Bertrand Vs. Cournot: An evolutionary approach

Pier Luigi Sacco
Department of Economics
European University Institute
Badia Fiesolana
50016 S. Domenico (Fi)
Italy
and
Department of Economic Sciences
University of Florence
Via Curtatone, 1
50100 Firenze
Italy

and

Carlo Scarpa
Department of Economic Sciences
University of Bologna
Piazza Scaravilli, 2
40100 Bologna
Italy

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Send correspondence to: Pier Luigi Sacco, Dept. of Economics, EUI, Badia Fiesolana, Via dei Roccettini, 9, 50016 S. Domenico (Fi), Italy.

In this paper we study an oligopoly game with a differentiated product using a dynamic evolutionary approach. Firms are allowed to choose between quantity setting and price setting behavior. We find that, under both classical interaction structures, namely 'random mating' and 'playing the field', quantity setting behavior (i.e., 'Cournot' behavior), is selected as an asymptotically stable state for the dynamics. *Journal of Economic Literature* Classification Numbers: C79, D43.

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1. Introduction.

A long debated problem in industrial economics is the choice of the strategic variable by an oligopolist who could set either his quantity or his price. The initial contributions by Cournot and Bertrand have recently been followed by a number of important papers, in the attempt to identify a criterion for the selection of one possible strategy. For instance, Singh and Vives (1984) consider a model with substitute goods in which firms can precommit themselves to the use of a certain strategic variable before playing the market game, showing that Cournot (quantity–setting) behavior prevails in equilibrium. On the other hand, Klemperer and Meyer (1986) introduce uncertainty, showing that the equilibrium behavior depends on the type of uncertainty observed [1].

In this paper we take a different approach, in that we use the concept of asymptotic dynamic stability of an evolutive game to determine whether in the long run there is a kind of behavior which tends to prevail on the other, or whether firms of both types can coexist. More precisely, this requires assuming an initial distribution of firms, some of which play á la Cournot, and some of which play á la Bertrand in a market for a differentiated product. Firms tend to change their behavior over time, choosing the strategy that guarantees larger profits. The main finding of the paper, obtained in two different contexts, is that Cournot behavior is the one that tends to be selected on evolutionary grounds.

Our paper can be related to a new and growing stream of game–theoretic literature, which can be synthetically labelled as the 'evolutive' approach, as opposed to the traditional eductive approach (Binmore, 1987). This approach to the dynamic analysis of economic phenomena is strictly linked to, but not quite exhausted by, recent developments in the field of theoretical biology [see among others Selten (1983), Samuelson (1987), (1991), Vega Redondo (1988), Nachbar (1990), Friedman (1991)].

The paper is organized as follows. Section 2 introduces the standard oligopoly model we adopt, which for expositional convenience is discussed in the case of a small number of firms (i.e., under a 'playing the field' interaction structure). Section 3 analyzes its dynamic properties. Section 4 goes along the same lines in the case of a large number of firms (which defines the 'random mating' interaction structure). Section 5 concludes the paper.

2. The 'playing the field' model.

Consider a market for a differentiated product where $N \geq 3$ oligopolistic firms compete. We assume that the number of firms is small enough that firms are able to identify all of their potential opponents [2]. The demand function for the i–th firm ($i = 1, \ldots, N$) is given by

$$x_i = a - bp_i + \sum_{j \neq i} sp_j \tag{1}$$

where $j = 1, \ldots, N$ and $a > 0$. The parameter s, $0 \leq s < b$, denotes the degree of substitutability. The inverse demand function corresponding to (1) can be written as [see Vives (1984)]

$$p_i = \alpha - \beta x_i - \gamma \sum_{j \neq i} x_j \tag{2}$$

where

$$\alpha = \frac{a}{b - (N-1)s}$$

$$\beta = \frac{b}{b^2 - (N-1)^2 s^2}$$

$$\gamma = \frac{s}{b^2 - (N-1)^2 s^2}$$

Clearly, this requires

$$b > (N-1)s \tag{3}$$

The cost function is linear and there are no fixed costs, that is $C_i = cx_i$. However, it is well known that the linear structure of the model entails no serious limitations (Singh and Vives, 1984).

We turn now to the characterization of the behavior of firms. Firms maximize profits. To this end, they may fix either quantity (Cournot behavior) or price (Bertrand behavior). If a firms sets its quantity, it maximizes

$$\pi_i = (\alpha - \beta x_i - \gamma \sum_{j \neq i} x_j - c) x_i \tag{4}$$

In a symmetric equilibrium, the profit maximizing quantity is

$$x^C = \frac{\{a - c[b-(N-1)s]\}[b+(N-1)s]}{2b+(N-1)s} \tag{5}$$

If a firm sets its price, it maximizes

$$\pi_i = (p_i - c)(a - bp_i + \sum_{j \neq i} sp_j) \tag{4'}$$

In a symmetric equilibrium, the profit maximizing price is given by

$$p^B = \frac{a+bc}{2b-(N-1)s} \tag{6}$$

We want to analyze a situation in which firms of both types coexist in the market. Assume therefore that there are n Cournot players and $N - n \equiv k$ Bertrand players. Without loss of generality, we identify the first n firms as the Cournot players, and the last k firms as the Bertrand players. Unlike the pure cases in which there are players of one type only, here the characterization of the behavior of firms may be problematic. If we tackle the issue from an eductive point of view, we should conclude that firms are playing different, and mutually inconsistent, games. In fact, the very definition of an equilibrium becomes problematic when firms are considering different strategy spaces [3].

This difficulty vanishes when we consider the issue from an evolutive point of view. In an evolutionary game, possible behavioral types are identified *a priori* by the available strategies; therefore, strategies *must* be different across behavioral types. The problem now becomes rather that of understanding the conditions under which each player embraces a given behavioral type, i.e., decides to play in a certain way. In our case, we assume that two different behavioral types coexist in the market: a Cournot type player, that sets a quantity $x = x^C$, and a Bertrand type player which sets his price at the level $p = p^B$.

This characterization of behavioral types may be considered too simplistic. In particular, we are assuming that each player keeps his strategic variable at the equilibrium level of the pure case even when facing players of a different type. There is no particular presumption of rationality behind this assumption, in line with the evolutive approach, where the specification of available strategies is somewhat exogenously fixed. The advantage of paying less attention to individual choices is that we can thus concentrate on the structure of the interaction among individuals. We could of course consider more complex specifications of the available behavioral types; in this respect, the present analysis can be considered a first, and in our opinion relevant, step.

A first obvious consequence of the above assumptions is that the 'equilibrium' will be asymmetric [4]. Let us therefore consider each behavioral type separately. As to Cournot players, we must determine the price at which they sell their fixed quantity x^C. We will denote this market clearing price by p^{CB}. The demand function for a Cournot player is

$$x_i^C = a - bp_i^{CB} + s\sum_{j=2}^{n} p_j^{CB} + skp^B \tag{7}$$

Exploiting symmetry of Cournot players and using (5) and (6), this expression can be solved for p^{CB}. Easy but lenghty calculations show that

$$p^{CB} = \frac{ab + c[b^2 - (N-1)^2 s^2]}{[2b + (N-1)s][b - (n-1)s]} + \frac{sk(a+bc)}{[2b - (N-1)s][b - (n-1)s]} \tag{8}$$

It is possible to show that, very intuitively, p^{CB} decreases as the number k of Bertrand players rises. Obviously, 'equilibrium' profits π^{CB} are defined as $(p^{CB} - c)x^{C}$.

We consider now a Bertrand player. His demand function is

$$p_i^B = \alpha - \beta x_i^{BC} - \gamma(k-1)x^{BC} - \gamma \sum_{j=1}^{n} x_j^C \qquad (2')$$

By taking account of this, we easily obtain the 'equilibrium' quantity x^{BC} for the Bertrand firms as

$$x^{BC} = \frac{a[b+(N-1)s]}{b+(k-1)s} - \frac{[b^2-(N-1)^2s^2]p^B + snx^C}{b+(k-1)s} \qquad (9)$$

where x^C and p^B are defined, respectively, in (5) and (6). It is interesting to notice that the 'equilibrium' quantity for Bertrand firms x^{BC} decreases when the number of Bertrand players rises. The reason is that, as already noted, when k increases the 'equilibrium' price for Cournot firms is reduced [5].

The profit for Bertrand type firms is $\pi^{BC} = (p^B - c)x^{BC}$.

3. Replicator dynamics for the 'playing the field' model.

In the previous section we have considered a one–shot formulation of the oligopoly game. In this section we want to analyze the changes over time in the distribution of firms across the two behavioral types as the relative performance of the two available strategies is observed. In other words, we address the following question. Given an initial distribution of firms across the two behavioral types, we want to determine whether firms maintain their initial behavior when time goes on, or rather switch to the alternative behavioral type. In this way, we can ask whether there is an invariant stationary distribution of behavioral types that attracts the economy, i.e., a distribution which is an asymptotically stable stationary point for the market dynamics. In particular, it will be interesting to characterize such distribution(s), and to see whether Bertrand and Cournot type firms can coexist in the market in the long run. If this is not the case, we shall ask whether a type tends to prevail on the other, and which one.

To this end, we shall make use of a dynamic model which has found a wide range of applications in the literature on evolutionary games (Nachbar, 1990). This model consists of the specification of a dynamical system, known as replicator dynamics, in which the distribution of the various behavioral types tends to change according to the relative performance of the corresponding strategies. In particular, it is assumed that the change in the number of individuals embracing a given behavioral type is continuously proportional to the number of individuals embracing that type [6]. The factor of proportionality depends on the performance of the corresponding strategy w.r.t. the average performance. It must be emphasized that the relative performance of the various behavioral types depends in turn on the number of individuals embracing each type. This makes the replicator dynamics nonlinear in the distribution of behavioral types.

In our case, the relative performance is given by the profit differential $\pi^{CB} - \pi^{BC}$. The interpretation of the replicator dynamics is therefore as follows. The number of firms which decide to play a certain strategy increases whenever playing that strategy has yielded above average profits. More specifically, with two behavioral types, the number of firms which adopt a certain strategy increases if and only if that strategy guarantees larger profits. Note however that even if a strategy yields larger profits given a certain distribution of firms across behavioral types, it does not necessarily follow that this strategy keeps on being the more rewarding as a larger and larger number of firms tends to adopt it.

In conclusion, our replicator dynamics can be written as

$$\dot{k} = k[\pi^{BC} - \frac{1}{N}(n\pi^{CB} + k\pi^{BC})] \tag{10}$$

Here we have assumed that the number of firms adopting each strategy may vary continuously. In fact, n and k can take integer values only. Following Seade (1980) we may simply interpret this procedure as taking the value of $\dot{k}$ at the relevant points of the state space.

A simple rearrangement of (10) gives

$$\dot{k} = \frac{kn}{N}(\pi^{BC} - \pi^{CB}) \tag{11}$$

Looking for a stationary distribution of behavioral types means looking for values of k such that $\dot{k} = 0$. This clearly shows that extreme distributions of behavioral types (i.e., all firms playing either Bertrand ($k = N$) or Cournot ($k = 0$)) are always stationary points for the replicator dynamics.

In general we might have more stationary points for $0 < k < N$, depending on the actual form of (11), i.e. of the profit differential as a function of k.

After some tedious algebraic manipulations it can be shown that

$$\pi^{BC} - \pi^{CB} \propto \frac{\zeta_0 + \zeta_1 k}{\xi_0 + \xi_1 k + \xi_2 k^2} \tag{12}$$

where $\zeta_0 = s(N-1)[a(4b^3 + 4b^2 s(N-1) + bs^2(N^2-1) - Ns^3(N-1)^2) + c(4b^4 + 4b^3 s(N-1) + b^2 s^2(N-3)(N-1) + bs^3(N-2)(N-1)^2 - s^4(N-2)(N-1)^3)]$; $\zeta_1 = 2s^3(a+bc)(N-1)^2$; $\xi_0 = b^2 + s^2(N-1)$; $\xi_1 = s(2b - Ns)$; $\xi_2 = s^2$.

These coefficients are always positive. This is obvious as regards ζ_1, ξ_0 and ξ_2. The positivity of ξ_1 is a consequence of (3). The parameter ζ_0 is positive iff the term in square brackets is positive, which is always the case [7]. It is easy to check that $\xi_0 + \xi_1 k + \xi_2 k^2$ is always positive for $k \geq 0$. The only possible interior stationary point could be at $k^* = -\zeta_0/\zeta_1$. Since both ζ_0 and ζ_1 are positive, k^* is not admissible as a stationary point. We have therefore proved

PROPOSITION 1. *The evolutionary game described by (11) admits as its only stationary points the distributions $k = 0$ (all players are Cournot) and $k = N$ (all players are Bertrand).*

The real issue, however, is to determine whether the stationary points are (asymptotically) stable, that is whether the market tends to approach it as time passes. Since the dynamics (12) are defined on the 2–dimensional simplex $k + n = N$, in order to reconstruct the global dynamics it is sufficient to study the local asymptotic stability of the stationary points. In view of (12),

$$\dot{k} \propto \frac{\zeta_0 + \zeta_1 k}{\xi_0 + \xi_1 k + \xi_2 k^2} \equiv F(k) \tag{11'}$$

One can prove

PROPOSITION 2. *The only asymptotically stable stationary point for (11') is $k = 0$. Therefore, in the small numbers case Cournot behavior tends to prevail on Bertrand behavior on evolutionary grounds.*

PROOF: $F'(0) \propto \zeta_1\xi_0 - \zeta_0\xi_1$. On the other hand, $F'(N) \propto \zeta_1\xi_0 - \zeta_0\xi_1 - 2\zeta_0\xi_2 N - \zeta_1\xi_2 N^2$, that is, $F'(N) < F'(0)$. Moreover, from Proposition 1 and from elementary geometrical considerations, $F'(0)$ and $F'(N)$ must have opposite signs. Therefore, $F'(0) \geq 0$. This completes the proof.

4. Replicator dynamics for the 'random mating' model.

In the previous section, we have considered the case where the number of firms in the market is small enough to justify the assumption that firms are able to identify all of their potential opponents. In this section we analyze the opposite polar case of the literature on evolutionary dynamics, known as 'random mating' or 'pairwise contests' [see Maynard Smith (1982) and also Vega Redondo (1988)]. In this case, the market is so large that firms are not able to identify all of their potential opponents. In particular, we are considering a fairly large population of firms which are assumed to match randomly two by two; firms are drawn from an uniform distribution over the population.

A possible interpretation of this mechanism is that firms need a public license to be able to sell, and a one shot license is given in each instant to two firms selected at random. An alternative rationalization could be the following. Suppose that firms can compete in a limited number (say, one) of small, identical local markets that can accommodate two firms only. Each firm chooses its market without knowing the rival' s identity, so its choice is random. Thus, each firm meets only one, randomly selected rival.

Clearly, given the stochastic nature of the mechanism, payoffs for the firms are now defined on an expected (i.e. ex ante) basis. The new model can be obtained from that of the former section by setting $N = 2$. In particular, one has

$$x^C = \frac{[a - c(b - s)](b + s)}{2b + s} \tag{5'}$$

and

$$p^B = \frac{a + bc}{2b - s} \tag{6'}$$

When a Cournot player meets a Bertrand player, his price will be

$$p^{CB} = \frac{a + sp^B - x^C}{b} = \frac{a + bc}{2b - s} + \frac{s^2[a - c(b - s)]}{b(4b^2 - s^2)} \tag{8'}$$

Not surprisingly, it is possible to check that $p^B < p^{CB} < p^C$. On the other hand, when a Bertrand player meets a Cournot player, the quantity he supplies will be

$$x^{BC} = \frac{\alpha - p^B - \gamma x^C}{\beta} = \frac{b[a - c(b - s)]}{2b - s} + \frac{s^3[a - c(b - s)]}{b(4b^2 - s^2)} \tag{9'}$$

where α, β and γ are as in (2), setting $N = 2$. Once again, one gets the rather intuitive result that $x^B < x^{BC}$: facing a less aggressive opponent, the Bertrand player is able to sell a larger quantity than in the Bertrand equilibrium.

The ex post profit level of Cournot players is $\pi^C = (p^C - c)x^C$ when the rival is another Cournot player and $\pi^{CB} = (p^{CB} - c)x^C$ when the opponent is a Bertrand player. Analogously, we can define $\pi^B = (p^B - c)x^B$ and $\pi^{BC} = (p^B - c)x^{BC}$. It is possible to prove the following:

LEMMA 1. *In the 'random mating' model,* $\pi^C > \pi^{BC} > \pi^{CB} > \pi^B$.

The proof, obtained after simple algebraic manipulations, is omitted. Notice, however, that profits in Lemma 1 are to be meant as ex post ones. Therefore, these are not the relevant payoffs for our evolutionary game.

To derive the dynamics for the 'random mating' model, we need define expected profits. If we denote by μ the initial proportion of the

Bertrand players across the population, a Cournot player' s expected profit is given by

$$E(\pi^C) = (1-\mu)\pi^C + \mu\pi^{CB} \tag{13}$$

Whereas, a Bertrand player' s expected profit is

$$E(\pi^B) = \mu\pi^B + (1-\mu)\pi^{BC} \tag{14}$$

Our replicator dynamics can thus be written as

$$\dot{\mu} = \mu\{E(\pi^B) - [\mu E(\pi^B) + (1-\mu)E(\pi^C)]\} =$$

$$\mu(1-\mu)[E(\pi^B) - E(\pi^C)] \equiv G(\mu) \tag{15}$$

Of course, $\mu = 0$ and $\mu = 1$ are stationary points of (15). We must moreover check whether there is an internal stationary point. This could be the case when $E(\pi^B) = E(\pi^C)$, i.e. when $\mu\pi^B + (1-\mu)\pi^{BC} = (1-\mu)\pi^C + \mu\pi^{CB}$. This means that if the internal stationary point exists, it is given by $\mu^* = (\pi^{BC} - \pi^C)/[(\pi^{BC} - \pi^C) - (\pi^B - \pi^{CB})]$. As it can be checked using Lemma 1, either $\mu^* < 0$ or $\mu^* > 1$, and therefore no internal stationary point exists.

This proves

PROPOSITION 3. *The evolutionary game described by (15) admits as its only stationary points the distributions $\mu = 0$ (all players are Cournot) and $\mu = 1$ (all players are Bertrand).*

Notice that this finding hinges only upon the fact that $\pi^B < \pi^{CB}$, which is quite a general result.

Proceeding as before, now we have to determine the global dynamics of (15). It turns out that

PROPOSITION 4. *The only asymptotically stable stationary point for (15) is $\mu = 0$. Therefore, in the large numbers case Cournot behavior still tends to prevail on Bertrand behavior on evolutionary grounds.*

PROOF: $G'(\mu) = (1-2\mu)[\mu\pi^B + (1-\mu)\pi^{BC} - (1-\mu)\pi^C - \mu\pi^{CB}] + \mu(1-\mu)(\pi^B - \pi^{BC} + \pi^C - \pi^{CB})$. $G'(0) = \pi^{BC} - \pi^C$, which is negative from Lemma 1. On the other hand, $G'(1) = -(\pi^B - \pi^{CB})$, which is positive from Lemma 1. This completes the proof.

Note that, from Lemma 1, when two players of different types meet, the Bertrand player is the one that makes larger (ex post) profits. On the other hand, if the evolution is driven by average (expected) profits, it is the Cournot strategy that attracts the whole population of firms. The intuition is the following. When two different players meet, the Cournot type player would not gain by changing his behavior, given the opponent's behavioral type, because $\pi^B < \pi^{CB}$. On the other hand, a Bertrand type player, although his profit is larger than his rival's, would be better off by choosing a different strategy, given that his opponent remains a Cournot player ($\pi^C > \pi^{BC}$). Notice that just these inequalities are needed to determine the signs of $G'(0)$ and $G'(1)$, and thus the result in Proposition 4.

REMARK. *The asymptotic stability of Cournot behavior could also have been proved by noting that, from Lemma 1, one has that $\mu = 0$ is an evolutionarily stable state for the 'random mating' model (whereas $\mu = 1$ is not) and by invoking the standard result that when payoffs are linear w.r.t. the distribution of behavioral types every ESS is an asymptotically stable state of the replicator dynamics [see e.g. Hofbauer and Sygmund (1988)].*

5. Conclusions.

It is interesting to stress that under both the 'playing the field' and the 'random mating' interaction structures, our analysis leads to the same result, i.e., that, Cournot behavior is always selected by our evolutionary dynamics. Obviously, our analysis is just a preliminary step. A generalized model of a population of firms including more sophisticated behavioral types could give further insights.

FOOTNOTES

[1] Kreps and Scheinkman (1983) present an interesting attempt to reconcile the two kinds of behavior. Another relevant analysis is Delbono and Mariotti (1990).

[2] This amounts to postulate a 'playing the field' interaction structure. See Maynard Smith (1982) and also Schelling (1978) for a pioneering contribution.

[3] Friedman (1988) analyzes the case where firms can choose quantity *and* price, stressing how the existence of equilibria is extremely problematic in this instance.

[4] As behavioral types are exogenously given, the word 'equilibrium' here simply means a situation of *market clearing*.

[5] Furthermore, it can be shown that the total quantity sold by Bertrand firms kx^{BC} increases with k.

[6] This rules out the possibility that at any given moment there is a massive shift of the population towards the most rewarding strategy.

[7] This term can be rewritten as: $4(a+bc)[b^3+b^2s(N-1)]+as^2(N-1)[b(N+1)-sN(N-1)]+cs^2(N-1)\{b^2(N-3)+s(N-1)(N-2)[b-(N-1)s]\}$. The first term is obviously positive. The second and the third term are both positive from (3).

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