Self-Reinforcing Mechanisms and Market Information

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Abstract

We consider the possibility of switching between two technological standards when there are network externalities and imprecise market information. Multiple equilibria in terms of market shares can arise. The main result is that lock-in to one of multiple equilibria is not a permanent outcome when the source of lock-in is network externalities. The market lingers at prevalence of one standard with intermittent transitions to prevalence of the other. In other words, lock-in is a temporary occurrence.

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1 Introduction

Path dependent dynamical systems of self-reinforcing (i.e., positive feedback) mechanisms tend to possess a multiplicity of possible asymptotic states. The initial state combined with early random events or fluctuations acts to push the dynamics into the domain of one of these asymptotic states and thus to select the structure that the system eventually locks into.

Some economic examples have recently been analysed. Lock-in may occur in the case of sequential choice between competing technologies with increasing returns to adoption [1]. If one technology gets ahead by good fortune, it gains an advantage, with the result that the adoption market may 'tip' in its favour and may end up dominated by it. Given other circumstances, a different technology might have been favoured early on and eventually dominated the market. Ordinarily, in the problem of competition between technologies with increasing returns to adoption there are multiple equilibria. As to which actual outcome is selected from the multiple candidates, it is argued that the prevailing outcome depends heavily on the initially chosen path. In particular, the resulting outcome can be inefficient, i.e., the market can be locked-in to the 'wrong' technology.

A question arising in this context is the following: If an economic system is locked-in to an inferior local equilibrium, is 'escape' into a superior one possible? Do we need policies for the economic system, or will spontaneous actions at a local level suffice?

An answer depends on the degree to which the advantages accrued by the inferior equilibrium are transferable to an alternative. Where learning effects and specialized fixed costs are the source of the self-reinforcing mechanisms, advantages are not usually transferable to an alternative equilibrium. Where coordination effects, which confer advantages from compatibility with other agents, are the source of lock-in, often advantages are transferable [11]. For example, users of a particular technological standard may agree that an alternative would be superior, provided that everybody switched. If the current standard is not embodied in specialized equipment and its advantage is mainly that of commonality of convention, then a changeover to a superior collective choice can provide an escape into the new equilibrium at negligible costs. It has been shown[3] that, in the presence of *network externalities*, i.e., when the benefits from adoption of a given technological standard increase with the number of other utilizers purchasing the same standard or compatible items (size of the network), and so long as agents know other agents' preferences, a co-ordinated changeover can occur (the band-wagon effect). However, if there is uncertainty of others' preferences and intentions, there can be *excess inertia*, resulting in firms remaining with the status quo even if they all favour switching, because they are unwilling to risk switching without being followed. In this case permanent lock-in to an inferior local equilibrium may occur.

The question about technological switching in the presence of benefits arising from network externalities seems to be particularly relevant, since the issues of compatibility and standardization have become more important than ever. This is especially true within the computer and telecommunications industries, which are characterized by urgent demands for compatibility and rapid innovation of new products and services. In some cases there can be a direct externality; the more subscribers to a given network, the greater the services provided by that network. Other examples entail indirect externalities associated with the provision of a durable good (hardware) and a complementary good or service (software). In these cases the externality arises when the amount and variety of software available increases with the number of hardware units sold. The cost that firms incur to achieve compatibility can be small, almost negligible compared with the benefits to be gained from compatibility [4]. This is especially true when it is possible to use standardized interfaces, by which we mean that each firm produces according to its own specifications, but that the products of different firms may use the same software or be capable of communicating with one another.

In the following pages we consider the problem of technological switching in the presence of network externalities by taking advantage of certain approximation results in the theory of population processes. Following a common approach in stochastic modeling, we will represent the actions of a firm by a random process in order to draw inferences about the 'macroscopic' behaviour of the system, which will be almost deterministic. Hence we will let transitions between technologies occur as a Markov process, with transition intensities depending on the market shares of each technology, i.e., on their *network*. This allows for self-reinforcement.

The remainder of this paper is organised as follows: In section 2 we construct a stochastic model for the decision rule of the firms, and derive a tractable approximation. Our main result is contained in section 3 where we define the stochastic attractiveness of each standard, and show how these determine the number and position of market equilibria. We find that permanent lock-in to one of multiple equilibria does not occur when the source of lock-in advantages is network externalities. Instead the system lingers at prevalence of one standard, with intermittent transitions to prevalence of the other. We conclude with section 4, where large deviations methods are used to calculate the sojourn time at one of a number of possible system equilibria.

2 A Stochastic Model

We will consider the case of two technological standards, denoted '0' and '1', which are substitutable. There are N firms in the industry which have to decide whether to stay with their present standard, or switch to an alternative, given that there are benefits to be obtained from compatibility. We assume that this is a market-mediated effect, i.e., a complementary good becomes cheaper and more readily available the greater the extent of the compatible market. Benefits from compatibility then arise if firms are able to exploit economies of scale in using a common supplier of a complementary good.

Our hypothesis is that the firms move between technologies according to identical (or at least very similar) rules. If $n(t) \in \{0, \ldots, N\}$ is the number of firms with standard '1', then since the remaining $N - n$ firms must have standard '0', n suffices as a description of the industry as a whole. In what follows it will be more convenient to deal in proportions and so set $z_N \equiv n/N$ as the proportion of the N firms which have standard '1'.

If the standard employed a firm is denoted j then, assuming benefits from compatibility, the firm has an action rule $A(j, z_N) \in \{S, M\}$ where S is 'stay with the current standard' and M is 'move to the other standard' respectively, of the form

$$A(j, z_N) = \begin{cases} S & \text{if} \quad (j = 0, z_N \leq c_0) \quad \text{or} \quad (j = 1, z_N \geq c_1) \\ M & \text{otherwise} \end{cases} \tag{1}$$

where the thresholds c_0 and c_1 may differ due to cost differentials incurred in switching standards. As an example, if we suppose that firms seek to maximise their one-period profit, where $p_j(z_N)$ is the price charged by the common supplier for one period consumption of standard j, and γ_j is the cost to switch from standard $1-j$ to standard j (which must be sufficiently small), then

$$A(j, z_N) = \begin{cases} S & \text{if} \quad p_j(z_N) \leq p_{1-j}(z_N) + \gamma_{1-j} \\ M & \text{otherwise.} \end{cases} \tag{2}$$

While definitive, the rule (1) (i) supposes that all firms have perfect information as to the state of the industry, and (ii) if all firms are interchangeable, results in lock-in to a local equilibrium. Farrell and Saloner [3] dealt with the second problem by allowing firms to have a preference, which was known to all the remaining firms. This allowed for a coordinated changeover to a superior collective choice (the band-wagon effect). We will show that supposing that firms have imprecise information about the market position is equivalent to a stochastic formulation, and that this allows for 'escape' when multiple equilibria are possible.

For a single firm we now want to represent the decision process (or more accurately the actions resulting from it) by a stochastic process, specifically a two-state (indexed by standard) Markov chain in continuous time. Let the transition intensities (see e.g., [14, page 41]) for the transitions '0'→'1' and '1'→'0' be

$$q(0,1) \equiv \lambda(z_N) \qquad q(1,0) \equiv \mu(z_N). \tag{3}$$

We will now show that this stochastic formulation is equivalent to operating the decision rule (1) on the basis of imprecise knowledge of z_N. Suppose that a firm is in state '0', then formally, in a small time interval Δt (identical for all firms), in which a firm makes exactly one decision,

$$\text{Prob}(S) = 1 - \lambda(z_N)\Delta t + o(\Delta t), \qquad \text{Prob}(M) = \lambda(z_N)\Delta t + o(\Delta t). \tag{4}$$

Observe that this defines a dimensioning relationship between the decision interval and the transition intensities.

If we define a correct action by the rule (1) the stochastic firm can make two types of error, namely $\{M \mid A = S\}$ and $\{S \mid A = M\}$. Let us denote the probability of an error by ε_0, then from (4) we obtain

$$\varepsilon_0(z_N) = \begin{cases} \lambda(z_N)\Delta t & z_N \leq c_0 \\ 1 - \lambda(z_N)\Delta t & z_N > c_0. \end{cases}$$

This is equivalent to operating the decision rule (1) with the 'perfect information' z_N replaced by 'imperfect information' in the form of a random variable X with distribution given by

$$\text{Prob}(X \le x) = \begin{cases} \varepsilon_0(c_0 - z_N + x) & 0 \le x \le z_N \\ 1 - \varepsilon_0(c_0 - z_N + x) & z_N < x < 1 \\ 1 & x = 1. \end{cases}$$

A similar random variable Y can be defined for a firm in state '1', with analogous dependence on $\mu(z)$.

We will now note some properties of the transition intensities (3).

(A1) For technical reasons we will require that λ and μ be Lipschitz continuous. Formally, $\lambda(\cdot)$ and $\mu(\cdot)$ are only defined on the points $\{z_N\} = \left\{0, \frac{1}{N}, \ldots, \frac{N-1}{N}, 1\right\}$, so we will take $\lambda(z)$ and $\mu(z)$ to be Lipschitz continuous functions through the points $\{\lambda(z_N)\}$ and $\{\mu(z_N)\}$.

(A2) Since firms benefit from increasing compatibility we have

$$\frac{\partial}{\partial z}\mu(z) < 0 < \frac{\partial}{\partial z}\lambda(z).$$

(A3) We will assume that

$$\lambda(0) > 0, \qquad \mu(1) > 0,$$

i.e., that there exists a positive probability of switching from a universally operated standard. This can be identified as error and/or a willingness on the part of firms to experiment with new technology.

Obviously $n(t)$ is a birth-death process, hence is reversible, with an easily obtained (static) equilibrium distribution [8, pages 10–14]. However, this result is not amenable to interpretation and worse, tells us nothing about the dynamics of the process, which are the object of interest. Further, the birth-death process solution is impossible to implement in the case of more than two standards, while the method we will propose can be extended, although some numerical computation may be required.

The process $z_N(t)$ is a Markov process with generator [2],

$$\mathcal{L}_N f(z) = N(1-z)\lambda(z)\left[f\left(z+\frac{1}{N}\right)-f(z)\right] + Nz\mu(z)\left[f\left(z-\frac{1}{N}\right)-f(z)\right],$$

which is very nearly first order for large N. Formally, Taylor's expansion yields

$$\mathcal{L}_N f = a(z)\frac{\partial}{\partial z}f + \frac{1}{2N}b(z)\frac{\partial^2}{\partial z^2}f + O\left(\frac{1}{N^2}\right), \tag{5}$$

where

$$a(z) = (1-z)\lambda(z) - z\mu(z), \qquad b(z) = (1-z)\lambda(z) + z\mu(z).$$

Now observe that the operator $a(z)\partial/\partial z$ is also the generator of a Markov process, specifically the (unique) deterministic process $z(t)$ which solves the ordinary differential equation

$$\frac{d}{dt}z = (1-z)\lambda(z) - z\mu(z), \qquad z(0) = z_N(0). \tag{6}$$

We then have the result that the continuous flow approximation $z(t)$ is asymptotically close to the stochastic process $z_N(t)$.

Theorem 1 (Kurtz[10]) *If the transition intensities $\lambda(z)$, $\mu(z)$ are bounded and Lipschitz continuous then, for any $t > 0, \epsilon > 0$ there exist positive numbers C_1, C_2 such that*

$$\text{Prob}\left(\sup_{0\le s\le t} \mid z_N(s) - z(s) \mid > \epsilon\right) < C_1 e^{-NC_2}.$$

Corollary *For all* $t \geq 0$

$$\lim_{N\to\infty} \mathsf{E}[z_N(t)] = z(t).$$

Notwithstanding theorem 1, there exists the possibility of $z_N(t)$ making excursions far from $z(t)$, which will be examined in section 4. This is of particular interest if $z(t)$ has multiple stable fixed points, since the system may be able to 'tunnel' between them. In the main, convergence of the $z(t)$ will imply similar convergence of $z_N(t)$, and hence our problem is reduced to solving the ordinary differential equation (6).

The fixed points of (6) are the solution(s) $\bar{z}$ of the equation

$$\bar{z} = \frac{\lambda(\bar{z})}{\lambda(\bar{z}) + \mu(\bar{z})}. \tag{7}$$

If we denote the right side of this equation by $F(\bar{z})$, (A1) and (A2) imply that $F(\cdot)$ is a continuous monotonically increasing function. Thus solutions of (7) occur due to $F(z)$ alternately crossing the diagonal from above and below, and (A3) then yields that (7) has an odd number of solutions (we adopt the convention that tangential (grazing) contact is a double root where no crossing is made). These observations yield the following result concerning the stability character of the fixed points of $z(t)$.

Theorem 2 *Let the solutions of the fixed point equation (7) be* $0 < \bar{z}_0 < \bar{z}_1 < \cdots < \bar{z}_{2m} < 1$. *Then the points* $\bar{z}_{2j}$, $j = 0, \ldots, m$ *are asymptotically stable in the regions* $D_0 = [0, \bar{z}_1), \ldots, D_j = (\bar{z}_{2j-1}, \bar{z}_{2j+1}), \ldots, D_m = (\bar{z}_{2m-1}, 1]$. *The points* $\bar{z}_{2j-1}$, $j = 1, \ldots, m$ *are unstable.*

Proof. First note that in a one-dimensional system the only possible fixed points are asymptotically stable or unstable. If we take a fixed point $\bar{z}$ of (6) and perturb it we obtain the linear perturbation equation

$$\frac{d}{dt}\xi = \left((1-\bar{z})\frac{\partial}{\partial z}\lambda(\bar{z}) - \lambda(\bar{z}) - \bar{z}\frac{\partial}{\partial z}\mu(\bar{z}) - \mu(\bar{z})\right)\xi.$$

Appealing to (7) we can rewrite this as

$$\frac{d}{dt}\xi = -\left(1 - \frac{\partial}{\partial z}F(\bar{z})\right)(\lambda(\bar{z}) + \mu(\bar{z}))\,\xi.$$

Hence $\bar{z}$ is stable (unstable) if $\partial F(\bar{z})/\partial z < 1$ (> 1). Observing that $F(\cdot)$ crossing from above (below) implies $\partial F(\bar{z})/\partial z < 1$ (> 1) completes the proof. □

We will conclude this section by examining the shape of the equilibrium probability distribution. Consider (5), taking $\mathcal{L}_N$ to $O(N^{-1})$ terms, thus obtaining a diffusion approximation. Then [14, page 51] the probability density $\pi(z,t)$ of $z(t)$ obeys the Kolmogorov forward equation

$$\frac{\partial}{\partial t}\pi = -\frac{\partial}{\partial z}(a\pi) + \frac{1}{2N}\frac{\partial^2}{\partial z^2}(b\pi),$$

an equation of Fokker-Planck type, which has the stationary solution

$$\pi(z) \propto b(z)^{-1}\exp\left[2N\int_0^z \frac{a(y)}{b(y)}dy\right]. \tag{8}$$

Since

$$a(z) = (\lambda(z) + \mu(z))\,(F(z) - z)\,,$$

it is easy to see that the integral is maximised at the stable solutions of the fixed point equation (7) and minimized at the unstable solutions. Hence $\pi(z)$ is peaked (with increasing sharpness as N becomes large) around the stable fixed points of (6).

3 Pseudo Network Externality

We will now seek to quantify the effect of compatibility. In our formulation there are two components to the attractiveness of each standard. These are network externalities, and the possibility of error which is intrinsic to a stochastic formulation. The following definition provides a measure, via the transition intensities, of the 'stochastic attractiveness' of each standard in terms of these components.

Definition The *pseudo network externality* (henceforth PNE) of standard j is θ_j where

$$\theta_0 \equiv \log\left[\frac{\mu(0)}{\lambda(0)}\right], \qquad \theta_1 \equiv \log\left[\frac{\lambda(1)}{\mu(1)}\right].$$

Taking θ_0 for example, $\mu(0)$ is the maximum intensity for a transition to standard 0, while $\lambda(0)$ is the minimum intensity for a transition to standard 1, i.e., the probability intensity of making a mistake for a firm with standard 0. Thus θ_j is the difference of the logarithms of a network externalities term and an error term. For the remainder of the paper we will need the following assumption, in addition to those in section 2.

(A4) There exist constants $k_0 > 0$, $k_1 > 0$ such that

$$\frac{\partial}{\partial z}\mu(z) = -k_0\mu(z), \qquad \frac{\partial}{\partial z}\lambda(z) = k_1\lambda(z).$$

The assumption (A4) implies the identity $\theta_1 = k_0 + k_1 - \theta_0$, which together with (7) and (A4) enables us to write

$$F(z) = \frac{1}{1 + \exp[\theta_0 - z(\theta_0 + \theta_1)]}. \tag{9}$$

This has the following immediate consequence.

Theorem 3 *$F(z)$ is convex (concave) for $z < \omega$ ($z > \omega$), where*

$$\omega = \theta_0/(\theta_0 + \theta_1), \tag{10}$$

and hence the system has at most two stable equilibria.

Proof. The first assertion follows immediately from (9), thus (7) has at most three solutions by a simple convexity argument and theorem 2 does the rest. □

Corollary *If $\theta_0 + \theta_1 < 4$ then there is exactly one stable equilibrium.*

Proof. First note that if $\omega < 0$, or $\omega > 1$ then (7) has only one solution. Otherwise, by (9), (10) and the definition of ω,

$$\max_z \frac{\partial}{\partial z} F(z) = \frac{\partial}{\partial z} F(\omega) = \frac{(\theta_0 + \theta_1)\lambda(\omega)\mu(\omega)}{(\lambda(\omega) + \mu(\omega))^2} = \frac{\theta_0 + \theta_1}{4}.$$

Hence if $\theta_0 + \theta_1 < 4$, $\partial F(z)/\partial z < 1$ for all z and (7) can only have one solution. Finally, by theorem 2, a solitary fixed point is stable. □

The next result is a complement to the corollary to theorem 3. These two results together with theorem 5 demonstrate that the PNEs completely determine the number of stable equilibria.

Theorem 4 *If $\theta_0 = \theta_1$ then:*

a) *$\bar{z} = 1/2$ is a solution of the fixed point equation (7).*

b) *If* $\theta_0 + \theta_1 > 4$ *the system has two stable equilibria.*

Proof. Part a) is immediate from (9). By (10), $\omega = 1/2$, and so (see the preceding proof), $\partial F(1/2)/\partial z = (\theta_0 + \theta_1)/4$. An appeal to theorem 2 and theorem 3 completes the proof of part b). □

The interpretation of theorem 4 and the corollary to theorem 3 is straightforward. If the two standards are equally attractive then $z = 1/2$ (equal market shares) is the unique stable equilibrium provided that the network externalities are sufficiently weak relative to the probability of error. If the network externalities are strong then there are two stable equilibria, corresponding to prevalence of each of the standards. If the two standards are not equally attractive we have the following similar, but necessarily more complex, result.

Theorem 5 *If* $\theta_0 \neq \theta_1$ *then:*

a) *If* $\theta_0 > \theta_1$ $(\theta_0 < \theta_1)$*, the system has a stable equilibrium at* $\bar{z} < 1/2$ $(\bar{z} > 1/2)$*.*

b) *Let*

$$B_0 = \min_{0<\beta<\theta_1} \left[(\beta + \theta_0)(1 + e^{-\beta})\right],$$

$$B_1 = \max_{0<\beta<1} \left[\beta\theta_0(1 + e^{(1-\beta)\theta_0})\right].$$

Then the fixed point equation (7) has the following properties:

(i) *If* $\theta_0 < 0$ *or* $\theta_1 < 0$ *there is exactly one stable equilibrium.*

(ii) *If* $\min\{\theta_0, \theta_1\} > 0$ *and* $\min\{\theta_0, B_0 - \theta_0\} < \theta_1 < \max\{\theta_0, B_1 - \theta_0\}$ *then there are two stable equilibria.*

(iii) *If* $0 < \theta_1 < \min\{\theta_0, B_0 - \theta_0\}$ *or* $0 < \max\{\theta_0, B_1 - \theta_0\} < \theta_1$ *then there is exactly one stable equilibrium.*

Proof. Parts a) and b)(i) are immediate from theorem 3. Next let us suppose that $0 < \theta_1 < \theta_0$. Then, recalling (9) and (10), since $F(\omega) = 1/2$, (7) has a solution $\bar{z}_0 < \omega$. To consider the range $\omega < z < 1$, let

$$z = 1 - \left(1 - \frac{\beta}{\theta_1}\right)(1 - \omega), \qquad 0 < \beta < \theta_1.$$

Then it is easy to show that $F(z) - z > 0$ for some β (and hence (7) has solutions $\bar{z}_1, \bar{z}_2 \in (\omega, 1)$) if and only if $\theta_0 + \theta_1 > (\beta + \theta_0)(1 + e^{-\beta})$, which yields the lower bound involving B_0. Now suppose $\theta_0 < \theta_1$. Since $F(\omega) = 1/2$, (7) has a solution $\bar{z}_2 > \omega$. Again it is easy to show that $F(\beta\omega) - \beta\omega < 0$ for some $\beta \in (0,1)$ (and hence (7) has two solutions $\bar{z}_0, \bar{z}_1 \in (0, \omega)$) if and only if $\theta_0 + \theta_1 < \beta\theta_0(1 + e^{(1-\beta)\theta_0})$. This yields the upper bound involving B_1, and parts (ii) and (iii) are then consequences of theorem 2 and theorem 3. □

We will now consider the interpretation of theorem 5. Part a) simply tells us that there will be a stable equilibrium corresponding to prevalence of the more attractive standard. In part b)(i) the benefits of network externalities for the given standard are small compared with the probability of mistakenly switching, and hence the other standard will prevail. Parts b)(ii) and (iii) show that there will be two stable equilibria (see the paragraph following lemma 1) provided the stochastic attractiveness of the standards do not differ too greatly.

Finally, let us formulate a lemma which we will need in the following section. We will state and prove the result only for $\theta_0 < \theta_1$ but the obvious complement also holds.

Lemma 1 *If $\theta_0 < \theta_1$ and $\bar{z}_0 < 1/2$ solves the fixed point equation (7), then (7) has another solution $\bar{z}_2$ such that $\bar{z}_2 > 1 - \bar{z}_0$.*

Proof. By hypothesis $\bar{z}_0 = F(\bar{z}_0)$, so the lemma will be proved if $1-\bar{z}_0 < F(1-\bar{z}_0)$. From (9) this is true if and only if $\exp[\theta_0 - \theta_1] < 1$. □

Observe that lemma 1 and theorem 4 imply that if the system has two stable fixed points then one is in the interval $[0, 1/2)$ and the other in the interval $(1/2, 1]$. Thus if there is more than one stable equilibrium then each represents the prevalence of a different standard.

4 Large Deviations

In cases where more than one stable equilibria exists it is possible to obtain large deviation type estimates for the expected time taken for the system to make a transition between them. Following [13] the expected time taken to go from a stable equilibria $\bar{z}$ to an unlikely point η is $\exp[NI + o(N)]$ where I solves the variational problem

$$I = \inf_S \int_{t_1}^{t_2} h\left(z(t), \frac{d}{dt}z(t)\right) dt,$$

where

$$h(z, \alpha) = \sup_\alpha \left\{\alpha z - (1-z)\lambda(z)[e^\alpha - 1] - z\mu(z)[e^{-\alpha} - 1]\right\}$$

and $S = \{t_1, t_2, z(t) : z(t_1) = \bar{z}, z(t_2) = \eta\}$ is the set of all paths from $\bar{z}$ to η. For our model the solution takes the form

$$I = \int_{\bar{z}}^{\eta} \log\left[\frac{z\mu(z)}{(1-z)\lambda(z)}\right] dz,$$

which section 3 allows us to calculate explicitly as

$$I = (\eta - \bar{z})\theta_0 - (\eta^2 - \bar{z}^2)\frac{\theta_0 + \theta_1}{2} + \log\left[\eta^{\eta}(1-\eta)^{(1-\eta)}\right] - \log\left[\bar{z}^{\bar{z}}(1-\bar{z})^{(1-\bar{z})}\right]. \quad (11)$$

From theorem 2 we know that any two stable equilibria are separated by an unstable equilibria, which we can take as an unlikely point (from which convergence to a stable point is exponentially fast). If $\bar{z}_0, \bar{z}_1, \bar{z}_2$ are stable, unstable and stable respectively then the above theory enables us to express the system dynamics in the form of the 'transition intensities'

$$q_N(\bar{z}_0, \bar{z}_2) \simeq \exp\left[-N\int_{\bar{z}_0}^{\bar{z}_1} \log\left[\frac{z\mu(z)}{(1-z)\lambda(z)}\right] dz\right],$$

and

$$q_N(\bar{z}_2, \bar{z}_0) \simeq \exp\left[-N\int_{\bar{z}_2}^{\bar{z}_1} \log\left[\frac{z\mu(z)}{(1-z)\lambda(z)}\right] dz\right].$$

Treating this as two-state Markov process the relative likelihoods of the two system equilibria are given by

$$\frac{\pi_N(\bar{z}_2)}{\pi_N(\bar{z}_0)} \simeq \exp\left[N\int_{\bar{z}_0}^{\bar{z}_2} \log\left[\frac{(1-z)\lambda(z)}{z\mu(z)}\right] dz\right]. \quad (12)$$

With obvious modifications the same procedure is applicable to any number of stable equilibria.

We can now examine the effects of PNEs for large N. Again we will state and prove our result only for $\theta_0 < \theta_1$, but again the obvious complement holds.

Theorem 6 *Suppose the system has two stable equilibria* $\bar{z}_0 < 1/2 < \bar{z}_2$. *If* $\theta_0 < \theta_1$ *then*

$$\lim_{N\to\infty} \frac{\pi_N(\bar{z}_2)}{\pi_N(\bar{z}_0)} = \infty.$$

Proof. Consider (12). The theorem will follow from positivity of

$$I_N = \int_{\bar{z}_0}^{\bar{z}_2} \log\left[\frac{(1-z)\lambda(z)}{z\mu(z)}\right] dz = \int_{\bar{z}_0}^{\bar{z}_2} (\theta_0+\theta_1)z - \theta_0 + \log\left[\frac{1-z}{z}\right] dz.$$

From lemma 1, $1-\bar{z}_0 < \bar{z}_2$ and it can be shown that the integrand is positive over the interval $(1-\bar{z}_0, \bar{z}_2)$. Hence

$$I_N > \int_{\bar{z}_0}^{1-\bar{z}_0} (\theta_0+\theta_1)z - \theta_0 + \log\left[\frac{1-z}{z}\right] dz = \frac{\theta_1-\theta_0}{2}(1-2\bar{z}_0) > 0.$$

□

Thus the predominant equilibria in a large system is that which corresponds to a larger market share of the standard with greatest pseudo network externality.

5 Conclusion

In this paper we have studied the possibility of switching between standards in the presence of network externalities and imprecise market information. We considered a model which allows recontracting within the market once it has formed and where the transition probabilities between standards for each firm depend on the market share of each standard, inducing self-reinforcement. For the case of two alternative standards the relative asymptotic likelihoods of the equilibria can be calculated. The main result (section 3) is that if the network externalities are sufficiently strong, the switching costs sufficiently small, and both effects relatively balanced over the two standards, then permanent lock-in to one market position is not possible. Instead the market makes intermittent transitions, after a sojourn time whose mean increases exponentially with the number of firms, between prevalence of each standard.

A few remarks are in order. In this paper we considered unsponsored technological standards, i.e., standards that cannot be priced and manipulated. If competing standards are sponsored or proprietary, their sponsors may compete fiercely to have them adopted as the *de facto* standard, so that sellers may engage in strategic pricing or cross-subsidization between early and late users. In early periods competition may be very good for buyers. However, once one standard has 'won', the proprietary de facto standard may become a source of monopoly power [5]. In these circumstances, exit from lock-in to a market position becomes more and more difficult.

Moreover, when network externalities are embodied in an installed base (see [6,7]), an early start or a protected market could in principle lead to a lasting competitive advantage. In this case substantial changeover costs of switching from one standard to another are likely to be incurred [9], and can prevent escape from a lock-in position.