DEBT RESTRUCTURING WITH MULTIPLE CREDITORS: A PUBLIC GOOD APPROACH

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Abstract

This paper studies the restructuring of non-marketable debt, when the number of creditors is arbitrary, and there is asymmetric information. With multiple creditors, debt forgiveness has the character of a public good. As for the case of private provision of a public good, mechanism design methods allow to show that debt restructuring is not generally ex-post efficient (liquidation occurs too often). We also derive an indirect mechanism, in which equity offers to exchange old debt for new claims of lower face value, and creditors choose how much to tender. This mechanism can replicate the optimal restructuring scheme. Finally, by means of an example we study repeated offers, and we show that an initial failure reveals information that discourages free-riding, and efficiency improves.

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1. Introduction.

In recent years much debate has surrounded the issue of debt renegotiation. Developments both in economic theory and in the economy itself have fueled a burgeoning literature. For instance, the LDC debt crisis brought debt restructuring on the first pages of the newspapers in the early '80s. More recently, on the front of the U.S. corporate credit market, the sharp increase in leverage has forced several prominent companies to sit at the negotiating table with their creditors. Also, U.S. bankruptcy reform in 1978 introduced a new court–supervised procedure for debt restructuring, which quickly became very popular (Chapter 11). The effectiveness of Chapter 11 in promoting reorganization of healthy companies, without delaying the dismissal of non–viable enterprises, is now widely questioned (White, 1989, Bradley and Rosenzweig, 1992). The issue of debt renegotiation in relation to the design of optimal bankruptcy laws has also been addressed in the context of institutional reform in Eastern Europe (Mitchell, 1990, Aghion et al., 1992). On the theory side, advances in bargaining theory and in contract theory have provided new tools to analyze debt restructuring. Also, work on the role of debt (and of different types of debt) in the corporate capital structure has confronted the question of how efficiently debt contracts can be renegotiated (Haugen and Senbet, 1978, Hart and Moore, 1990, Bergman and Callen, 1991, Gorton and Kahn, 1992).

In most models, the need for debt renegotiation arises from some version of the underinvestment problem, identified by Myers (1977)[1]: When a company has a large debt outstanding, creditors are the main beneficiaries of new investment, so managers–shareholders tend to invest too little. In the absence of debt restructuring profitable investment opportunities are forgone, and in extreme cases viable companies can be

[1]In the debate over LDC debt, Myers underinvestment problem has become popular as the "debt overhang" problem (see, among many, Krugman 1988).

liquidated prematurely.

Models of corporate financial distress often assume that while some type of debt can be renegotiated relatively easily (bank loans, private placements), widely–held instruments such as bonds or commercial paper are costly or even impossible to renegotiate (Bulow and Shoven, 1979, White, 1980 and 1989)[2]. Bankruptcy laws to promote court–supervised renegotiation are often motivated by the need to overcome coordination problems among multiple creditors (Jackson, 1986). An extreme case of coordination failure is studied by Gertner and Scharfstein (1991), who consider renegotiation of publicly–held debt when bondholders are atomistic[3]. The hold–out problem is also often mentioned to explain the difficulties encountered by highly indebted LDCs in obtaining debt forgiveness from their private creditors (Sachs, 1990).

A general feature underlying the work described above is the public–good nature of debt forgiveness: By giving up one's own contractual rights, a debtholder favors the other claimants, because contract law does not allow to exclude from the benefits of debt forgiveness creditors who do not participate. In the present paper, we study debt restructuring with multiple creditors from the perspective offered by the literature on private provision of public goods, when the individual cost of provision is private information (Rob, 1989, and Mailath and Postelwaite, 1991). The problem is formulated in terms of mechanism design, using the methods developed by Myerson and others to study optimal auctions (see, for instance, Myerson, 1985). The revelation principle allows to show that, when the willingness to contribute to debt forgiveness is private information, the outcome of debt renegotiation is not generally ex–post efficient, and liquidation may occur even if continuation is Pareto–optimal. This is true for any static debt restructuring

[2]For a model in which it is optimal for firms to issue both types of debt, see Detragiache (1992).

[3]For empirical evidence on corporate debt workouts, see Weiss (1990), Gilson et al. (1990), Asquith et al. (1992), and D. Brown et al. (forthcoming).

game. Furthermore, as the number of creditors approaches infinity, ex–post inefficiency becomes the rule, as in the case of public goods. On the other hand, for a finite number of creditors Pareto–improving renegotiation can succeed with positive probability: Despite incentives to hold out, creditors contribute to debt forgiveness because it increases the probability of continuation.

The direct mechanism described above does not consider the possibility that creditors trade their claims among themselves. Hence, the results apply to restructuring of non–marketable debt, such as bank loans, trade credits, and so on. A full–fledged model of restructuring of publicly–traded debt would be very complex, because (i) trade among bondholders occurs under asymmetric information, (ii) the value of the bonds depends on the outcome of the renegotiation process, (iii) the restructuring scheme is designed based on the information revealed by trading activity[4].

The second part of our analysis is concerned with the description of a mechanism that could be implemented in practice. We characterize a direct revelation scheme that maximizes shareholders' expected profit, and then we show that this outcome can be replicated by an indirect mechanism (a mechanism that does not involve a mediator). The indirect restructuring scheme resembles quite closely an exchange offer, where equity proposes to creditors to exchange old debt for a new claim of lower face value. The offer specifies that the exchange will be successful only if the amounts tendered satisfy a certain condition, for instance if they exceed a minimum tendering requirement. Exchange offers have often been used by US corporations in the 1980s to restructure their public debt (see Gertner and Scharfstein, 1991). By inducing heterogeneous creditors to tender different fractions of their holdings, exchange offers allow management to price discriminate.

[4]Similar limitations apply to the case of public goods. For instance, Rob (1991) considers the problem of compensating landowners for allowing the construction of a polluting plant in a residential area. However, the model does not allow resident to trade land among themselves.

The model outlined above does not allow management to submit a new proposal, if an initial restructuring proposal fails. This is not realistic. Although we are not able to extend the general model to a multiperiod framework, we present a simple two–period game, in which creditors' strategies are restricted (each creditor can exchange either all of his claims or nothing). This example shows that the opportunity to repeat offers can be welfare–improving: Failure of the first round of tendering reveals to creditors that the others have a low willingness to contribute to debt forgiveness. Therefore, holding out becomes less attractive. Although creditors may or may not be more reluctant to offer forgiveness at the first stage, the overall probability of success increases. This example shows that the possibility of repeating offers may reduce the inefficiencies that arise in the static mechanism.

The paper is organized as follows: First the basic model is presented, and the outcome of the direct revelation mechanism is derived. In section 3 an indirect restructuring procedure is characterized. A brief discussion of alternative restructuring procedures follows. Section 4 discusses the implications of extending the model to the case of common values, i.e. the case in which different willingness to contribute depend on different estimates of the true liquidation value of the firm. In section 5 the issue of repeated offers is addressed. Section 6 concludes.

2. Debt Restructuring and Incentive–Compatibility.

Let's start with building a very simple model, in which a highly indebted firm confronts the problem of renegotiating its widely–held debt. The firm in question is owned by shareholders–managers, and it has a project that requires an amount I to be invested in the current period, and yields a cash–flow with a present discounted value of $C + I$, with $C > 0$. The firm has no cash on hand, but it has debts outstanding amounting to D, which are all due in the current period. The debt is so large, that shareholders expect their claims to become worthless if the firm is liquidated. To make the problem interesting, let $C + I - D < I$, so that it is not profitable for either shareholders or outside investors to undertake the project. This set–up insures that continuation of the business can occur if and only if existing debt is appropriately written down. The debt is in the hands of N creditors, which we will often refer to as debtholders, although they could also be trade creditors, banks, or other type of claimants. Creditors are all risk–neutral, they have equal priority, and they and hold an identical share of total debt $d = \frac{D}{N}$. For tractability, the distribution of claims across creditor is taken as exogenous with respect to debt renegotiation proposals, implying that no trade in claims can occur while restructuring takes place.

Creditors differ in the utility that they expect to receive if the firm is liquidated. Heterogeneity may stem from differences in the utility function or from differences in the estimate of the liquidation value of the firm. Under the first interpretation, the model is one of private values, while under the second it is one of common values. The analysis in this section is for the private values case, while the case of common values is treated in section 4. There is a continuum of possible types of creditor, with liquidation utility of $x^i \in [\underline{x}, \bar{x}]$. For convenience, we will assume that x^i is the creditor i' s payoff in liquidation if he owned claims to the total entire value of the firm. Since each creditor owns 1/N of the firm, the expected utility in liquidation is actually $\frac{x^i}{N}$.

Assumption 1. $\bar{x} = D$.

Remark. Since creditors cannot receive more than the contractual value of debt in a liquidation, it must be $\bar{x} \leq D$. Assumption 1 simply requires the set of possible types to be large enough.

If management knew the true x^i, debt restructuring would always be socially efficient, independent of the number of creditors: Management could propose a take–it–or–leave–it plan that gives creditor i his reservation value $\frac{x^i}{N}$, and all creditors accept. This strategy is individually rational for the firm if and only if

$$C \geq \sum_{j=1}^{N} \frac{x^j}{N}, \tag{1}$$

that is if and only if continuation is Pareto–efficient.

Asymmetry of information is a necessary ingredient to capture coordination problems in debt restructuring with multiple creditors, so we will assume that x^i is private information of each creditor[5]. From the point of view of the management and of the other creditors, x^i is a random variable, with distribution $F(x^i)$, and density $f(x^i)$. For simplicity, it will be assumed that the creditors liquidation values are identically and independently distributed. Let x denote the vector $(x^1, \ldots, x^N)$. For future reference, define

$$f(x) = \prod_{j=1}^{N} f(x^j) = [f(x^j)]^N$$

Also, let $x_{-i} \equiv (x^1, .., x^{i-1}, x^{i+1}, .., x^N)$, and

[5]If the debt consists of traded securities, the market price before restructuring reveals information about the valuations of security holders. This information is common knowledge. Our model is relevant for traded securities to the extent that residual private information remains.

$$f(x_{-i}) = \prod_{j \neq i} f(x^j) = [f(x^j)]^{N-1}$$

and similarly for the density $f(x_{-i})$.

Assumption 2. $\frac{F(x^i)}{f(x^i)}$ is monotonically non–decreasing for all x^i and all $i = 1, ..., N$.

This property holds for a several commonly used distribution functions (see Rob, 1989).

To analyze debt renegotiation we will assume that owners–shareholders choose the procedure through which renegotiation is to occur (for instance, an exchange offer with a minimum tendering requirement, a mechanism through which each creditor is asked to submit a debt forgiveness proposal, a vote, etc.), subject to participation constraints for the creditors. This approach implies that initial debt contracts do not contain specific provisions as to how renegotiation should occur, except that contracts cannot unilaterally be altered by a subset of the parties. We believe this approach to be quite realistic. Less realistic is the restriction to static renegotiation procedures, which we will use in this section. Section 5 discusses repeated restructuring offers.

We will characterize the outcome of debt renegotiation as the Bayesian equilibrium of a direct revelation game, in which creditors confidentially communicate the value of their signal to a mediator. The mediator then instructs the owners to choose continuation with a certain probability, and to make each creditor a payment. By the Revelation Principle, all equilibria of all static communication games can be obtained as a truthtelling equilibrium of a direct revelation game (Myerson, 1985). Since owners are assumed to choose the renegotiation procedure, we will focus on mechanisms that maximize their expected payoff.

Let $I^k \equiv \{ \underline{x} \leq x^j \leq \bar{x}, j = 1, 2, ..., k \}$, with $k = 1, 2, ..., N$. Then I^N is the set of all possible types of creditor. Let (ω, p) be the mechanism. ω is a vector of functions $\omega^i : I^N \rightarrow$

$\mathbb{R}^N$, assigning a payment to each creditor i as a function of the vector of reported types $\hat{x}$. The function $p : I^N \to [0, 1]$, gives the probability of continuation as a function of reported types. We will assume that the payments ω take place only if the firm continues to operate (the opposite assumption would not change the results). For the mechanism to be a truthtelling Bayesian equilibrium, it must be an equilibrium for each type to report the true x^i, and it must be individually rational for all players to participate. The participation constraints are, for all i

$$\text{(IR)} \qquad U(x^i, \hat{x}^i, \omega^i, p) = \int_{I^{N-1}} p(\hat{x}^i, x_{-i}) \left[\omega^i(\hat{x}^i, x_{-i}) - \frac{x^i}{N} \right] f(x_{-i})\, dx_{-i} \geq 0,$$

where $\hat{x}^i$ is the report of creditor i (if the firm is told to continue, creditors receive the payment ω^i, but they lose the liquidation value). The participation constraint for shareholders is

$$\text{(EXABB)} \qquad V(x, p, \omega) = \int_{I^N} p(x) \left[C - \sum_{j=1}^{N} \omega^j(x) \right] f(x)\, dx \geq 0.$$

This constraint is often referred to as "ex–ante budget balance" in the literature on public goods. We could also impose the more stringent ex–post budget balance constraint, namely

$$\text{(EXPBB)} \qquad C \geq \sum_{j=1}^{N} \omega^j(x) \;\; \forall\, x \in I^N.$$

However, in this context ex–ante budget balance is probably more appropriate. First of all, notice that if the funds needed to finance new investment are obtained from an outside lender, ex–post shareholders are always at worst indifferent between investing and liquidating, since limited liability prevents their payoff from falling below zero. If the project is financed by outsiders, it is therefore credible that management will invest whenever the mechanism recommends to continue. But management can arrange outside financing before debt renegotiation takes place, specifying in the contract that funds will be released if and only if renegotiation succeeds. If (EXABB) holds, risk–neutral lenders are willing to enter such a committment, even if they now that ex–post debt forgiveness may not be sufficient to make the loan profitable. Shareholdes are better off prearranging the financing of new investment, because (EXABB) is a weaker constraint than (EXPBB).

For truthful reporting to be an equilibrium it must be:

(IC) $$U(x^i, x^i, \omega^i, p) \geq U(x^i, x^j, \omega^i, p) \;\; \forall\, i, j.$$

Using standard techniques one can show that (IC) implies that for all i

(2) $$U(x^i, x^i, \omega^i, p) = U(\bar{x}, \bar{x}, \omega^i, p) + \int_{I^{N-1}} \int_{x^i}^{\bar{x}} p(u, x_{-i})\, du\, f(x_{-i})\, dx_{-i} \geq 0.$$

PROPOSITION 1. *Consider a mechanism such that $p(x)$ is decreasing in each x^i. Such a mechanism satisfies (IR), (EXABB), and (IC) if and only if*

(3) $$\int_I p(x) \left[C - \sum_{i=1}^{N} \left[\frac{x^i}{N} + \frac{F(x^i)}{(N) \;\; f(x^i)} \right] \right] f(x)\, dx \geq 0.$$

Proof. The proof is standard. For readers' convenience we report it in the Appendix.

Ex–post Pareto–efficiency requires $p(x) = 1$ if $C \geq \sum_{i=1}^{N} \frac{x^i}{N}$, and $p(x) = 0$

otherwise. Clearly, this rule does not satisfy condition (3) for all values of C, so ex–post efficiency cannot be insured in general. The loss is entirely due to the presence of the terms $\frac{F(x^i)}{N\ f(x^i)}$ on the LHS, which, in turn, appear because of the incentive–compatibility constraints. The inability to achieve ex–post efficiency does not depend on whether the mechanism maximizes shareholders' expected utility or not.

Next, following the work of Rob (1989), we will characterize an optimal mechanism, that is a mechanism that maximizes shareholders' expected utility. To this purpose, we impose the following assumption:

Assumption 2. $\frac{F(x^i)}{f(x^i)}$ is monotonically non–decreasing for all x^i and all $i = 1, ..., N$.

This property holds for a several commonly used distribution functions (see Rob, 1989). Also, let

(4) $$z(x^i) = \frac{F(x^i)}{f(x^i)}.$$

In analogy with auction theory, we will refer to $x^i + z(x^i)$ as creditor i's "virtual" liquidation value. Also, implicitly define

(5) $$\tilde{x}(x_{-i}) = N\ C - \sum\nolimits_{j \neq i} [x^j + z(x^j)] - z[\tilde{x}(x_{-i})].$$

As it will turn out, $\tilde{x}(x_{-i})$ is the liquidation value of the debtholder who has the pivotal virtual liquidation value, given the continuation rule chosen by management in equilibrium.

Proposition 2 (Rob, 1989). *Let $\hat{p}(x)$ be a continuation rule such that:*

$$\hat{p}(x) = 1 \text{ if } C \geq \sum_{i=1}^{N}\left[\frac{x^i}{N} + \frac{F(x^i)}{(N)\ \ f(x^i)}\right]$$

$$\hat{p}(x) = 0 \text{ otherwise,}$$

and let

$$\hat{\omega}^i(x) = (N)^{-1}\, \tilde{x}(x_{-i}).$$

Then $(\hat{p}, \hat{\omega})$ is an optimal mechanism.

Proof. See proof of Theorem 1 in Rob (1989).

We note here that the mechanism of Proposition 1 implies that creditor with the highest liquidation value receive an interim utility of zero. From (2), and using the definition of $\tilde{x}(x_{-i})$, the interim utility to debtholder i according to the mechanism is

$$U(x^i; \hat{p}, \hat{\omega}^i) = \int_{I^{N-1}} \hat{p}(x^i, x_{-i}) \left[\frac{\tilde{x}(x_{-i}) - x^i}{N}\right] f(x_{-i})\, dx_{-i}. \tag{6}$$

The outcome derived in Proposition 2 shows that debt renegotiation with multiple creditors can be successful, even if firm's management cannot unilaterally reduce the contractual value of creditor claims either directly (imposing a majority vote, for instance), or indirectly (issuing claims formally or de facto senior to existing ones). Creditors, although behaving non–cooperatively, are willing to agree to partial debt forgiveness, because that makes them better off than immediate liquidation.

2. 1 An Example.

To clarify what the mechanism of Proposition 2 implies, consider the following example. Let $N = 2$, $\underline{x} = 0$, $\bar{x} = 1$, and $F(x^i) = x^i$. Then the continuation rule $\hat{p}$ requires to continue with probability one if

$$C \geq x^1 + x^2.$$

In contrast, ex–post it would be efficient to continue whenever $C \geq (2)^{-1} (x^1 + x^2)$. The payoff to creditor 1 if the firm continues is

$$\hat{\omega}^1(x^1, x^2) = \frac{C}{2} - \frac{x^2}{2}.$$

Notice how the mechanism requires creditor 1 to make a larger contribution to debt forgiveness (accept a lower payoff) when the other creditor reports a high expected liquidation value.

2. 2. A Large Number of Creditors

A natural question at this stage is what is the impact on ex–post efficiency of increasing the number of borrowers. Models of bankruptcy and financial distress often argue that efficient renegotiation becomes more difficult, or even impossible, when the firm has a large number of creditors. Again, the results derived for the case of public goods are extremely helpful. In fact, Rob (1989, Theorem 2) has shown that, under some restrictions on the distribution of x,

$$\lim_{N \to \infty} \Pr \left\{ C \leq \sum_{i=1}^{N} \left[\frac{x^i}{N} + \frac{F(x^i)}{(N) \quad f(x^i)} \right] \mid C \geq \Sigma_{i=1}^{N} x^i \right\} = 1.$$

In words, according to the mechanism of Proposition 2, as N becomes large the probability that the firm will not continue, given that it is ex–post efficient to do so, goes to one. Mailath and Postlewaite (1990) extend this result to all possible mechanism that satisfy (IC), (IR), and (EXABB). Hence, in cases in which creditors are extremely numerous, and have private information about their liquidation value, it is legitimate to expect that potential gains from debt renegotiation cannot materialize.

3. Indirect Restructuring Procedures

The purpose of this section is to construct a mechanism that, although not relying upon the presence of a mediator, can replicate the outcome of the optimal restructuring procedure described in Proposition 2. In particualr, we shall construct a game palyed by equity on one side and all crediotrs on the other, that has a Bayesian–Nash equilibrium entailing the same payoffs as the the equilibrium of Proposition 2. This game resembles an exchange offer game, in which management offers a new asset (of lower face value) in exchange for the old one, and creditors simultaneously decide which fraction of their holdings (possibly zero) to exchange. The exchange goes through if and only if the bids collected satisfy certain conditions spelled out in the offer. In the game, payments to creditors if the firm continues only depend on the fraction of the portfolio that the creditor has tendered. Hence, in contrast with the direct mechanism, the payment is not a function of the type of the other creditors in the market[6].

[6]Cornelli (1992) shows that for a monopolist facing a fixed cost of production it is an optimal selling procedure to offer customers a menu of possible prices. As in our case, high valuation consumers are willing to pay a price above the minimum to increase the

Suppose that the terms of the new claim offered in the exchange are such that its present discounted value if the firm continues is $b < \frac{D}{N}$. Each creditor can exchange all or part of his holdings, or he can choose not to tender at all. Let $h = (h_1, \ldots, h_N)$ be the vector of the portfolio shares tendered by each creditor. Define

$$w^i = d - h^i (d - b).$$

w^i is the value of the claim of a creditor who tenders a fraction h^i of his holdings, if the exchange succeeds and the firm continues to operate. For given b, choosing w^i is equivalent to choosing h^i. Since h^i must be in the [0, 1] interval, w^i can range between d and b. To replicate the direct mechanism, we have to find the amounts tendered that induce the same interim utility as the direct mechanism, and then we have to express the probability of continuation in Proposition 2 as a function of the amounts tendered, instead of the reported types.

PROPOSITION 3. *The optimal renegotiation procedure can always be replicated by an exchange offer, in which a new debt contract is offered in exchange for the old ones, creditors can exchange any fraction of their holdings, and the exchange is void unless the amount tendered satisfies a condition specified in the offer.*

Proof. The expected payoff to creditor i from tendering an amount h_i, that yields a payment w^i is

$$\int_{I^{N-1}} p(x^i, x_{-i}) \left[w^i - \frac{x^i}{N} \right] f(x_{-i})\, dx_{-i}.$$

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probability of the good being produced.

To replicate the direct mechanism, w^i must satisfy (see equation (6))

$$\text{(7)} \qquad w^i \int_{I^{N-1}} \hat{p}(x^i, x_{-i})\, f(x_{-i})\, dx_{-i} = \int_{I_{-i}} \left[\frac{\tilde{x}(x_{-i})}{N} \right] f(x_{-i})\, dx_{-i}.$$

where $\hat{p}(x^i, x_{-i})$ is the probability function implied by the decision rule derived in Proposition 2. Define

$$\Gamma(x^i) \equiv \{ x_{-i} \mid \hat{p}(x^i, x_{-i}) = 1 \}.$$

The equilibrium payment becomes

$$\text{(8)} \qquad \hat{w}^i = \int_{\Gamma(x_i)} \left[\frac{\tilde{x}(x_{-i})}{N} \right] f(x_{-i} \mid \Gamma(x_i))\, dx_{-i}, = \hat{W}(x^i)$$

where

$$f(x_{-i} \mid \Gamma(x_i) = f(x_{-i}) \left[\int_{\Gamma(x^i)} f(x_{-i})\, dx_{-i} \right]^{-1}.$$

Under Assumption 2 $\hat{w}^i$ is monotonically increasing in x^i: As x^i increases, the set $\Gamma(x^i)$ loses some elements. Specifically, it loses the larger values of x_{-i}. If the function under the integral sign was constant, the integral would not change, since we are integrating the conditional density. However, $\tilde{x}(x_{-i})$ is decreasing in x_{-i}, so the value of the integral must increase with x^i. Since also h^i is monotonic in w^i, There exists a function G: $[0, 1] \rightarrow [\underline{x}, \bar{x}]$ such that $x^i = G(\hat{h}^i)$. Define

$$\hat{b} = \hat{W}(\underline{x}).$$

$\hat{W}(\underline{x})$ is the payment that gives the creditor with the lowest liquidation value the same interim utility as the direct mechanism of Proposition 2. Of course, if $\hat{b}$ is the value of the new asset, to obtain a payment $\hat{b}$ the creditor must exchange his entire portfolio, and payments below $\hat{b}$ are not feasible. On the other hand, if a creditor chooses not to exchange any of his holdings, the payment if the offer succeeds is the face value of the old claim, $d = \frac{D}{N}$. Notice that under Assumption 1 $\bar{x} = D$, so if a type $\bar{x}$ chooses not to exchange at all his interim utility is equal to zero for any probability of success $p(\cdot)$.

We can now characterize the equilibrium strategies of the exchange offer game, that yield the same interim utilities to creditors and the same ex–ante utility to shareholders as the direct revelation mechanism of Proposition 2. These strategies are

(i) Management: Offer a new asset with present discounted value $b = \hat{b}$. Announce that the exchange offer will be valid if and only if the amounts tendered by creditors satisfy $C \geq (N)^{-1} \sum_{i=1}^{N} \{ G(h^i) + z[G(h^i)] \}$.

(ii) Creditor i: tender $\hat{h}^i = \left[d - N\,\hat{w}^i \right] \left[d - \hat{b} \right]^{-1}$.

From Proposition 2 we know that these strategies are incentive–compatible and individually rational for creditors, and that the continuation rule maximizes equity's expected payoff. Notice that the choice of b and the fact that $D = \bar{x}$ insure that the set of equilibrium values of $\hat{h}^i$ is the whole interval [0, 1]. Hence, incentive–compatibility is sufficient for the $\hat{h}^i$s to be mutual best responses[7]. ◇

This results show that in spite of the incentives to free–ride, exchange offers of junior assets can succeed whenever the number of creditors is finite. Incentives to accept a claim worth less than the old one arise because each creditor knows that he will influence the probability of success. This mechanism allows management to extract more debt forgiveness from creditor with lower expected liquidation value, effectively engaging in price discrimination[8].

In a special case, management's continuation rule as a function of the amounts tendered can be expressed as a minimum tendering requirement. In particular, if there exists a monotonically increasing function $R(\cdot)$ such that

$$\sum_{i=1}^{N} \{ [G(h^i) + z [G(h^i)] \} = R \left[\sum_{i=1}^{N} h_i \right],$$

then the minimum tendering requirement is $R^{-1}(C)$, where R^{-1} is the inverse function of R. If $W(x^i)$ and $z(x^i)$ are linear functions, the condition is satisfied. This is the case, for instance, if the distribution function is of the class $F(x^i) = k\,(x^i)^{\alpha}$, with α and k constant.

3. 1. An Example with Two Creditors.

Using the results of Section 2, it's easy to compute the equilibrium strategies of the

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[7]If shareholders chose $b < \hat{W}(\underline{x})$ the exchange offer would still replicate the direct mechanism of Proposition 2. In this case, all bondholders would exchange a smaller fraction of their holdings, and no type would exchange everything. On the other hand, $b > \hat{W}(\underline{x})$ would require $h(\underline{x}) > 1$, which is not feasible.

[8]For a recent analysis of price discrimination, see Spulber (1993).

exchange offer, in the case of $N = 2$, $\underline{x} = 0$, $\bar{x} = 1$, and $F(x^i) = x^i$. According to the continuation rule $\hat{p}$, the set $\Gamma(x_2)$ is $[0, C - x^1]$, and the probability of a success given x^1 is $C - x^1$. Hence,

$$\hat{w}^1 = (C - x^1)^{-1} \left[\int_0^{C-x^1} (1\backslash 2)\,(C - x^2)\, dx^2 \right] = \frac{C}{4} + \frac{x^1}{4}.$$

As expected, the payment is increasing in the expected liquidation value (recall that interim utility is decreasing, however). Then, the present discounted value of the new asset should be

$$\hat{b} = W(\underline{x}) = \frac{C}{4},$$

and the amounts tendered as a function of the x^is are

$$\hat{h}^i = \left[\frac{1}{2} - \frac{C}{4} - \frac{x^i}{4} \right] \left[\frac{1}{2} - \frac{C}{4} \right]^{-1} = 1 - x^i \left[\frac{1}{2} - \frac{C}{4} \right]^{-1}$$

(recall that $D = \bar{x} = 1$). So all creditors except those with the highest liquidation value tender a positive fraction of their holdings. From this formula, we can retrieve the relationship that links the x^is to the amounts tendered

$$x^i = 2\,(1 - \hat{h}^i) \left[1 - \frac{C}{2} \right].$$

Recalling that the optimal continuation rule is $p = 1$ if $x^1 + x^2 \leq C$ and $p = 0$ otherwise, we can now compute the minimum tendering requirement to be $\frac{4 - 6\ C}{2 - C}$.

An important issue are the incentives for the parties to break the initial commitment: For some realizations of (x^1, x^2) the amount tendered may fail to satisfy the minimum tendering requirement, even though the amount of debt forgiveness received is sufficient to make continuation optimal ex–post. In the context of the example, ex–post it would be optimal for equity to continue whenever $w^1 + w^2 \leq C$. Using the equilibrium values of the w^is, this condition becomes $x^1 + x^2 \leq 2\ C$. So whenever $C < x^1 + x^2 \leq 2\ C$, shareholders would like to renege on the commitment to the minimum tendering requirement. Notice, also, that for those parameter values both creditors are better off ex–post if the commitment is broken, since $\hat{w}^i = \frac{C + x^i}{4} \geq \frac{x^i}{2}$ when $x^1 + x^2 \leq 2\ C$. So, in this example, when equity receives bids leading to payments that are less than the continuation value of the firm, it is common knowledge that continuation Pareto–dominates liquidation. This is true even if bids are anonymous, or if equity only knows the sum of the bids. Clearly, the incentives to renegotiate are strong in these circumstances. The problem is similar to the issue of whether it is credible for a seller to set a minimum price above her reservation price in an auction (see for instance Milgrom, 1987).

We note here that in the context of the example it is easy to determine the equilibrium of the game, if equity cannot commit not to accept the offer whenever it is ex–post Pareto–optimal to do so. Suppose creditors expect the offer to be accepted if and only if $x^1 + x^2 \leq (2\backslash 3)\ C$. Using the same formula as before, it is a best response for creditor i to choose a payoff

$$w^i = (1\backslash 6)\ C + \frac{x^i}{2}.$$

Ex–post it is optimal to continue if and only if

$$(2\backslash 3)\ C + \frac{x^1 + x^2}{2} \leq C,$$

which is consistent with the continuation rule hypothesized. In this equilibrium, continuation is more likely even though creditors grant less debt forgiveness. Nonetheless, ex–post efficiency is not achieved in general.

As pointed out in the introduction, the static mechanism design framework neglects not only the issue of ex–post renegotiation, but also that of repeated offers. After a first round of bidding, not only management could consider breaking the initial commitment, but it could also choose to make another offer. For instance, take the mechanism just presented, in which management has no incentives to violate the commitment to the minimum tendering requirement. Suppose the bids received are such that the offer is rejected. By observing w^i, however, management infers the true liquidation values of the creditors. In all states in which continuation is Pareto–efficient but the offer fails, management has an incentive to make a take–it–or–leave it offer equal to the liquidation value. But then, of course, initial tendering behavior would change. So the problem should be posed in a multiperiod framework, in which repeated offers are possible. The discussion of this case is in section 5.

3. 4. Majority Voting

In the previous sections it was assumed that debt could be restructured only with unanimous approval of all creditors. In terms of the model, the participation constraints of creditors had to be satisfied with probability one. Sometimes, however, a restructuring plan may be subject to majority voting. In the U.S., for instance, firms can restructure under Chapter 11 of the bankruptcy code, if they obtain approval by a 2/3 majority of each class of claimants. We claim that also under majority voting the indirect mechanism described above remains useful as a tool for price discrimination.

Consider the following mechanism: Management proposes a plan, that includes a flat minimum debt forgiveness rate, as well as an offer to exchange the old debt for another asset, with an even lower face value. Creditors vote on the plan. If the majority is in favor, all debt is scaled down, and the exchange offer takes place. If the minimum tendering requirement is met, the firm continues, otherwise liquidation follows. This mechanism certainly allows shareholders to replicate the outcome of a simple plan, that gives all debtholders the same payoff rate: It's sufficient to set the flat rate of debt forgiveness high enough to insure that continuation is profitable even if nobody participates to the exchange offer. On the other hand, shareholders may grant themselves a higher expected profit by asking for a lower flat rate (thereby increasing the probability that the plan will be approved), and extracting further debt forgiveness from low liquidation value creditors through the exchange offer. As in the model of sections 2 and 3, some creditors are willing to contribute more than the flat rate to reduce the probability of liquidation.

4. The Model with Common Values.

The set–up proposed in the preceding sections assumes that differences in the liquidation value of the firm to a debtholder reflect differences in preferences. Another, perhaps more appealing, interpretation is that creditors are trying to forecast the market value of the firm in liquidation, and that the x^i represent different estimates. In this case, however, it is realistic to allow creditors to revise their estimate, if they could learn other creditors' estimates. In the language of auction theory, the model could be cast in terms of common values. Following Myerson (1985), suppose that the expected liquidation value of the firm according to creditor i is

$$x^i + \sum_{j \neq i} e^j(x^j),$$

where the functions $e^j(x^j)$ capture how the debtholder would revise his estimate if he knew the estimate of debtholder j. For the sake of realism, these functions are increasing in their arguments. Also, to maintain our interpretation of x^i as creditor i' s estimate, we will assume that for all j

$$(9) \qquad \int_{\underline{x}}^{\bar{x}} e^j(x^j)\, f(x^j)\, dx^j = 0.$$

To keep notation simple, define $e(x_{-i}) = \sum_{j \neq i} e^j(x^j)$. Now we can trace the changes how the assumption of common values changes the results in the previous sections. Consider first Proposition 1. It is straightforward to show that condition (3) now becomes

$$(3') \qquad \int_I p(x) \left[C - \sum_{i=1}^{N} \left[\frac{x^i + e(x_{-i})}{N} + \frac{F(x^i)}{(N)\ \ f(x^i)} \right] \right] f(x)\, dx \geq 0.$$

If p(x) is zero for high realizations of x, the terms $\int_I p(x)\, e(x_{-i})\, f(x)\, dx$ are negative (recall that the functions e^j are increasing, and that (9) has been assumed). Hence, the constraint is less strict than under private values. The intuition is the following: Debtholders know that the offer will succeed only if other debtholders have rather low estimates of the liquidation value, so that the true liquidation value is indeed likely to be low. This allows shareholders to extract more debt forgiveness than under private values. Notice how this is the reverse of the "winner's curse" in common value auctions (Milgrom, 1987). In an auction, obtaining the object reveals to the bidder that his estimate of the value was too high, so bidding behavior is more conservative than under private values.

Next, consider the optimal mechanism of Proposition 2. Redefine

$$\tilde{x}(x_{-i}) \equiv \left\{ \min x^i \mid N\,C = \Sigma_{i=1}^{N} [x^i + e(x_{-i}) + z(x^i)] \right\}.$$

Then it is an optimal restructuring procedure to set $p(x) = 1$ if

$$C \geq \sum_{i=1}^{N} \left[\frac{x^i + e(x_{-i})}{N} + \frac{F(x^i)}{(N) \quad f(x^i)} \right],$$

and to set $p = 0$ otherwise. Let $\hat{p}$ be this rule. Then the payment to creditor i according to the mechanism is

$$\hat{\omega}^i(x^i, x_{-i}) = (N)^{-1} [\tilde{x}(x_{-i}) + e(x_{-i})].$$

Notice that, while $\tilde{x}(x_{-i})$ tends to be low for large values of x_{-i}, the opposite is true for $e(x_{-i})$. The first term captures what could be called the extent of free–riding opportunities: If other creditors have high valuation, the potential for free–riding is reduced, so creditor i has to contribute more to debt forgiveness. With common values, however, high reported liquidation values by the other types also induce creditor i to revise his estimate upwards, reducing the contribution to debt forgiveness. In general, either effect can prevail. In fact, $\hat{\omega}^i$ may not be monotonic in x_{-i}. This affects the equivalence result in Proposition 3.

Recall that the payment in the indirect mechanism of Proposition 3 was

$$\hat{w}^i = \hat{W}(x^i) = \int_{\Gamma(x_{-i})} \hat{\omega}^i(x^i, x_{-i})\, f(x_{-i} \mid \Gamma(x_{-i}))\, dx_{-i}.$$

This function may not be monotonic in x^i, if low $\hat{\omega}^i(\cdot, \cdot)$ are not necessarily associated to large x_{-i}, and therefore, it cannot be inverted to express the continuation rule $\hat{p}$ in terms of the amounts tendered h^i (which are monotonic functions of the w^is). Hence, the equivalence between exchange offers and the optimal direct mechanism breaks down.

If debtholders do not assign much weight to the forecast of the others, it is more likely that the optimal restructuring procedure can be replicated by an exchange offer. Consider the following example. As in Section 3.1, let $N = 2$, and let x^i be uniformly distributed on [0, 1]. Furthermore, assume that the revision functions are of the form:

$$e^j(x^j) = \alpha\left[x^j - \frac{1}{2}\right],$$

where α is a constant. If $\alpha < 1$ a creditor gives less weight to the estimate of his opponent, and vice versa. In this case, the optimal continuation rule is to set $p = 1$ if

$$C \geq \left[\frac{1+\alpha}{2}\right](x^1 + x^2) - \frac{\alpha}{2},$$

hence

$$\tilde{x}(x^j) = \left[\frac{2}{2+\alpha}\right]\left[C - \frac{\alpha}{2}\right] - x^i.$$

The payment to debtholder 1 under the direct mechanism is

$$\hat{\omega}^1(x^1, x^2) = (2)^{-1}\,[\tilde{x}(x^2) + e(x^2)] = \left[\frac{2\,C + \alpha}{4 + 2\,\alpha} - \frac{\alpha}{4}\right] - \frac{x^2}{2}(1-\alpha).$$

When creditors give the same weight to other creditors' estimate ($\alpha = 1$), the free–riding effect and the estimate–revision effect exactly offset each other, and the payment is constant. But then $\hat{W}(x^i)$ is also constant in x^i, and it cannot be inverted.

5. Repeated Offers

In the previous sections we assumed that management could make take–it–or–leave–it offers. In practice, however, offers are often repeated, therefore a more realistic approach should try to take this option into account.

The extension of the general static model to a multiperiod framework is not straightforward. First, one has to make appropriate assumptions as to what actions are observable by the various agents. If all actions are observable, then incentives no to play according to fully revealing strategies in the early stages are large, and pooling behavior tends to prevail. Hence, the equilibrium of the constituent game of a multiperiod game may be very different than the equilibrium of a one–shot game. The literature on the "ratchet effect" studies this problem.

If, instead, information is only partially revealed (for instance, agents can observe only the sum of the amounts tendered), another difficulty arises. The posterior beliefs of agent i over x_{-i} depend on x^i. Consider, for instance, the case of N = 3. In the first round, the exchange offer fails, revealing that $x^1 + x^2 + x^3 \geq K$. Now agent 1 infers that $x^2 \geq K - x^1 - x^3$, where $x^3 \in [\underline{x}, \bar{x}]$. Expected utility of agent 1 in the second stage reads as

$$U(x^1, \hat{x}^1, \omega^1, p) = \int_{\underline{x}}^{\bar{x}} \int_{(K-x^1-x^3)}^{\bar{x}} p(\hat{x}^1, x^2, x^3) \left[\omega^1(\hat{x}^1, x^2, x^3) - \frac{x^1}{N} \right] f(x^2)\, dx^2\, f(x^3)\, dx^3.$$

Since the lower bound of integration is a function of x^1, interim utility is no longer linear in the "true" type, and standard methods cannot be used to characterize the solution.

5. 1. An "All–or–Nothing" Example

To shed some light on the implications of repeated offers, we analyze a two–period example with two debtholders, whose strategies are constrained to be of the 'all of nothing' type, that is exchange either zero or their entire portfolio. The example shows that the strategies of debtholders evolve due to the information that is revealed in the first round of bidding. Therefore, an offer that fails in the first round can succeed in the second, even if the terms are unaltered. The second result is that the probability of success is higher in the two–stage game than in the one–shot version. Hence, the ability of repeating the offers can reduce losses in ex–post efficiency[9].

The One–Shot Game

Let b denote the value of the new claim, and suppose that the minimum tendering requirement is 50%. Each creditor owns half of the debt, so that participation of one creditor is sufficient for the offer to succeed. It can be shown that the equilibrium strategies are of the form: Tender if and only if the liquidation value is $x^i \leq \tilde{x}$, where $\tilde{x}$ is the indifferent type, i.e. the cut–off liquidation value (Fudenberg and Tirole, 1991, p. 333). We will only study symmetric equilibria. To determine the value of $\tilde{x}$, consider that the payoff to a player x^i from tendering is b with probability one, while the payoff for not tendering is $d\ F(\tilde{x}) + [1 - F(\tilde{x})]\ x^i$. Hence, $\tilde{x}$ is the solution to the equation $b = (d - \tilde{x})\ F(\tilde{x}) + \tilde{x}$, i.e. of

[9]This game has several features of the game of repeated provision of a public good studied by Fudenberg and Tirole (1991, Example 8.3). However, here the good is provided only once, and the offer is extended only if it fails in the first round. This differences are important. Fudenberg and Tirole use their example to show how repetition may reduce efficiency, because players initially bid less to create a reputation for being high–cost. On the other hand, we find that repetition, by allowing information transmission, improves efficiency.

$$F(\tilde{x}) = \frac{b - \tilde{x}}{d - \tilde{x}}.$$

The Two–Stage Game

Let us assume that the first stage strategies of the repeated game have the cutoff property, and let $\tilde{x}_1$ be the cutoff type of creditor in the first stage. If the offer fails in the first round, than the posterior cumulative distribution of player i about the type of player j is

$$\frac{F(x^j) - F(\tilde{x}_1)}{[1 - F(\tilde{x}_1)]}.$$

Let $\tilde{x}_2$ be the cutoff point at the second stage, and assume that $\tilde{x}_2 \geq \tilde{x}_1$. The payoff from tendering at the second stage is b for sure, while the payoff from not tendering is

$$V_2(x^i, \tilde{x}_1) = d\left[\frac{F(\tilde{x}_2) - F(\tilde{x}_1)}{[1 - F(\tilde{x}_1)]}\right] + x^i\left[1 - \frac{F(\tilde{x}_2) - F(\tilde{x}_1)}{[1 - F(\tilde{x}_1)]}\right].$$

Therefore, the cutoff type in the second stage satisfies the equation: $b = V_2(\tilde{x}_1, \tilde{x}_1)$, i. e.

$$\frac{b - \tilde{x}_2}{d - \tilde{x}_2} = \frac{F(\tilde{x}_2) - F(\tilde{x}_1)}{[1 - F(\tilde{x}_1)]}. \tag{10}$$

Now, to see that indeed $\tilde{x}_2 \geq \tilde{x}_1$, note that the LHS of equation (10) is monotone decreasing function of $\tilde{x}_2$, which is positive for $\tilde{x}_2 < b$, and zero when $\tilde{x}_2 = b$. On the other hand, the RHS of (10) is an increasing function of $\tilde{x}_2$, taking the value of zero at $\tilde{x}_2 = \tilde{x}_1$, and being positive for $\tilde{x}_2 > \tilde{x}_1$. Hence, the point at which the two function cross is larger than $\tilde{x}_1$ if and only if $\tilde{x}_1 < b$. But this must be the case: A creditor for which b is lower than the liquidation value strictly prefers not to exchange, hence for $\tilde{x}_1$ to be indifferent it must be $\tilde{x}_1 \leq b$. These results straightforwardly extend to the case in which the second period claim has a higher face value than the first period claim.

Result 1. In equilibrium, an offer that is rejected in the first round may be accepted in the second round.

After observing an initial failure, each debtholder is more pessimistic about the other type, and this induces them to rely less on free–riding. We still need to derive the value of $\tilde{x}_1$, the first–period cutoff point in the two–stage game. As before, this value is the liquidation value that makes a creditor indifferent between tendering and not tendering, hence it is the solution to

$$b = d\, F(\tilde{x}_1) + \delta\, V_2(\tilde{x}_1, \tilde{x}_1)\, [1 - F(\tilde{x}_1)],$$

where δ is the discount factor, used by debtholders to discount the future. Since we already know that $\tilde{x}_2 > \tilde{x}_1$, it is clear that a player of type $\tilde{x}_1$ is sure to tender in the second period. Hence, $V_2(\tilde{x}_1, \tilde{x}_1) = b$, and the first–period cutoff point is the solution to

$$F(\tilde{x}_1) = \frac{(1 - \delta)\, b}{d - \delta\, b}.$$

Notice that for all $b < d$, $\tilde{x}_1 \in [\underline{x}, \bar{x}]$, and that the probability that the offer succeeds in the first round is positive as long as $\delta < 1$, i.e. waiting is costly. To show that the ability to repeat the offer increases the probability of success, it is sufficient to show that $\tilde{x}_2 > \tilde{x}$. To see this, notice that $F(x) \geq \frac{F(x) - F(\tilde{x}_1)}{[1 - F(\tilde{x}_1)]}$ (with strict inequality for all $x < \bar{x}$). Therefore, F(x) intersects the function $\frac{b - x}{d - x}$ below $\tilde{x}_2$ (see Figure 1).

Result 2. The probability of success of the repeated offer is larger than that of a one–shot offer with the same terms.

A final remark concerns the role of the discount factor δ, which can be interpreted as the cost of waiting. It can be shown that when $\delta < \frac{\tilde{x}}{b}$, $\tilde{x}_1 > \tilde{x}$, i.e. in the first stage of a repeated offer debtholders are more likely to tender than they are in a one–shot offer, and vice versa. This is because the cost of waiting is high when δ is large.

6. Conclusions.

Rather than summarizing the results, we here briefly discuss some open issues, and some directions for future reasearch, that are suggested by our approach.

As for the case of public goods, the results from the direct mechanism indicate that there can be large inefficiencies from renegotiating with a large number of creditors. We have not, however, addressed the issue whether these inefficiencies can be reduced if firm liabilities can be traded. Also, inefficiencies could be reduced by selling the liabilities to specialized agencies, thereby reducing the number of creditors.

Considerations of ex–post efficiency, however, should not mislead judgement about the ex–ante optimality of firm capital structure. The decision of how to raise capital, whether through debt or equity, or through widely–held debt as opposed to bank loans or private placements, is an endogenous decision, that is made to maximize the firm's ex–ante profits. As shown in Detragiache (1992), ex–ante it may be optimal to raise some funds through a security that is costly to renegotiate ex–post.

The study of a repeated exchange offer game, shows that dynamic renegotiation procedures are very different from static ones. Because information is revealed through time, the inefficiencies due to asymmetric information can be at least partially overcome. This is a new area of research, and we have only provided an example of a multiperiod game. Further work in this area is needed to obtain more general results.

Finally, in the model studied here all creditors are assumed to have equal priority. Extending the model to allow for creditors with different seniority status, or may be secured, would bring further insights on the nature of the conflicts among claimants that hinder efficient renegotiation.