THE MODERN THEORY OF FINANCE: SUGGESTIONS FOR AN OVERALL VISION[1]

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1. Purpose of this essay

Traditionally the term "finance" has been used to to indicate the art of raising funds to realize both public and private spending programs. There was a distinction in the two cases between public finance and private finance. More recently a new formalized discipline called "theory of finance" has been developed, which concerns several problems certainly not independent of the traditional financial subjects, but especially connected by a common feature, the valuation and comparison of future risky payment (or income) flows. What attributes the character of a unitary discipline to the modern finance is consequently the valuation method.

In this essay we propose to illustrate how several questions that are in the domain of finance actually involve valuation and comparison of risky and time distributed payment flows. Portfolio selection, valuation and financial structure of the firm, maturity structure of interest rates, forward (in relation with spot) exchange rates, relation between insurance prize and loss valuation, as well as relation between taxes and public debt -to mention the most known- belong to these questions[2].

2. Common characteristics of financial assets

We now intend to illustrate, by means of a simple example, how different looking financial assets actually share important characteristics.

Let us consider initially two very simple financial assets, a bond coupon and a dividend warrant, both maturing in a year. The bond coupon, even if in principle likely not to produce any future payment in case of default of the debtor, may be considered as an example of future riskless "capital" (obviously in nominal terms), whereas the dividend warrant may be considered as an example of future risky capital. In fact we cannot know with certainty what the value at maturity of a dividend warrant will be. Yet it is possible to think of such a value as generated by a *normally distributed random variable* whose mean and variance are known. For the time being let us assume that such characteristic parameters of the random variable may be known from the values of the past dividends.

In turn the value of a bond coupon may be thought of as generated by a particular random

2 A representative indication of the content of a modern treatise of finance is offered by Fama (1976).

variable, similar in principle to the former, but characterized by zero variance. The same assumption might apply to other financial assets like ordinary treasury bills.

A mixed portfolio made of bonds and shares may be treated like a dividend warrant: the characteristic parameters of the associated random variable may be considered known.

An insurance contract by which the insurer receives a premium today from the insured party and may have to bear the consequences of a risky event that may or may not take place during a certain period of time can conveniently be associated with a random variable.

If we look carefully, a random variable may be associated also with an *option*. Consider the simple option sold today for a given price from the owner of a dividend warrant that gives the bearer the right to buy the dividend warrant at maturity for a price equal to the average past dividend. It is obviouly possible that the buyer of the option will receive an income at maturity equal to the excess current value (at that date) of the dividend with respect to the average past dividend, as it is possible that he receives no income at all. Again, in front of the payment of a certain amount of money today, there is a future risky income whose probability distribution may be obtained from the one of the dividend warrant.

The simple conclusion is that a certain random variable may be associated with every financial asset.

It may in addition be observed that financial assets may be assembled in combinations called *portfolios*. If markets exist for all the considered financial assets (assumption of *complete markets*) and if all the agents have access to the financial market at perfectly even conditions, i.e. they can be both issuers or buyers of financial assets, their position having no influence on prices and no transaction cost having to be incurred (assumption of *perfect financial markets*), everybody can undo and recompose portfolios at will and, ideally, in a costless way. In other words everybody may tailor the portfolio he likes mostly in terms of characteristics. If financial assets are therefore required in view of their characteristics and if these are well reflected in the characteristics of the associated random variables, we may think of using these characteristics for the purpose of evaluating the financial assets. If this is the case, relationships among random variables will imply relationships also among prices.

An example may clarify this. Consider once more the dividend warrant and the coupon of a bond. It is interesting to notice that the first one, conveniently supplemented, may be transformed into a portfolio which is perfectly corresponding, as to the characteristics of the random variable, to the bond coupon. Such a portfolio will have to have equal value (price) to the one of the bond coupon.

The portfolio to which we refer includes actually three components: (a) a dividend warrant having expected value (mean of the random variable) equal to the one of the bond coupon; (b) an "insurance" in favour of the bearer of the dividend warrant, that compensates him fully at maturity if the dividend turns out less than the bond coupon; (c) an option, this time sold by the bearer of the dividend warrant, that gives the buyer the right to receive, always at maturity, the difference, if any, between the dividend and the bond coupon[3].

The example suggests that the price of any future risky capital (dividend warrant) is equivalent to the price of a future riskless capital (bond coupon), minus the price of the combination of an insurance (against the future unfavourable risk) with an option (related to a future favourable risk). These two components of the price (valuation of the riskless future capital and of the combination insurance-option) may be interpreted as prices of the two characteristics *expected value* and *variability*.

3. The market price of a future risky capital

Since we have seen that every financial asset shows specific quantitative characteristics (expected future value and risk), one useful and the other one generally disuseful[4], it seems reasonable to assume that the one who buys a financial asset buys actually these characteristics[5]. If financial markets are *complete* and *perfect* we have to expect that each of these measurable characteristics receives the same price from the market, independently of the relative proportion in which the different characteristics are represented. The price of the useful characteristic will be positive; the one of the disuseful characteristic will be negative. The price of the financial asset as such will result from the summation of the valuations of its characteristics, as the price of a dinner at a restaurant may be reckoned from the summation of the chosen items.

Suppose for a moment that all the considered financial assets have the same given maturity (time interval between the valuation moment and the moment in which the realization of the

3 In technical terms both the insurance contract described under (b) and the contract described under (c) are called *options*. In the first case we have a *put* option; in the second a *call* option.

4 The assumption of prevailing *risk aversion* does not exclude the existence of risk loving agents.

5 This view draws from Lancaster (1966).

random variable will be known). Imagine you have a portfolio of financial assets and you give it an increment equal to the marginal quantity γ of the financial asset described by the random variable x. We may suppose that the market attributes to the marginal quantity of the financial asset described by x a price proportional to the valuation of the *marginal* changes of its characteristics:

$$\gamma P_x = \alpha \quad marginal \quad expected \quad value - \beta \quad marginal \quad expected \quad variability$$

where the new symbols have the following meaning: P_x price of the financial asset described by the random variable x; α and β positive parameters that represent the prices (in absolute value) of the characteristics for the maturity t. Due to the assumption of risk aversion of the market the valuation of the risk will have a negative sign.

In order to determine the expected value and expected variability of a marginal portfolio represented by the quantity γ of the financial asset described by x we may imagine adding such a marginal portfolio to the whole *market portfolio* (the collection of all existing financial assets), that we assume described by the random variable y. The random variable $\gamma x + y$ may now be associated to the above mentioned compound portfolio. From the expected value and from the variance of such last random variable it is possible to obtain the expected value and a measure of the expected variability of the random variable associated to the marginal portfolio:

$$marginal \quad expected \quad value = m(\gamma x + y) - m(y) = \gamma m(x)$$

where the symbol $m(x)$ represents the expected value of the random variable x and

$$marginal \quad expected \quad variability = var(\gamma x + y) - var(y) = \gamma^2 var(x) + 2\gamma cov(x, y)$$

Take notice that we have been using the variance of the random variable associated with every portfolio as a measure of the expected variability.

By substituting in the previous expression for γP_x we obtain:

$$P_x = \alpha m(x) - \beta\gamma var(x) - 2\beta cov(x, y)$$

and for γ going to zero

$$P_x = \alpha m(x) - 2\beta cov(x,y)$$

We are now in a condition to forget about the assumption of one maturity for all financial assets. If we assume that financial assets are grouped by maturity t and that the realizations of the random variables take place at such maturities as well, we will generally have a distinct valuation function for every maturity:

$$P_{x,t} = \alpha_t m(x) - 2\beta_t cov(x,y)$$

If the covariance between x e y is null and t is equal to one, the parameter α_1 comes to represent the discount factor of a riskless future capital, $1/(1+i)$, and defines implicitly the yearly interest rate for riskless capitals. Similarly if the covariance between x and y is null and t is equal to two, the parameter α_2 defines implicitly the yearly interest rate for riskless capitals maturing in two years and so on for the whole maturity structure of interest rates on riskless capitals.

$cov(x,y)$ may be positive or negative. One should not be surprised by the fact that if the random variable associated with a financial asset shows negative covariance with the random variable associated with the whole market portfolio, the valuation of the risk carried by that financial asset,$-2\beta cov(x,y)$, is positive. In effect such a financial asset is characterized by a price variability which is particularly apt to diversify the portfolio and to reduce its overall risk.

If the expected value $m(x)$ is equal to zero, whereas $cov(x,y)$ is not null, our formula lends itself to the valuation of an insurance-option portfolio maturing in t years, of the same kind described in the previous section.

The suggested valuation formula looks clearly similar to the mean-variance model proposed by Markovitz (1952) and Tobin (1958) and may be shown to coincide with the so called *capital assets pricing model* (CAPM) originally proposed by Sharpe (1964). However it may be pointed out that the present formula represents a simple expression of the *law of one price* in the sense that we expect that every unit of a certain characteristic is evaluated at the same price by the market in a certain moment of time. As a matter of fact in every perfect market the law of one price is enforced by *arbitrage* operations that would immediately take place if different prices for the same good appeared.

The valuation formula must not consequently be confused with a demand function for a financial asset, obtained by constrained maximization of a utility function and connecting the price of the financial asset with the quantity demanded and the prices of financial substitutes and complements[6].

It is usually assumed that similar demand functions are characterized by some degree of stability over time and may be used to solve problems of comparative statics within general equilibrium systems (for example when the exogenous money supply changes), whereas our formula must be conceived as empirically testable among different financial assets of the same maturity with reference to the *same moment of time*.

4. Parameter estimate and test of the theory

The parameters α_t, β_t may be estimated on a sample of prices (and their covariances) of financial assets taken in the same moment of time. Percentage of "explained" variability and limited dispersion of the parameters represent indirect confirmations of the *efficiency* of financial markets[7].

There are obvious limits to the explaining power of the model. Some of them are the following. First of all the set of considered characteristics may not be complete (or the relevant random variables may not be normally distributed). Financial assets of the same kind (shares, bonds), and to a greater extent several *kinds* of financial assets, may encounter obstacles to substitutability in terms of the characteristics that we have considered, due to insufficient standardization and lack of organized markets. In other words real markets may not be fully complete and perfect.

In addition transitory factors, such as temporary excess demand or supply, may push the market far from an equilibrium position.

Correct specification of the expected values and expected covariances of the relevant

6 However it might be pointed out that the mere existence of market prices for the different characteristics of financial assets imply the interaction of individual demand and supply functions and therefore, according to the usual interpretation of such functions, the implicit solution of a great number of individual maximization processes.

7 For these testing problems see Fama (1976).

random variables is one of the big estimation problems. The easiest solution is obviously the one of adopting *backward looking* expectations, i.e. expectations based on past realizations of the random variables. But there is no garantee that similar procedures are adequate.

5. Intermediation

Intermediation may be defined as the production of financial assets by means of financial assets. It is in fact possible to sell financial "products" obtained by combining together other financial assets or simply characteristics, and then to invest the income of such sales, acting so at the market value of the bought financial assets (assets of the intermediaries) is not less than the market value of the sold financial assets (liabilities of the intermediaries). Banks for instance "sell" deposits and invest the corresponding incomes in other financial assets such as loans, bonds and liquid assets. Investment funds and other financial organizations issue shares and invest in bonds and shares of other institutions.

Thanks to the intermediation activity, which is obviously not offered for free, every individual investor has the possibility of making his choices according to his own preferences, as every financed agent may offer financial instruments that are compatible with the specific characteristics of his economic activity. In principle if the preferred instruments to finance a business activity or to invest financial wealth are not available in adequate supply, every agent, enterprise or financial investor might directly behave like an intermediary and select his own assets and liabilities so to obtain the desired structure of characteristics. In such a way he would avoid the intermediation costs. In practice however the lack of standard financial assets, scarsity of information and probably also the lack of an organized market would make individual portfolio management much less efficient.

By means of the previously described financial assets pricing model it is possible to give an abstract representation of both portfolio selection and intermediation activity which, if there are no intermediation costs, come to coincide. For this purpose let us disregard the distinction among maturities and equate the second member of our valuation formula to a constant:

$$\alpha m - 2\beta cov = cost$$

It is possible to interpret this relation as the equation of a plane in the variables m, cov. It encompasses all possible combinations of future expected prices and risks, which are compatible with a given value of wealth (positive constant) or a given value of liabilities (negative constant) and with the market prices of the two characteristics. Any cople of points on such a plane identify two "objects" that may be exchanged on a fair base. In general shifting on such a plane holding the wealth constant means recomposing the portfolio in terms of characteristics; shifting on the plane holding the assets (liabilities) constant means changing the *composition* of assets (liabilities).

6. Typical problems

The theory that allows to attribute a price to a flow of future incomes and expenditures suject to risky conditions represents a powerful instrument for the rigurous formulation and solution of several problems belonging to various areas of economic analysis.

First comes the problem of the valuation of the firm, of its shares and of its debts, proposed by Modigliani-Miller (1958 and 1961). These authors connect three distinct flows to every firm: the flow of gross profits (before deduction of interest payments on debts), the flow of earnings from the property of shares (dividend payments plus capital gains) and the flow of interest payments on debts. An elementary intuition leads to formulate the so called *indifference theorem* of the financial structure on the valuation of the firm. It states that in general (i.e. except the cases in which the financial structure generates allocative distortions) the flow of expected gross profits is independent of its distribution to share owners or to lenders of capital[8]. In other words the *market value of the firm* is equal to the market value of its shares plus the market value of its bonds (debts). In terms of the problems introduced in the previous sections we may think of three different random variables associated to gross profits, earnings from share property and interest payments on debts[9]. We know how to evaluate every flow of future risky incomes and we can consequently say that the current value of the shares of a firm may be obtained from the difference between the market value of the firm and the market value of its debt.

It is immediately obvious that if the firm were to reduce its debt, for instance by issuing new

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8 In the ideal world of Modigliani-Miller the dividend policy of earnings is therefore irrelevant.

9 Of course the risk measure connected with interest payments on debts will generally be smaller than the ones connected with the other flows.

shares, the flow of future interest payments would also reduce, whereas the flow of future share earnings would increase. In any case the change in the structure of its debt would not influence the flow of the expected gross profits. A firm can therefore choose its own degree of *leverage* without influencing its market value.

If the indifference theorem of Modigliani-Miller is to hold in the real world, ideal conditions must obviously take place. A vast literature belonging to the domain of information theory attributes the meaning of a *signal* about the prospects of future profit to the financial structure. In the real world it may therefore happen that we pay careful attention to the financial structure. Nevertheless the intuition at the origin of the theorem remains of the utmost importance.

The discussion about the theorem of Modigliani-Miller suggests a general remark on several problems that imply discount or capitalization operations in a risky context. Recourse to similar operations tipically takes place in formulating intertemporal balance equations between resourses and uses, or between incomes and expenditures in risky conditions. Think for instance of the consumer utility maximization in a multiperiod and uncertain world. This clearly corresponds to the saving problem in the *life cycle* (Modigliani 1949 and Modigliani-Brumberg 1954), when it is formulated with reference to risky conditions (Leland 1968 and Modigliani-Drèze 1972). The equality between future consumption and future risky income must in effect hold in terms of present values with explicit valuation of risk.

A related and only slightly different problem is raised by the so called *Ricardian equivalence theorem*, originally due to Ricardo (1817), cap. XII and recently revisited by Barro (1974). According to these authors an equivalence may be established between the tax financing and the public debt financing of a government deficit. The core of the Ricardian equivalence is the correspondence between the present value of the future payments (and related future taxation) for the service of the debt and the tax that should immediately be levied, as an alternative, in order to balance the government budget[10]. Obviously both future payments for the service of the debt and future taxation for the financing of the service may be characterized by probability distribution functions[11]. Consequently we meet again the discount problem in conditions of risk.

Other cases are even more elementary. Think of an insurance against fire, theft, accident or death. The risk against which the insurance is made may take place (once or in other cases more than once) within a certain period of time with a likelihood that once more may be described by a convenient probability distribution. Consequently we see the possibility to compute the present value of the damage likely to take place. The insurance *premium* is obviously proportional to such a present value[12].

Different only at first sight is the problem of pricing future contracts (i.e. contracts for future delivery), including forward exchange rates. The relationship between future prices and spot (for immediate delivery) prices is a complex one, in the sense that it includes both systematic and stochastic factors. It is very likely for instance that the future exchange rate is influenced by inflation expectations on prices measured in various currencies as suggested by the theory of the *purchasing power parity*, but our attention must here focus on its stochastic components. Let us therefore consider a situation where the relationship between future and spot prices is free from systematic components. It is obvious that the party that "insures" another party against the risk of a future (spot) price change with respect to the current (spot) price requires that the insured party pays an adequate insurance premium that explains the difference between forward and current price[13].

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10 A straightforward implication of the Ricardian equivalence is that from a macroeconomic point of view government bonds are not wealth. This conclusion does not generally hold at a microeconomic level because the personal distribution of the public debt within the community does not usually correspond to the distribution of taxation. If this were to happen government bonds would cease to represent wealth even at an individual level. About the empirical relevance of the Ricardian equivalence and of its implications we must obviously be cautious.

11 This is for example the assumption made by Handa (1990), even if his intent was the one of finding cases in which the Ricardian equivalence does not hold.

12 On the economics of insurance see the recent textbook by Borch (1990).

13 On the theory of future prices see Stein (1986).

This short survey of examples shows how the modern theory of finance provides economic analysis with a powerful instrument for the correct stochastic and intertemporal formulation of problems that traditionally fall within the domain of micro and macroeconomics.

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