

**THE CONTESTABILITY PARADIGM IN  
THE PRESENCE OF VERTICAL  
DIFFERENTIATION: ENTERING THE  
INCUMBENT'S QUALITY NICHE**

**LUCA LAMBERTINI**

**No 135**

**The Contestability Paradigm  
in the Presence of Vertical Differentiation:  
Entering the Incumbent's Quality Niche\***

Luca Lambertini  
Dipartimento di Scienze Economiche  
Università degli Studi di Bologna

April 1992

*Abstract*

*The aim of this paper is to analyse the interaction between vertical differentiation and contestability in the context of a repeated game with discounting. It is shown that, if consumers retaliate after a hit-and-run entry and suppliers have reasonably low discount rates, contestability may not hold.*

*\* I am extremely grateful to Patrizio Bianchi, Roberto Cellini, Roberto Censolo, Flavio Delbono, Paolo Onofri, Gianpaolo Rossini and Carlo Scarpa for fruitful discussions and most valuable comments on an earlier draft. I am also greatly indebted to Michele Burattoni and Dario Sermasi for their computational help. The usual disclaimer applies.*

## Introduction

In a previous paper (1991), we tackled the issue of contestability in the context of vertical differentiation, suggesting that contestability vanishes since product differentiation gives rise to a mechanism of self-selection among consumers.

The final outcome is that the market looks like a natural oligopoly, with a limited number of firms (possibly two) selling vertically differentiated products and enjoying positive profits at equilibrium.

The possibility of entry with the same quality as the incumbent's remains to be investigated: that is, potential entrants still have the chance of contesting every single niche in the quality spectrum. We are now going to show that, under some additional and very reasonable assumptions about consumers' information, contestability is not likely to emerge as a reasonable outcome in the repeated game with low discounting, though it is a possible outcome for the one-shot game.

The paper is structured as follows: in the first three sections, we summarize the framework adopted in the previous one and its main results; in the fourth, we compare the profits gained by an incumbent who sells a good of the highest quality to those gained by the same producer in duopoly; then, in the fifth section we assess the possibility of contestable entry by a competitor adopting a *hit-and-run* strategy in a repeated game with discounting.

## 1. Contestability and Natural Oligopoly as a reinterpretation of Bertrand's competition

The Contestability Theory may be considered as a rediscovery of the classic concept of competition, revisited under a perspective *à la Bertrand*.<sup>1</sup>

The distinctive feature of the contestability approach is to be identified in the fact that potential freedom of entry is strengthened by specific conditions (no sunk costs and possible supernormal profits) which render such entry attractive and, thus, effective. This leads the market to reach an efficient configuration. Thus, contestability looks like a reference paradigm as regards efficiency and welfare, and this holds for oligopolistic and monopolistic outcomes as well.

When natural oligopolies are taken into consideration, the Bertrand strategy in prices is coupled to competition in quality, which radically modifies the outcome reached by such a model.<sup>2</sup> In this sense, the Natural Oligopoly Theory may be seen as a possible solution to the Bertrand paradox.<sup>3</sup>

Thus, both approaches may appear as a reinterpretation of Bertrand's competition; yet, while the former gravitates around the pole of perfect competition, the latter reaches the monopolistic outcomes implicitly brought about by a price strategy *à la Bertrand*.

Our aim is to integrate these perspectives. This amounts to saying that we must put into question the robustness of contestability as against a particular class of oligopolies and, at the same time, the reliability of the criteria usually adopted to assess the efficiency and the social welfare (i.e., the desirability) of a specific market configuration.

We may first ask ourselves if the contestability concept can resist to structural changes that may be extremely far from any technological consideration. In other words, technological contestability is a **necessary** condition to effective contestability; the question now is: is it also (and always) a **sufficient** condition? And, if not, how does the market look like?

Within the Contestability Theory, the existence of entry barriers is linked to the presence of capital specificity and sunk costs; moreover, possible scope economies induce firms to enter contiguous industries, leaving completely out of consideration these industries' structure.

Thus, we have to investigate the changes induced in this framework by the introduction

---

1. Baumol (1982); Baumol, Panzar and Willig (1982).

2. Shaked and Sutton (1982, 1983, 1987); Sutton (1986).

3. For an assessment of Bertrand's oligopoly, see Friedman (1977, ch.3).

of the following elements:

- (a) uncertainty (ex ante, about market's contestability);
- (b) vertical (i.e., quality) differentiation;<sup>4</sup>
- (c) first mover advantage.

We will try to show that market's actual contestability cannot be assured by cost considerations alone. The observed outcome, in terms of market form, depends heavily on the qualitative as well as informational issues we just mentioned, combined with demand conditions - that is, the preference structure and the income distribution assumed for consumers.

The basic idea we will refer to, firstly developed by Shaked and Sutton (1982, 1983, 1987) and Sutton (1986), is that if we quit the horizontal differentiation model to adopt a vertical differentiation one, the 'atomistic' or fragmented outcome reached by Hotelling (1929) may be invalidated.

On the contrary, vertical differentiation would yield a natural oligopoly in which a very small number of firms (possibly two) make positive profits by selling a superior quality good,<sup>5</sup> even though they don't necessarily control a large market share.

---

4. The concept of 'quality' has been widely discussed. In a pure horizontal differentiation context, if we conceive a particular good as a bundle of characteristics, that good will be preferred because of a specific feature that the others haven't (Lancaster, 1979). This meaning of 'quality' is linked to the heterogeneity of consumers' preferences, while in case of vertical differentiation preferences are homogeneous (cfr. assumption 1 below).

5. By this, we mean that, *ceteris paribus*, a personal computer with the very same functions of any PC, is better if it works faster; the quality of a camera is higher if its lenses are manufactured with superior glasses, etc., even if it 'does the same things' that any other camera can do.

## 2. The basic model

Let's consider an 'empty' market, roughly speaking, a market that is only 'potential'. In activating it, firms are nonetheless conditioned by ex ante uncertainty on the cost side, while they know what the behaviour of demand is (i.e., consumers' preferences and income distribution are known).

Thus, we have to imagine a firm acting as a 'pioneer' to identify such technological (cost) conditions. Let us assume that the market shows ex post perfect contestability, which means that other producers may enter the market with positive (expected) profit.

At time  $t_i$ , we can describe this market as follows:

$$X_T \gg x_i \geq 0 \quad (1)$$

$$x_i = f^{-1}(p_i) \quad (2)$$

$$p_i x_i - c(x_i) \geq 0 \quad (3)$$

$$\exists p_e \leq p_i \wedge x_e \leq X_T - x_i \ni x_e p_e - c(x_e) \geq 0 \quad (4)$$

$$t_i > \tau_e. \quad (5)$$

Subscripts  $i$  and  $e$  are respectively referred to the pioneer, which actually becomes incumbent, and to a potential entrant;  $X_T$  is the whole supply for the industry in perfect competition;  $t_i$  is the time the incumbent takes to react to new entries by lowering his/her price;  $\tau_e$  is the time period the entrant takes to quit the market before the incumbent's reaction.

Condition (1) shows the quantity supplied by the pioneer; condition (2) says that this supply is totally sold. By (3), we know that incumbent's profits are non negative. Condition (4) states that there exists a pair  $(x_e, p_e)$  such to yield non negative profits to an entrant. At last, condition (5) states that the entrant can get these profits adopting a hit-and-run strategy, and quit the market before the incumbent's reaction.<sup>6</sup>

---

6. Labelled the sunk costs as  $\sigma$ , we could assume  $\sigma = 0$ . We prefer, however, the formulation given in the text, since the definition itself of 'sunk costs' implies a temporal dimension.

Let's now assume that product quality,  $q$ , is defined over the closed interval  $[L, H]$ ,  $L > 0$ , and that consumers' preferences are synthesized by a parameter  $\theta_q$  - defined over the closed interval  $[\theta_L, \theta_H]$  - increasing in income and uniformly distributed across the population between  $\theta_L$  and  $\theta_H = \theta_L + 1$ ,  $\theta_H \geq \theta_L$ , with total density  $F(\theta) = 1$ .<sup>7</sup> The expression  $\theta_q q$  turns consumers' preferences into money values.

For reasons that will be clear later on, we shall now make the following basic assumptions:

**ASSUMPTION 1:** every individual ranks the existing goods in the same order, that is:  $q_H > q_{H-\alpha} > \dots > q_{L+\alpha} > q_L$ . This amounts to saying that preferences are homogeneous across consumers: prices and income being equal, every consumer would buy the same product, i.e., the top one.

**ASSUMPTION 2:** production involves variable costs only.<sup>8</sup>

**ASSUMPTION 3:** unit (variable) production costs are constant; specifically, they don't increase with quality. Thus, every increase in product quality will turn into increased sales. Formally:

$$c(q) = c = \theta_L L - \varepsilon \quad \forall q. \quad (6)$$

This is the crucial hypothesis: the unit variable costs are strictly less than the marginal evaluation, i.e., the marginal willingness to pay for quality by the richest consumer, for each income class, even the lowest.<sup>9</sup> This seems to provide the strongest incentive to produce a high-quality good.

Thus, although in a preliminary way, we stress the interaction between cost conditions

---

7. This is the same morphology assumed by Gabszewicz and Thisse (1979, 1980) and subsequently by Shaked and Sutton (1982, 1983). The assumptions about parameter  $\theta$  hide an analogous assumption about income distribution: income  $y$  is defined over the interval  $0 < a \leq y \leq b$ , with total density equal to 1, and is uniformly increasing over the population of consumers.

8. What may seem an unrealistic hypothesis at first sight, is actually meant to accentuate the contestability degree of the market.

9. In the following section, we are also going to assess the magnitude of  $\varepsilon$ , conditional on the covering of market's demand.

on the supply side and willingness to pay by the public on the demand side; this leads to think that there should be a ‘move’ by which a firm could achieve a permanent market share and positive profits of monopolistic flavour, without paying any attention to competitors’ prices.

Let us finally assume that the pioneer chooses to enter on a small scale (condition (1)), selling a high-quality product,<sup>10</sup> indeed, the highest:  $q_i = H$ . On the one hand, this choice relies on  $q$ ’s **immediate observability** by consumers; thus, it testifies pioneer’s honesty, since it signals his/her intention to stay in the market for long. The quality choice is assumed to be fixed.<sup>11</sup> On the other, if the game is characterized by a sufficiently complete degree of information (in the usual game theoretical sense) to allow any player to know in advance the payoff vector of the game itself, then the pioneer, as we will show, has a strong incentive to supply the highest possible quality. This means that the piononer is given an advantage in the Stackelberg sense, as far as the quality choice is concerned.<sup>12</sup>

Now, let  $U$  be the consumer’s utility function; assume he/she can choose between a) buying a single unit of product from the seller or b) not buying at all. His utility will be, respectively:  $U = \theta_q q - p$ ,  $U = 0$ . Given this representation of preferences,  $\theta_q$  may be considered as the marginal willingness to pay for quality, as the quality-price tradeoff of any consumer, or else as the inverse of the marginal rate of substitution between income and quality. Generally speaking, consumers will buy if the quality-price combination yields them a non negative net surplus; in the case under analysis, this means:

$$U = \theta_H H - p_H \geq 0. \tag{7}$$

---

**10.** This implicitly means to assume that a) vertical differentiation deals strictly with the intensity of a certain characteristic (that is, the product at stake remains horizontally homogeneous with the others); and b) any quality increase doesn’t involve sunk costs (this is stated in the most extreme terms in assumption 2 above).

**11.** This is the *extrapolability hypothesis* introduced by von Weizsäcker (1980), which states that the quality chosen in the first period is a signal of the quality to be supplied in the future. Since the definition of  $q$  involves an immediate positive fixed cost, there is no incentive to reduce it later on; on the other hand, if there is a reputation-effect of any kind and if this effect relies on the first period performance, there isn’t any incentive to increase quality in the subsequent periods, either.

**12.** It is a simplifying assumption to think that he/she cannot supply more than one quality at the same time, to avoid the possibility of prevention of any further (stable) entry by potential competitors.



The above condition is satisfied by (6), but the total demand available to the pioneer remains to be evaluated. If a good of quality  $q$  is supplied at a price  $p$ , demand will consist of the consumers whose preferences are such that  $\theta_q q \geq p$ . If  $N$  is the total number of consumers, we have:

$$x(p) = N[1 - F(p/q)].$$

In terms of the present model, this means that the demand facing the pioneer is:

$$x_{iH}^M = \theta_H - \frac{p_{iH}}{H}, \quad (8)$$

where the apex  $M$  stands for *monopoly*, to distinguish the quantity sold and the profits gained by the pioneer when he/she is alone on the market from the same magnitudes in oligopoly.

The pioneer, or the incumbent, maximize the following objective function:

$$\pi_{iH}^M = (p_{iH} - c)x_{iH} \quad (9)$$

with respect to price,  $p_{iH}$ . That is:

$$\max_{p_{iH}} \pi_{iH}^M = (p_{iH} - c) \left[ \theta_H - \frac{p_{iH}}{H} \right]; \quad (9')$$

the first order condition is:

$$\frac{\delta \pi_{iH}^M}{\delta p_{iH}} = \theta_H H - 2p_{iH} + c = 0, \quad (10)$$

which yields:

$$p_{iH}^M = \frac{(\theta_H H + c)}{2}, \quad (11)$$

while the incumbent's profits are:

$$\pi_{iH}^M = \frac{(\theta_H H - c)^2}{4H}. \quad (12)$$

At the end of the first period, potential competitors face the perspective of a non negative ‘unfulfilled demand’, which the exact amount of will be specified later, to be satisfied at a price that seemingly allows positive profits. However, the heart of the matter is the choice of  $q$ ; the entrant faces three alternatives:

- 1) a strategy *à la Shaked and Sutton*, selling a good of quality  $h < H$  with positive profits, in a natural duopoly framework;
- 2) a contestable strategy, entering the market niche of the existing product(s), trying to obtain highest profits by stealing the market share of the incumbent(s) and quit before any reaction;
- 3) a perfectly competitive strategy: the entrant can sell a low-quality product ( $L$ ), at the price  $p_e = c_L$ . But, since he/she has the chance (1) above, he/she won’t behave this way.

### 3. From Contestability to Natural Oligopolies

In this section, we investigate the market outcome implied by alternative (2) mentioned above. That is, we assume that the entrant engages a competition in quality and prices with the entrant.

At the turn of the second period, instant  $t_2$ , the entrant supplies a good of quality  $q_e = h = H - \delta$ ,  $0 < \delta \leq H - L$ . The reason is quite simple: should he/she choose quality  $H$ , we would observe a case of competition with homogeneous products, which would finally yield zero profit for both competitors.<sup>13</sup>

The final configuration of the market is the outcome of a competition we can describe as a multistage non cooperative game, each stage of which is to be solved looking for the Nash Equilibrium in the relevant variable. In the first stage, firms have to choose whether or not to enter the market. At the second, once they have checked the number of firms actually present, they face the option regarding the kind of good to be supplied, that is, quality choice. Then, having observed its rivals' qualities, at the third stage each firm has to choose its price. The solution of a sequence like this, if it exists, is a Perfect Equilibrium for the game.<sup>14</sup> Here, we will proceed as usual by backward induction. In our case, given the quality chosen by the incumbent, by virtue of his/her Stackelberg advantage, the second stage obviously reduces to the selection of the best response from below by the entrant.

Consumers are indifferent between the two products (the incumbent's and the entrant's) if and only if the following condition is verified:<sup>15</sup>

$$\theta_H H - p_i = \theta_h h - p_e. \quad (13)$$

---

13. The least-differentiation choice would then turn out in a 'Bertrand paradox'. Moreover, given the incumbent's choice, the entrant is compelled to reply 'from below'. This, in our simple model, is a consequence of the hypothesis that  $q$  is bounded above. For the general case in which  $q \in [0, \infty)$ , see Lambertini (1991, appendix) and Shaked and Sutton (1982, 1983).

14. More formally, we say that an  $n$ -tuple of strategies is a Perfect Equilibrium for the game if, after any stage, the part of the firms' strategies pertaining to the game consisting of the remaining stages, forms a Nash Equilibrium for that game (cfr. Selten, 1975).

15. Cfr. Bresnahan (1981, 1987) and Tirole (1988, p.296).

Market demands for the two goods are respectively:<sup>16</sup>

$$x_i(\hat{p}) = \theta_H - \left( \frac{\Delta p}{\delta} \right), \quad (14)$$

$$x_e(\hat{p}) = \left( \frac{\Delta p}{\delta} \right) - \theta_L. \quad (15)$$

In order to reach the Nash Equilibrium, each firm is required to:

$$\max_{p_j} (p_j - c)x_j, \quad j = i, e. \quad (16)$$

Thus, the incumbent's objective function is:

$$\max_{p_{iH}} \pi_{iH}^d = (p_{iH} - c) \left[ \theta_H - \frac{p_{iH} - p_{eH}}{\delta} \right]; \quad (17)$$

and the entrant's:

$$\max_{p_{eH}} \pi_{eH}^d = (p_{eH} - c) \left[ \frac{p_{iH} - p_{eH}}{\delta} - \theta_L \right]. \quad (18)$$

The first order conditions are:

---

**16.** With several differentiated goods, every consumer shall decide not only **whether or not** to buy, but also **which** good to buy. Our case shows the most interesting situation, in which there is no *dominant* good. In fact, given  $p_1 > p_2 \wedge q_1 > q_2$ , the consumers whose preferences are such that  $\tilde{\theta} \geq (p_1 - p_2)/(q_1 - q_2)$  will buy good 1, while those consumers whose preferences are such that  $p_2/q_2 < \theta < \tilde{\theta}$  will buy good 2. The remaining consumers won't buy at all. Thus, the demanded quantities will be:

$$x_1(\hat{p}) = N[1 - F((p_1 - p_2)/(q_1 - q_2))],$$

$$x_2(\hat{p}) = N[F((p_1 - p_2)/(q_1 - q_2)) - F(p_2/q_2)].$$

$$\frac{\delta \pi_{iH}^d}{\delta p_{iH}} = \delta \theta_H - 2p_{iH} + p_{eh} + c = 0; \quad (19)$$

$$\frac{\delta \pi_{eh}^d}{p_{eh}} = p_{iH} - 2p_{eh} - \delta \theta_L + c = 0. \quad (20)$$

And the reaction functions will be respectively:

$$p_{iH} = R_i(p_{eh}) = (p_{eh} + c + \delta \theta_H)/2; \quad (21)$$

$$p_{eh} = R_e(p_{iH}) = (p_{iH} + c - \delta \theta_L)/2. \quad (22)$$

The Nash Equilibrium in prices requires:

$$p_{eh}^* = c + (\theta_H - 2\theta_L) \frac{\delta}{3} \quad (23)$$

and:

$$p_{iH}^* = c + (2\theta_H - \theta_L) \frac{\delta}{3} > p_{eh}^*, \quad (24)$$

where the star denotes the equilibrium values. Since

$$U = \theta_L L - p_{eh}^* \geq 0, \quad (25)$$

we obtain

$$c \leq \theta_L L - (\theta_H - 2\theta_L) \frac{\delta}{9}, \quad \varepsilon = (\theta_H - 2\theta_L) \frac{\delta}{9}. \quad (26)$$

The quantities sold at equilibrium will be:

$$x_{eh}^* = \frac{\theta_H - 2\theta_L}{3} \quad (27)$$

and:

$$x_{iH}^* = \frac{2\theta_H - \theta_L}{3}; \quad (28)$$

$$x_{iH}^* > x_{eh}^*, \quad x_{iH}^* + x_{eh}^* = 1 \quad (29)$$

and the profits:

$$\pi_{iH}^d = (2\theta_H - \theta_L)^2 \delta / 9; \quad (30)$$

$$\pi_{eh}^d = (\theta_H - 2\theta_L)^2 \delta / 9; \quad (31)$$

$$\pi_{iH}^d > \pi_{eh}^d > 0. \quad (32)$$

Let's summarize the results: the higher-quality supplier is allowed to set a higher price, this way making higher profits. This is not, however, the end of the story: since  $\pi = \pi(\delta)$ , both firms will gain from differentiation, as in the horizontal model *à la Hotelling*. Hence, the logical consequence is that the entrant will locate himself/herself at the lower bound of the quality spectrum, i.e., in  $L$ .

Moreover, given the structure of the model, two is the largest number of firms enjoying positive profits by selling differentiated goods, and the market is completely covered.<sup>17</sup> This is the so-called *finiteness property*.<sup>18</sup>

---

17. These are, obviously, somewhat standard results for a model of natural oligopoly, as displayed, for instance, by Jean Tirole (1988, appendix to chapter 7).

18. Cfr. Shaked and Sutton (1982, 1983).

#### 4. The high-quality supplier in the two contexts

Before starting with the analysis the following section is dedicated to, we have to compare the profits accruing to the high-quality supplier in the two market configurations. That is, we must ask ourselves if, and under what conditions

$$\pi_{iH}^M > \pi_{iH}^d; \quad (33)$$

or

$$\frac{(\theta_H H - c)^2}{4H} > (2\theta_H - \theta_L)^2 \frac{\delta}{9}; \quad (34)$$

which is not straightforward, given the opposite price and quantity effects involved by the Bertrand competition that takes place in the duopolistic configuration, even softened as it may be by quality differentiation.<sup>19</sup>

Normalising  $\theta_q$  over the interval  $[0, 1]$ , which satisfies the above assumptions (cfr. section 1), and substituting into (34) the expression for  $c$ , we obtain:

$$\frac{(4H - L)^2}{36H} > \frac{4}{9}(H - L) \quad (35)$$

$$\frac{16H^2 + L^2 - 8HL}{36H} > \frac{16H(H - L)}{36H} \quad (36)$$

$$\frac{L^2 + 8HL}{36H} > 0, \quad (37)$$

which is always true. Thus, the profits accruing to the ‘monopolist’ selling a high quality good, are always greater than those accruing to the same individual in duopoly. This is not a trivial

---

<sup>19</sup> For the comparison of prices and quantities in the two market configurations, see the appendix. The calculations displayed in this section as well as in the appendix are developed adopting the upper bound of the cost variable,  $c = \theta_L L - (\theta_H - 2\theta_L) \frac{(H-L)}{9}$ .

proposition, as it may seem at first sight. In general, a monopolist is expected to perform as least as good as a duopolist, but this isn't necessarily true if the monopolist decides to supply  $q=H$  from the outset, since this quality choice doesn't maximize his/her first period profits, though it maximizes the discounted stream of profits accruing to the same seller in the repeated game, as the reader can easily verify by inspection of the next section. Hence, this decision relies on the incumbent's expectation of further entries.



## **5. *Hit-and-run* entry into the incumbent's quality niche**

The above analysis leaves untouched the contestability of any quality niche. That is, given the cost structure and the reaction timing initially assumed, it must be possible for a competitor to adopt a hit and run strategy at any quality level; but intuition suggests us that the outcome of a repeated game could be quite far from contestability, if a reputation mechanism of any kind is involved.<sup>20</sup>

In order to investigate what happens in the repeated game, let us assume that (a) the consumers who buy 'read' the contestable behaviour adopted by the entrant and communicate it to the uninformed ones (either immediately or with some lag), exerting a positive externality, and (b) they retaliate, i.e., they stop buying his/her product in the future, so that the competitor can contest a niche (or an income class) no more than once.

We can outline four different cases, which will be analysed in detail in the following subsections:

- 1) instantaneous information; one incumbent;
- 2) lagged information; one incumbent;
- 3) instantaneous information; two incumbents;
- 4) lagged information; two incumbents.

### **5.1. CASE 1: instantaneous information; one incumbent**

In the simplest setting, information spreads instantaneously among consumers, and the entrant faces an incumbent selling a high quality good. The entrant contests the incumbent's niche, at a price slightly lower than the incumbent's (by a positive quantity, arbitrarily close to zero), stealing his/her total demand, and obtaining (almost) the same profits. At the end of the period, he/she quits before the incumbent's reaction. The game ends here, since (a) the incumbent, in the next period, can react by lowering his/her price, and (b) the consumers in the

---

<sup>20</sup>. This is nothing but a further application of the Folk Theorem to a game of complete information. For its original formulation, see Friedman (1971).

highest income class inform the others about the entrant's behaviour.

Thus, the entrant will be incentivated to behave as a natural oligopolist from the very beginning, if his/her discount rate satisfies the following condition:

$$\pi_{IH}^M < \sum_{t=0}^{\infty} \pi_{eL}^d \frac{1}{(1+r)^t}; \quad (38)$$

Normalising  $\theta_q$  over the interval  $[0, 1]$ , the above condition becomes:

$$\frac{(16H^2 + L^2 - 8HL)}{36H} < \frac{(1+r)(H-L)}{9r}, \quad (39)$$

since

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \left( \frac{1+r}{r} \right).$$

Thus we have:

$$\frac{r(16H^2 + L^2 - 8HL)}{36Hr} < \frac{4H(1+r)(H-L)}{36Hr} \quad (40)$$

that is:

$$36H^2r + rL^2 - 8HLr < 4H^2 + 4H^2r - 4HL - 4HLr \quad (41)$$

and

$$r(12H^2 + L^2 - 4HL) < 4H(H-L); \quad (42)$$

$$r < \frac{4H(H-L)}{12H^2 + L^2 - 4HL}. \quad (43)$$

When the distance between  $L$  and  $H$  is very large, i.e., for instance, when  $L = 0$ ,  $H = 1$ , this

expression gives

$$r < \frac{1}{3}, \quad (44)$$

that is

$$\alpha = \frac{1}{1+r} > \frac{3}{4}. \quad (45)$$

When  $H \rightarrow L$ ,

$$r \rightarrow 0, \quad (46)$$

and

$$\alpha \rightarrow 1. \quad (47)$$

## 5.2. CASE 2: lagged information; one incumbent

In this setting, information takes one period to spread among consumers, so that the entrant has the possibility to contest two quality levels. Condition (38) now becomes:

$$\pi_{iH}^M + \frac{\pi_{eL}^d}{(1+r)} < \sum_{t=0}^{\infty} \pi_{eL}^d \frac{1}{(1+r)^t}; \quad (48)$$

and, following an analogous procedure:

$$\frac{(16H^2 + L^2 - 8HL)}{36H} + \frac{(H-L)}{9(1+r)} < \frac{(1+r)(H-L)}{9r}, \quad (49)$$

$$\frac{(16H^2 + L^2 - 8HL)}{36H} < \left( \frac{H-L}{9} \right) \left( \frac{(1+r)^2 - r}{r(1+r)} \right), \quad (50)$$

$$\frac{(16H^2+L^2-8HL)}{36H} < \frac{(H-L)(r^2+r+1)}{9(r^2+r)}, \quad (51)$$

$$\frac{(r^2+r)(16H^2+L^2-8HL)}{36(r^2+r)H} < \frac{4H(H-L)(r^2+r+1)}{36(r^2+r)H}. \quad (52)$$

The roots of this expression are:

$$r_1 = \frac{-3 + \sqrt{9 + \frac{144(H-L)H}{12H^2+L^2-4HL}}}{6}, \quad r_2 = \frac{-3 - \sqrt{9 + \frac{144(H-L)H}{12H^2+L^2-4HL}}}{6}, \quad (53)$$

and since the first term of the polinomial we can derive from (52) is positive, the latter is satisfied if

$$r_2 < r < r_1. \quad (54)$$

But, since  $r_2 < 0 < r_1$ , (54) becomes

$$0 < r < r_1. \quad (54)$$

What can we say about the magnitude of  $r_1$ ? At first sight, as the distance between  $H$  and  $L$  tends to infinity (or when  $L = 0$ ,  $H = 1$ ),  $r_1 = \frac{-3 + \sqrt{21}}{6} \sim \frac{1}{4}$ , while if  $H \rightarrow L$   $r_1 \rightarrow 0$ , so that (52) is satisfied if

$$0 < r < \frac{1}{4}. \quad (55)$$

More formally, we can simulate graphically the function implicit in (52), fixing the value of  $L$  and a given interval for  $H$  and  $r$ . This is what we do in graphic 1 below, which is drawn for  $L = 10^{-3}$ ,  $H \in [10^{-3}, 1]$ ,  $r \in [0, 1]$ . The condition is satisfied in the shaded region of the surface; for values of  $H$  very close to  $10^{-3}$ ,  $r$  approaches 0 and the function does not exist.

[PICTURE]

Graphic 1

### 5.3. CASE 3: instantaneous information; two incumbents

If the contestable entry is adopted when a natural duopoly already exists, and information spreads immediately among consumers, condition (38) becomes:

$$\pi_{iH}^d < \sum_{t=0}^{\infty} \pi_{eL}^d \frac{1}{(1+r)^t}, \quad (56)$$

which, normalising, looks as follows:

$$\frac{4(H-L)}{9} < \left( \frac{1+r}{r} \right) \left( \frac{H-L}{9} \right) \quad (57)$$

$$4r(H-L) < (1+r)(H-L), \quad (58)$$

that is satisfied for

$$r < \frac{1}{3}, \quad i.e., \quad \alpha > \frac{3}{4}, \quad \forall H, L. \quad (59)$$

### 5.4. CASE 4: lagged information, two incumbents

If the information regarding entrant's behaviour takes one period to reach all consumers, condition (38) modifies as follows:

$$\pi_{iH}^d + \frac{\pi_{eL}^d}{(1+r)} < \sum_{t=0}^{\infty} \pi_{eL}^d \frac{1}{(1+r)^t}, \quad (60)$$

which becomes:

$$\frac{4(H-L)}{9} + \frac{(H-L)}{9(1+r)} < \left( \frac{1+r}{r} \right) \left( \frac{H-L}{9} \right). \quad (61)$$

The roots of this expression are:

$$r_1 = \frac{-3 + \sqrt{21}}{6}, \quad r_2 = \frac{-3 - \sqrt{21}}{6}, \quad \forall H, L. \quad (62)$$

Thus, the above condition is satisfied for

$$0 < r < r_1 = \frac{-3 + \sqrt{21}}{6} \sim \frac{1}{4}, \quad \text{i.e., } \alpha > \frac{4}{5}. \quad (63)$$

## Conclusions

The aim of this paper was to investigate the relationship between natural oligopoly (i.e., vertical differentiation) and contestability. We showed that, notwithstanding the contestability of every single quality level, as far as a reputation effect or a retaliation by consumers is involved, the adoption of a contestable strategy turns out to be unprofitable and short-lived for reasonable values of the discount rate. On the other hand, the natural oligopoly outcome seems to hold even adopting the rather extreme assumption that the supply of a superior quality item does not involve any fixed cost, which is of course meant to give the market the greatest degree of contestability.

Obviously, it remains true that, in absence of such a retaliation mechanism (which may be the case if consumers retain no memory of the past, or if the good at stake needs no safeguard of the reciprocal identity of buyers and sellers), contestability works and is more attractive than a stable position as a natural oligopolist in every circumstance.

## Appendix

As far as quantities are concerned, we must ask ourselves if

$$x_{iH}^M \leq x_{iH}^d;$$

normalising, we obtain:

$$\frac{(H-c)}{2H} \leq \frac{2}{3},$$

that is

$$\frac{(4H-L)}{6H} \leq \frac{2}{3},$$

$$4H-L \leq 4H, \Rightarrow L \geq 0,$$

that is always true.

As for prices, we must verify if

$$p_{iH}^M \geq p_{iH}^d;$$

that is

$$\frac{(H+c)}{2} \geq \frac{(H-L)}{3},$$

or

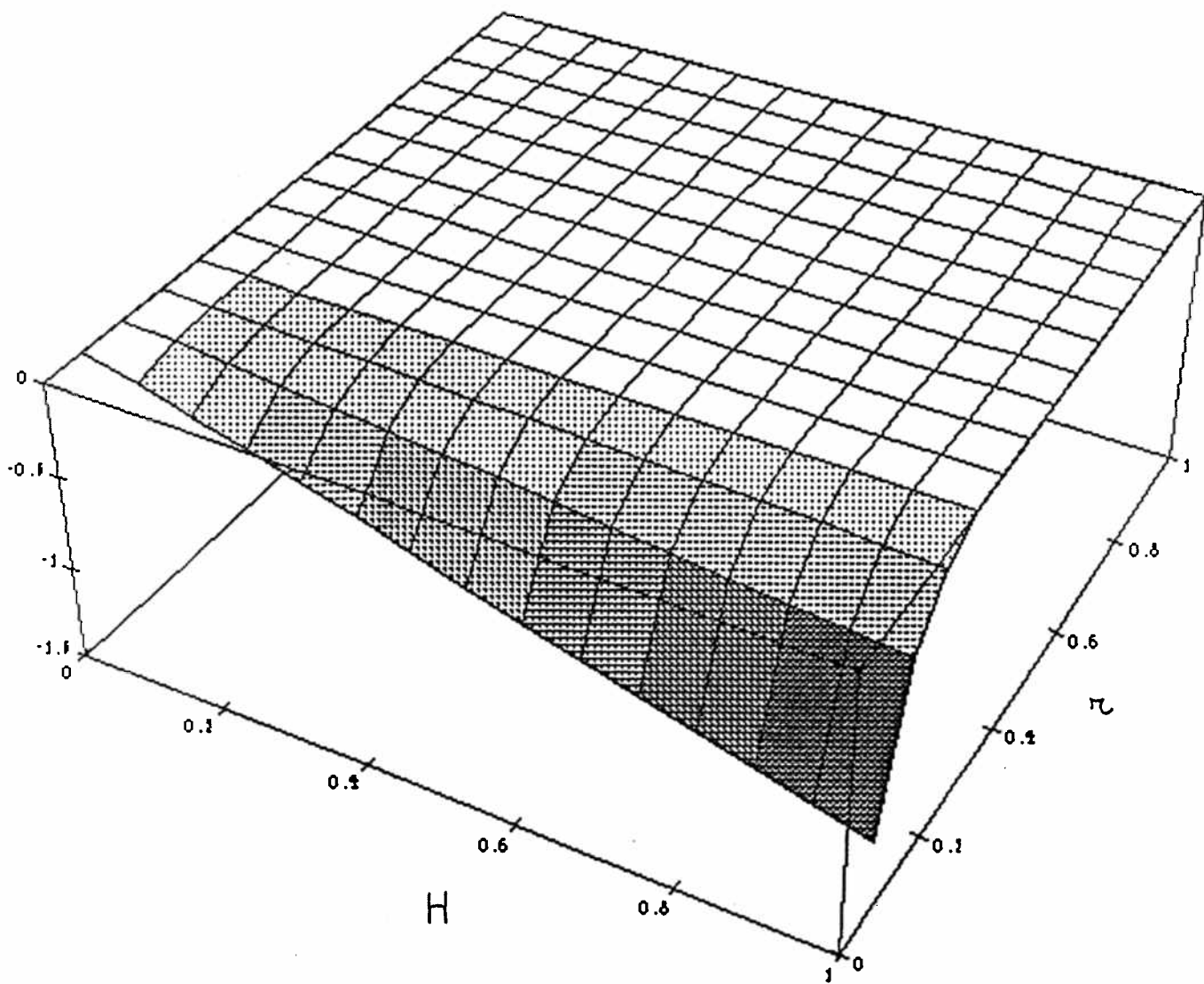
$$\frac{(2H-L)}{6} \geq \frac{(H-L)}{3}$$



and finally

$$2H - L \geq 2H - 2L \Rightarrow L \geq 0,$$

which is always true.



## BIBLIOGRAPHY

Abreu, D. (1988), *On the Theory of Infinitely Repeated Game with Discounting*, *Econometrica*, 56, n.2, pp.383-96.

Akerlof, G. (1970), *The Market for 'Lemons': Quality and the Market Mechanism*, *Quarterly Journal of Economics*, 84, pp.488-500.

Allen, F. (1984), *Reputation and Product Quality*, *Rand Journal of Economics*, 15, pp.311-27.

Appelbaum, E. and C. Lim, *Contestable Markets under Uncertainty*, *Rand Journal of Economics*, 16, pp.28-39.

Aumann, R. (1985), *Repeated Games*, in G. Feiwel (ed.), *Issues in Contemporary Microeconomics and Welfare*, London, Macmillan, pp.209-42.

Baumol, W. (1982), *Contestable Markets: An Uprising in the Theory of Industry Structure*, *American Economic Review*, 72, pp.1-15.

\*\*\*\* (1986), *Williamson's "The Economic Institutions of Capitalism"*, *Rand Journal of Economics*, 17, pp.279-86.

\*\*\*\*, J. Panzar and R. Willig (1982), *Contestable Markets ant the Theory of Industry Structure*, New York, Harcourt Brace Jovanovich.

Eaton, C. and R. Lipsey (1989), *Product Differentiation*, in Schmalensee, R. and R. Willig (eds.), *Handbook of Industrial Organization*, Amsterdam, North-Holland, pp.723-68.

Farrell, J. (1986), *Moral Hazard as an Entry Barrier*, *Rand Journal of Economics*, 17, pp.440-9.

Friedman, J.W. (1971), *A Noncooperative Equilibrium for Supergames*, *Review of Economic Studies*, 38, n.1, pp.1-12.

\*\*\* (1977), *Oligopoly and the Theory of Games*, Amsterdam, North-Holland.

Fudenberg, D. and E. Maskin (1986), *The Folk Theorem in Repeated Games with Discounting or with Incomplete Information*, *Econometrica*, 54, n.3, pp.541-68.

\*\*\* and J. Tirole (1986), *A Theory of Exit in Duopoly*, *Econometrica*, 55, pp.943-60.

\*\*\*\*, \*\*\*\* (1987), *Understanding Rent Dissipation: on the Use of Game Theory in Industrial Organization*, *American Economic Review*, 77, pp.176-83.

Gabszewicz, J.J., A. Shaked, J. Sutton and J.F. Thisse (1986), *Segmenting the Market: The Monopolist's Optimal Product Mix*, *Journal of Economic Theory*, 39, pp.273-89.

Gabszewicz, J.J. and J.F. Thisse (1979), *Price Competition, Quality, and Income Disparities*, *Journal of Economic Theory*, 20, pp.340-59.

\*\*\*\*, \*\*\*\* (1980), *Entry (and Exit) in a Differentiated Industry*, *Journal of Economic*

- Theory, 22, pp.327-38.
- Hotelling, H. (1929), *Stability in Competition*, Economic Journal, 39, pp.41-57.
- Kreps, D. and R. Wilson (1982), *Reputation and Imperfect Information*, Journal of Economic Theory, 27, n.2, pp.253-79.
- \*\*\*, P. Milgrom, J. Roberts and R. Wilson (1982), *Rational Cooperation in the Finitely Repeated Prisoner's Dilemma*, Journal of Economic Theory, 27, n.2, pp.245-52.
- Lambertini, L. (1991), *The Contestability Paradigm in the Presence of Vertical Differentiation. Efficiency Evaluation of a Natural Oligopoly*, working paper n.111, Dipartimento di Scienze Economiche, Università degli Studi di Bologna; paper presented at the 18th EARIE Congress, Ferrara, 1-3 September 1991.
- Lancaster, K. (1979), *Variety, Equity and Efficiency*, New York, Columbia University Press.
- Lane, W. (1980), *Product Differentiation with Endogenous Sequential Entry*, Bell (now Rand) Journal of Economics, 11, pp.237-60.
- Maskin, E. and J. Riley (1984), *Monopoly with Incomplete Information*, Rand Journal of Economics, 15, pp.171-96.
- Milgrom, P. and J. Roberts (1982), *Predation, Reputation, and Entry Deterrence*, Journal of Economic Theory, 27, n.2, pp.280-312.
- Mussa, M. and S. Rosen (1978), *Monopoly and Product Quality*, Journal of Economic Theory, 18, pp.301-17.
- Nelson, P. (1970), *Information and Consumer Behavior*, Journal of Political Economy, 78, pp.311-29.
- Salop, S. (1977), *The Noisy Monopolist*, Review of Economic Studies, 44, pp.393-406.
- Salop, S. and J. Stiglitz (1977), *Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion*, Review of Economic Studies, 44, pp.493-510.
- Schmalensee, R. (1982), *Product Differentiation Advantages of Pioneering Brands*, American Economic Review, 72, pp.349-65.
- Selten, R. (1975), *Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Games*, International Journal of Game Theory, 4, pp.25-55.
- Shaked, A. and J. Sutton (1982), *Relaxing Price Competition through Product Differentiation*, Review of Economic Studies, 49, pp.3-13.
- \*\*\*\*, \*\*\*\* (1983), *Natural Oligopolies*, Econometrica, 51, pp.1469-83.
- \*\*\*\*, \*\*\*\* (1987), *Product Differentiation and Industrial Structure*, Journal of Industrial Economics, 36, pp.131-47.
- Sutton, J. (1986), *Vertical Product Differentiation: Some Basic Themes*, American

Economic review, 76, P&P, pp.393-8.

Tirole, J. (1988), *The Theory of Industrial Organization*, MIT Press.

von Weizsäcker, C. (1980), *Barriers to Entry*, Springer Lecture Note Series No. 185, Berlin, Springer Verlag.

Williamson, O. (1975), *Markets and Hierarchies: Analysis and Antitrust Implications*, New York, Free Press.

\*\*\*\* (1985), *The Economic Institutions of Capitalism*, New York, Free Press.