + \frac{B_v}{\theta}S_1(p;\theta,m) -$$

$$-\frac{B_V}{\theta}\left[(V(p;\theta,m) - \theta V_\theta(p;\theta,m)) - (V(p_L;\theta,m) - \theta V_\theta(p_L;\theta,m))\left(\frac{p}{p_L}\right)^{\gamma_2}\right] +$$

$$+\frac{B_u}{1-\theta}\left[\frac{(1-\theta)\gamma_2}{p_L}F(p_L;\theta,m) - (1-\theta)\frac{dp_L}{d\theta}F'(p_L;\theta,m)\right]\left(\frac{p}{p_L}\right)^{\gamma_2} +$$

$$+K\left[\frac{B_u}{1-\theta}\left(1 - \frac{(1-\theta)\gamma_2}{p_L}\right) + \frac{B_V}{\theta}\right]\left(\frac{p}{p_L}\right)^{\gamma_2} + \xi_1 + \xi_2 = 0 \qquad (B3)$$

which is equal to (34) if ξ_1 and $\xi_2 = 0$.

Moreover, applying the results of Appendix A we can note that:

$$V(p;\theta,m)-\theta V_\theta(p;\theta,m)=\left(\frac{p}{\rho-\alpha}-\frac{c}{\rho}\right)-\int_0^\infty H(p_t;c-m,t)e^{-\rho t}dt \qquad (B4)$$

$$=\int_0^\infty [pe^{\alpha t}(1-N(d_2))-c(1-N(d_2))]e^{-\rho t}dt=$$

$$=-\int_0^\infty W(p;c-m,t))e^{-\rho t}dt$$

where $W(p;\, c\text{-}m,t) = w(p;\, c\text{-}m,\, t) + m\, N(-d_2)$ and

$w(p;c-m,t)e^{-\rho t}=-pe^{-(\rho-\alpha)t}N(-d_1)+(c-m)e^{-\rho t}N(-d_2)$

is the price of a European put option with expiration date t and exercise price $(c\text{-}m)$ written on an asset which pays continuous dividends at rate $(\rho-\alpha)$ and spot price p.

Again applying the strong Markov properties of the stochastic process $\{p_t\}$ it can be shown that[15]

$$V(p;\theta,m)-\theta V_\theta(p;\theta,m)-[V(p_L;\theta,m)-\theta V_\theta(p_L;\theta,m)]\left(\frac{p}{p_L}\right)^{\gamma_2}= \qquad (B5)$$

$$=-E_0\int_0^T W(p;c-m,t)e^{-\rho t}dt$$

where the expected value is taken with respect to the stopping time T. Apart from term $m\,N(-d_2)$, (B5) represents the sum, till the shut down of the firm, of European put options with expiration date $t \leq T$.[16]

Obviously, if we let $m = c - p_L$ then p is always greater than p_L; the puts will never be exercised and they will be allowed to expire, i.e.

$$E_0\int_0^T We^{-\rho t}=0.$$

15 Harrison (1985), proposition 11 p.47.

16 W represents a trasformation of a put option with two strike price c and $c\text{-}m$ (see also footnote 7).

The above results can be slightly simplified assuming constant relative risk aversion (CRRA). In particular let

$$u(LE) = LE^{1-R}, \qquad 0 < R < 1$$

be the representative employee utility function, and

$$v(S_1) = S_1^q, \qquad 0 < q < 1,$$

the manager's utility function. In this case the bargaining power indicators become:

$$B_u = \frac{u'(LE)}{u(LE)} = \frac{1-R}{LE} \quad \text{and} \quad B_v = \frac{v'(S_1)}{v(S_1)} = \frac{q}{S_1},$$

and the equilibrium condition (B3) reduces to:

$$L_\theta = \frac{1-R}{\theta}\left\{-\frac{\theta}{1-\theta} + \frac{q}{1-R} + \phi_1(\theta;K,m) + \phi_2(\theta;K,m)\right\} + \xi_1 + \xi_2 = 0 \qquad (B3')$$

where

$$\phi_1(\theta;K,m) = -\frac{q}{1-R}\frac{1}{S_1(p;\theta,m)}\left\{[V(p;\theta,m) - \theta V_\theta(p;\theta,m)] - [V(p_L;\theta,m) - \theta V_\theta(p_L;\theta,m)]\left(\frac{p}{p_L}\right)^{\gamma_2} - K\left(\frac{p}{p_L}\right)^{\gamma_2}\right\} =$$

$$= \frac{q}{1-R}\frac{E_0\int_0^T W(p;c-m,t)e^{-\rho t}dt + K\left(\frac{p}{p_L}\right)^{\gamma_2}}{S_1(p;\theta,m)}$$

$$\phi_2(\theta;K,m) = \frac{\theta}{1-\theta}\frac{1}{LE(p;\theta,m)}\left[\frac{(1-\theta)\gamma_2}{p_L}F(p_L;\theta,m) + K\left(1 - \frac{(1-\theta)\gamma_2}{p_L}\right) - (1-\theta)\frac{dp_L}{d\theta}F'(p_L;\theta,m)\right]\left(\frac{p}{p_L}\right)^{\gamma_2}$$

Since we are looking for a distributional equilibrium with the option to shut down alive, the SOC for a local maximum is

$$L_{\theta\theta} = \frac{1-R}{\theta}\left\{-\frac{1}{(1-\theta)^2} + \frac{d\phi_1}{d\theta}(\theta;k,m) + \frac{d\phi_2}{d\theta}(\theta;k,m)\right\} < 0. \qquad (B6)$$

For $\hat{\theta} < \theta < 1$ we have that $\xi_1 = \xi_2 = 0$ and (B3') reduces to

$$L_\theta = \frac{1-R}{\theta}\left\{-\frac{\theta}{1-\theta} + \frac{q}{1-R} + \phi_1(\theta;K,m) + \phi_2(\theta;K,m)\right\} = 0. \qquad (B3'')$$

For the case of $0 < \theta \le \hat{\theta}$ we get $\xi_1 = 0$ and $\xi_2 < 0$, which implies that $p_L \le 0$ and the option becomes worthless. Therefore (B3') reduces to

$$L_\theta = \frac{1-R}{\theta}\left\{-\frac{\theta}{1-\theta} + \frac{q}{1-R}\right\} = 0 \qquad (B3''')$$

and the SOC reduces to

$$\frac{1-R}{\theta}\left\{-\frac{1}{(1-\theta)^2}\right\}<0$$

which is always satisfied.

Notice that by the non linearity of ϕ_1 and ϕ_2 the sufficient condition for a local maximum, for the case $\hat{\theta}<\theta<1$, cannot lead to clear-cut results. We have found it hard to reach an overall specification of the existence of an internal solution. However the following analysis presents enough results that point out how the maximum depends on the various characteristics of the model.

First it is worth noting that ϕ_1 is positive for all $\theta \in (\hat{\theta}, 1)$, whilst for the positivity of ϕ_2 it is sufficient to have a large K and $dp_L/d\theta<0$.

In addition to that, if we let θ go to $\hat{\theta}$ this implies that $p_L \to 0$. Since in this case the European put option *w()* will never be exercised, then

$$\lim_{\theta \to \theta^+} \phi_1(\theta, K, m) \equiv \lim_{p_L \to 0} \phi_1(\theta, K, m) = 0$$

whilst, since

$$F(p_L;\theta,m) \geq F(p_L;\theta) = (1-\theta)\left(\frac{p_L}{\rho-\alpha}-\frac{c}{\rho}\right), \frac{dp_L}{d\theta}\big|_{p_L=0} > 0,$$

it can be shown that:

$$\lim_{\theta \to \theta^+} \phi_2(\theta;K,m) \equiv \lim_{p_L \to 0} \phi_2(\theta;K,m) = \lim_{p_L \to 0} \phi_2(\theta;K,c-p_L) =$$

$$= \lim_{p_L \to 0} \frac{\theta}{1-\theta}\frac{1}{LE}\left[\frac{(1-\theta)^2\gamma_2}{(\rho-\alpha)}-\frac{(1-\theta)^2\gamma_2}{\rho}\frac{c}{p_L}+K-K\frac{(1-\theta)\gamma_2}{p_L}-\frac{(1-\theta)^2}{\rho-\alpha}\frac{dp_L}{d\theta}\right]\left(\frac{p}{p_L}\right)^{\gamma_2} =$$

$$= \lim_{p_L \to 0} f\left(\frac{1}{p_L}\left(\frac{p}{p_L}\right)^{\gamma_2}\right) =$$

$$= \begin{Bmatrix} 0 & \text{if} & \gamma_2 < -1 \\ \infty & \text{if} & \gamma_2 > -1 \end{Bmatrix} \qquad (B7)$$

According to (B7) the sum $\phi_1+\phi_2$ tends either to zero or to infinity.

On the contrary, if $\theta \to 1$ the sum $\phi_1+\phi_2$ tends inevitably to infinity.

Pulled together these results confirm that, if an interior solution exists, for $\theta \in (\hat{\theta}, 1)$, it will be to the right of θ^A, (the solution obtained by Aoki), that is

$$\frac{\theta^A}{1-\theta^A}=\frac{q}{1-R}.$$

We can see two cases reported in Figure 4 and 5.

In fig.4 line OS represents the ratio $\theta/(1-\theta)$ starting from the origin, passing through the point corresponding to the pure Aoki solution where $q/(1-R)=\theta/(1-\theta)$. Then we can draw the curve described by $\phi_1+\phi_2+q/(1-R)$ when $\gamma_2>-1$. The equilibrium level of θ will be below the point F', where the two curves cross, as (B3") requires.

In Fig.5 we draw the same diagram when $\gamma_2<-1$. The equilibrium in F" still corresponds to a level of $\theta>\hat{\theta}$. This is the situation described by (B7).

Figure 4 if $\gamma_2 > -1$

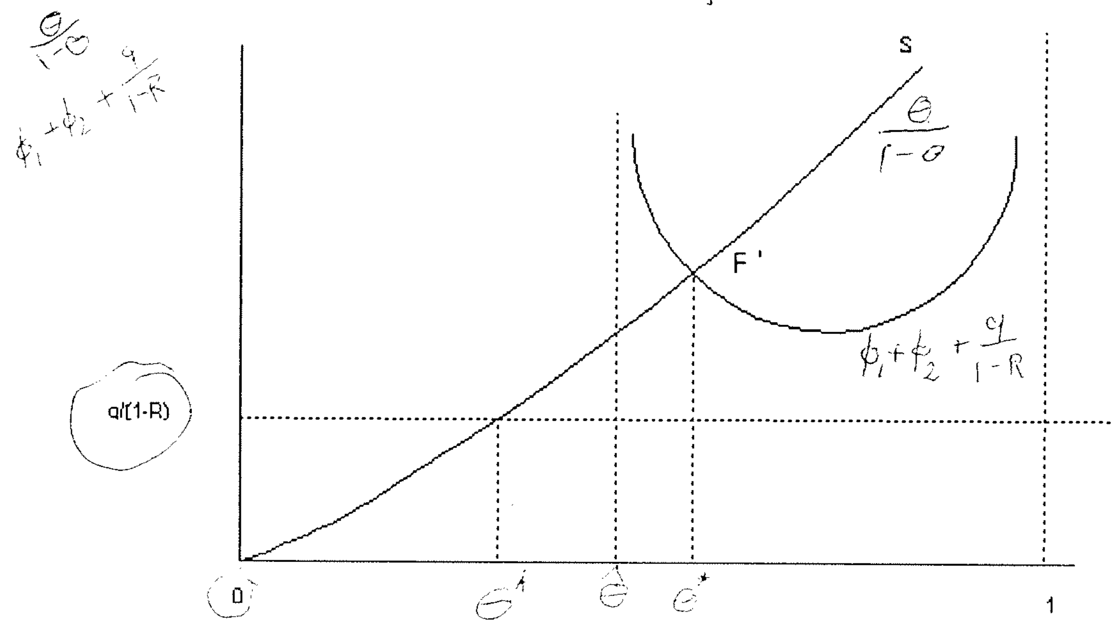
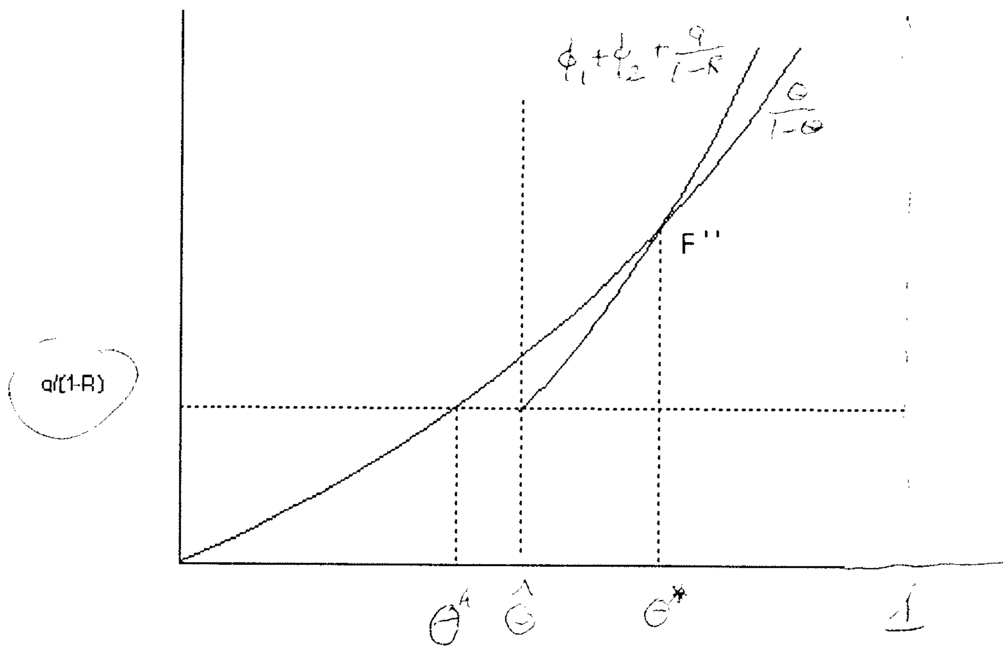


FIGURE 5



if $\gamma_2 < -1$

References

Aoki M.(1980) A Model of the Firm as a Stockholder-Employee Cooperative Game. *American Economic Review*, vol.70, n. 4, 600-610.

Aoki M. (1982) Equilibrium Growth of the Hierarchical Firm: Shareholder - Employee Cooperative Game Approach. *American Economic Review*, vol.72, n.5, p.1097-1110.

Aoki M. (1984) *The Cooperative Game Theory of the Firm*. Clarendon Press, Oxford.

Bös D. (1991) *Privatization. A Theoretical Treatment*. Clarendon Press, Oxford.

Dixit A. (1989) Entry and Exit Decisions under Uncertainty. *Journal of Political Economy*, v.97, n. 3, p.620-638.

Dixit A. (1992) Investment and Hysteresis. *Journal of Economic Perspectives* v.6, n.1; 107-132.

Harrison J.M. (1985) *Brownian Motion and Stochastic Flow Systems*. Wiley, New York.

Harsanyi J. (1956) Approaches to the Bargaining Problem Before and After the Theory of Games. *Econometrica*, v.24, p.144-156.

Harsanyi J. (1977) *Rational Behaviour and Bargaining Equilibrium in Games and Social Situations*. Cambridge University Press, Cambridge.

Ingersoll J.E.(1987), *Theory of Financial Decision Making*. Rowman and Littlefield, New Jersey.

McDonald R. - D. Siegel (1985) Investment and the Valuation of Firms when there is an Option to Shut Down. *International Economic Review*, v. 26, n.2, p.331-349.

McDonald R. - D.Siegel (1986) The Value of Waiting to Invest. *Quarterly Journal of Economics*, v.CI, n.4 , November, p.707-727.

Merton R.(1973) An Intertemporal Capital Asset Pricing Model. *Econometrica*, V.41, n.5, p.867-887.

Pindyck R. (1991) Irreversibility, Uncertainty and Investment. *Journal of Economic Literature*, Sept. v.26, n.3, 1110-1148.

Weitzman M. (1984) *The Share Economy: Conquering Stagflation*. Harvard University Press, Cambridge MA.