MARKET LEADERSHIP IN MULTISTAGE GAMES WITH SEQUENCES OF TECHNOLOGICAL RACES

by

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CONTENTS

1. MODELS WITH A SEQUENCE OF INNOVATIONS

1.1 Introduction and Taxonomy

Most of the existing literature on the relationship between market structure and technological competition may be classified according to the way in which the following features are modelled:

(i) type of R&D competition;

(ii) number of types of players;

(iii) number of stages of the game.

With respect to (i) previous contributions adopted:

a) auction (for instance, Gilbert and Newbery (1982), Dasgupta (1982), Vickers (1986)).

b) non-cooperative game with uncertainty where the strategies are fixed costs of R&D (for instance, Loury (1979), Dasgupta and Stiglitz (1980));

c) non-cooperative game with uncertainty where the strategies are variable costs of R&D (for instance, Lee and Wilde (1980), Harris and Vickers (1986));

d) race (for instance, Fudenberg et al. (1983), Harris and Vickers (1985) (1986)).

With respect to (ii) we may find models with

a) n ($\geq$2) symmetric players (Futia 1980), Loury (1979), Lee and Wilde (1980), Dasgupta and Stiglitz (1980), Reinganum (1982));

b) n ($\geq$2) asymmetric players, i.e., one versus (n-1) symmetric players (for instance, Gilbert and Newbery (1982), Reinganum (1985), Vickers (1986)).

Finally, with respect to (iii), there are

a) one-stage R&D competitions (one-shot games);[1]

b) multistage competitions (multistage games).

An excellent survey of previous contributions is Kamien and Schwartz (1982) and I shall not attempt to duplicate their account of the literature. Furthermore, in what follows I shall mainly focus on a recent development of this branch of study - models with sequences of competitions. A model with a sequence of competitions must not be confused with a multistage game (e.g. Harris and Vickers (1985) (1986)) in which a single innovation (prize) is won through a sequence of moves (decisions). While both areas represent interesting extensions of earlier works, I shall confine my attention to the case in which firms compete for a sequence of prizes (for instance, patents) and I study the interaction between the outcome of such a multiple competition over time and the dynamics of market structure (leadership).

1.2 Motivation

The purpose of modelling a sequence of innovative opportunities is easy to explain. Most of the existing literature cited in the previous Section has focused on one-shot games in which two or more firms strive to win a competition for a prize (for instance, a patent improving the technology and reducing the production cost). Strictly speaking, it is slightly misleading to talk about interaction in these models since decisions are taken once-and-for-all. At most, these models succeed in providing an explanation of a single change occurring in market structure; for instance, why a monopoly may become a duopoly, or why a firm may increase its leadership.[2] What these models do not explain is the complex way in which market structure evolves over time; this task may be carried out only in a dynamic context, where the interaction among a sequence of events is explicitly taken into account. Then, if we aim at investigating the relationship between market structure and technological competition, we ought to consider models which incorporate an explicit sequential structure. Moreover, for the sequential framework to matter in grasping the interaction between market structure and innovations, it is important to capture possible asymmetries among firms and the source of their different incentives to win (i.e., innovate). To model some of the complexities which arise when several innovative opportunities are in prospect is the primary purpose of next pages.

At present there exist two proper models which consider a sequence of innovations: Reinganum (1985) and Vickers (1986). Since I shall partly follow Vickers'model, it seems suitable to report his framework and his main results. Reinganum's work will be briefly outlined and discussed in Section 1.4.

1.3 Vickers'model

In Vickers'model there is a finite sequence of cost reducing innovations. Each innovation is protected by an infinitely long-lived patent so that imitation is not allowed. The timing and nature of the innovations are common knowledge from the outset. The innovation (prize) is assigned to the winner of a race. Each race is modelled as a deterministic bidding game where two firms compete for an indivisible prize allowing the winner to become technologically leader until the next patent is won. Firms are asymmetrically placed in so far as they have different costs: hence, in general they bid differently because they have different incentives to win each race. There is no discounting. Payoffs and technology are such that the duopolistic structure persists but the technological leadership may change. More precisely, four features characterize his model:

(i) a patent race occurs at the beginning of each period;

(ii) a patent race takes the form of a simple deterministic bidding game;

(iii) a winner of patent race does become (or remain) technologically superior to his rival;

(iv) a firm's current profit flow depends upon the levels of technology represented by the two firms' most recent patents.

The main question addressed in his model is how market structure evolves over time. More precisely, "... does one firm become increasingly dominant by winning most or all patent races? Or is there a process of action-reaction, in which market leadership is constantly changing hands?" (p. 1)

Let us introduce the technical details of his model. I shall use Vickers'notation.

The game has T ($<\infty$) stages which we number backwards; t=T, T-1,...,2,1, and the technology offers a cost level c_t for each period. We assume that

$$c_1 < c_2 < \dots < c_T < c_{T+1} < c_{T+2}$$

where $c_{T+1} = c_A$ is the initial cost of firm A and $c_{T+2} = c_B$ is the initial cost of firm B. Notice that c may be thought of more generally as any parameter of the cost function.

Each stage (period) a deterministic bidding game takes place giving the winner the right to employ the cost level associated with that stage. The prize is won by the higher bidder: he pays the maximum amount that the other player would be prepared to pay to get the innovation. In fact denoting b* the maximum amount that the other player would have paid, for any bid lower than b* there would be active rivalry for the prize. In other words, b* is the lowest bid to choke off that rivalry. This seems to be a reasonable condition to impose on an ascending bid auction. The loser does not forgo his bid.

$\pi(\alpha,\beta)$ is the profit of the firm during the period in which its cost is c_α and its rival has cost c_β. It is assumed that

$$\pi(\alpha,\beta) \geq 0 \quad \forall \alpha,\beta$$

$\pi(\alpha,\beta)$ is increasing in β and decreasing in α.

Joint profits when costs are c_α and c_β are defined as

$$\sigma(\alpha,\beta) = \sigma(\beta,\alpha) = \pi(\alpha,\beta) + \pi(\beta,\alpha).$$

There is no discounting and firms maximize their net profits (profits minus bid expenses) over time. Recalling that stages are numbered backwards, if firms have costs c_{t+1} and c_{t+k} $(k>1)$, let

$$h = \Omega(t,t+1) - \Omega(t+k,t)$$

be the incentive of the currently high cost firm (H) to win patent t, where $\Omega(s,t)$ is the net payoff of a firm when it has cost c_s and its rival has c_t in the subgame beginning immediately after the last race. Similarly, the currently low cost firm's incentive to win patent t is

$$\ell = \Omega(t,t+k) - \Omega(t+1,\ t)$$

and H wins the patent if $h \geq \ell$. Defining

$$\Upsilon(s,t) = \Upsilon(t,s) = \Omega(s,t) + \Omega(t,s)$$

as the joint payoffs, $h \geq \ell$ iff

$$\Upsilon(t,t+1) \geq \Upsilon(t,t+k)$$

If $h \geq \ell$, the winning bid of H is ℓ and his payoff in the subgame beginning after the race for patent t+1 is

$$\Omega(t+k,t+1) = \pi(t+k,t+1) + \Omega(t,t+1) - \ell = \pi(t+k,t+1) + \sigma(t,t+1) - \Omega(t,t+k)$$

whereas, if $h < \ell$

$$\Omega(t+k,t+1) = \pi(t+k,t+1) + \Omega(t+k,t)$$

Whithin this model, Vickers proved that

(A) $\sigma(t,t+1) > \sigma(t,t+k)$, $t=1,2,\ldots, T+2, k=2,\ldots,T+2-t$
is a sufficient condition for Action-Reaction (alternating winners) and that

(B) $\pi(s+k,s) = 0$, for all s and $k > 1$
is a sufficient condition for Increasing Dominance (the low cost firm wins the current race, for whatever couple of cost levels).

Thus, the way in which the market leadership evolves depends upon the profit in the product market. When condition (A) holds, the higher cost firm has a greater incentive to win the race and Action-Reaction results, i.e., the leadership switches from one firm to the rival. In other words, the duopolistic market structure persists, but the identity of the leader changes after each race. Condition (A) may be shown to be satisfied, under reasonable circumstances, if the competition on the product market is of the Cournot type (Vickers (1986)).

When condition (B) holds the leadership instead is made stronger. Intuitively, condition (B) might hold in a product market where competition à la Bertrand takes place. Indeed, if the lower cost firm charges the minimum between the monopolistic price (given his current cost level) and the rival's cost level, the rival's profit will be zero.

The surprising contrast between static and dynamic efficiency is one of the most interesting conclusions that emerge from the model. Bertrand (i.e., intensive, in static terms) competition in the product market seems to involve increasing dominance in dynamic terms. Cournot competition in the product market is instead responsible of a more dynamic competition, leading to Action-Reaction and to a falling price over time.

1.4 Reinganum's model

Reinganum (1985) models a noncooperative dynamic game in which at any time an incumbent monopolist (insider) is challenged by other firms (outsiders). There is a finite sequence of drastic innovations (see Section 2.1 for the implications of considering drastic innovations: the basic idea is that a drastic innovation offers such superiority on rivals that they cannot offer any effective competition) and each race is a tournament, i.e., the winner takes all the prize and the loser(s) get nothing. The monopolistic market structure therefore persists, but the identity of the monopolist may change over time. Thus, the monopolistic power (profit) is temporary and it lasts until an innovation is obtained by one of the rivals (outsiders). The only source of asymmetric behaviour among the firms lies in the fact that the incumbent-monopolist receives a profit flow, whereas the other firms do not. R&D costs are contractual and the presence of technological uncertainty is modelled by means of an exponential distribution function of the date of success. Firms maximize expected profits given by the discounted payoff minus variable and fixed costs. One of Reinganum's main results is that the incumbent invests less tnan each rival in the current stage. The subgame perfect Nash equilibrium, which is shown to exist - but see Vickers (1985), ch. 4, for an assessment of the assumptions required in the proof of this result - is symmetric among the (n-1) challengers, while the incumbent has less than 1/n chance of being the incumbent in the next stage. The result that at each stage the current monopolist commits fewer resources to R&D than the outsiders and therefore stands a lower chance of winning may be explained as follows. The monopolist is currently earning monopoly profits, whereas the other firms (outsiders) have nothing to lose by succeeding earlier rather than later. The interpretation of this result reveals also that the sequential structure is not relevant to Reinganum's findings. This is because if they lose the incumbent's and a rival's expected payoff after the current race are identical. Thus, their relative incentives for winning the current race cannot depend on what happens after it. There is not an effective difference in firms' incentives to innovate. The only difference is because the incumbent, but not the outsider, is currently receiving a profit flow and this makes it more costly (in terms of profit forgone) for the incumbent to innovate than it is for each challenger, just as in the models where only an innovation is in prospect (see Reinganum (1983)). It is this asymmetric position - jointly to the drastic nature of the innovations - that explains why the incumbent, to quote Reinganum (1985, p. 90), "has relatively less incentive to shorten the length of the current stage of incumbency".

2. TECHNOLOGICAL LEADERSHIP WITH A SEQUENCE OF INCREMENTAL INNOVATIONS

2.1 Drastic vs incremental innovations

In Reinganum (1985) it is implicit that each innovation is drastic in the sense that the firm innovating first gains a monopolistic position and the loser(s) get zero profits. This hypothesis is responsible for her main result, i. e., the incumbent (monopolist) invests less resources to R&D than his rivals and the market is thus always monopolised.

In Vickers (1986) each innovation is such that the firm getting the patent "becomes industry leader in terms of technology"; this is true irrespective of the cost distance from the rival.

In what follows I relax the above-said feature of Vickers' model, by considering <u>incremental</u> innovations. An innovation is said to be incremental with respect to a certain cost pattern of the firms when its adoption may not imply a change of leadership. This definition captures the idea of small innovations and introduces another aspect of dependence on the past which is absent in Vickers' (and Reinganum's) model. Indeed, while in Vickers' model the winner of race t switches from his cost (whatever it is) to cost c_t and becomes (or remains) the leader, here the cost level of the winner is reduced by Δc_t and the follower, although winning the race, may still fail in gaining the leadership. Formally, let

$$\Delta c_T, \Delta c_{T-1}, \ldots, \Delta c_2, \Delta c_1$$

be the finite sequence of cost reductions allowed by the T innovations which have been numbered backwards. For sake of simplicity we normalize cost reductions in such a way that for all $i \in [1,T], \Delta c_i = 1$. This means that each innovation brings about an unitary cost reduction for the innovator. Let $c_A = a$ and $c_B = b$ be the cost levels of firm A and B, respectively. If A wins a race his cost becomes a-1; if B wins a race his cost becomes b-1.

In next pages I shall adopt Vickers' framework and I shall find sufficient conditions on profits for there to be Increasing Dominance (ID) and Catching Up (CU), where ID means again that the low cost firm wins the next (and hence all) races, and CU means that the high cost firm wins the next race (and hence approaches the rival). Of course, the follower may catch the leader up if, for some $i \in [1,T]$,

$$a-b=\sum_{j=1}^{i} \Delta c_J = \Delta_i$$

whereas, if for some $i \in [1,T]$, $\Delta_i > a-b$ we have "overtaking" as we will see below.

In Vickers'notation, labelling H and L the currently high cost firm and the low cost firm, respectively, H's incentive to win an incremental innovation race is

$$h=\Omega(a-1,b) - \Omega(a,b-1)$$

and L's incentive to win is

$$l=\Omega(b-1,a) - \Omega(b,a-1)$$

when H's cost is a and L's cost is b. Thus, $h \geq l$ iff

$$\Upsilon(a-1,b) \geq \Upsilon(a,b-1)$$

and H wins if this inequality holds.

2.2 Results

This Section collects the two main results of this chapter.

Proposition 1. The following

$$\text{(H1)}\ \alpha > \beta \rightarrow \pi(\alpha,\beta) = 0$$

is a sufficient condition for ID.

Proof. The proof is by backwards induction. We know that L (whose cost is β) wins a race if

$$(1)\qquad \gamma(\alpha,\beta-1) > \gamma(\alpha-1,\beta)$$

Let us consider the last race, when L has cost level, say, $c_L=2$ and H has $c_H=k$ for some $k>2$: Thus (1) reduces to

$$(2)\qquad \gamma(k,1) > \gamma(k-1,2)$$

which reduces in turn, by (H1), to

$$(3)\qquad \pi(1,k) > \pi(k-1,2)$$

which holds because of the assumptions on π namely, because π is decreasing in its first argument and increasing in its second argument. Thus, L wins the last race.

Assume now that for races occurring at $t=2,3,\ldots,s-1 < T$, (1) holds, i.e.

$$(4)\qquad \gamma(t+k,t) > \gamma(t+k-1,t+1) \qquad k > 1$$

This means that L wins all races occurring at $t=1,2,\ldots,s-1$.

Let us now consider the race occurring at $t=s$. We have to prove that

$$(5)\qquad \gamma(s+k,s) > \gamma(s+k-1,s+1) \qquad k > 1$$

We have

$$\Omega(s,s+k) = \Pi(s,s+k) + \Omega(s-1,s+k)$$

and

$$\Omega(s+1,s+k-1) = \Pi(s+1,s+k-1) + \Omega(s,s+k-1)$$

because in both cases the cost of the bid for the winner (L) is zero. This is because our inductive hypothesis implies that $\Omega(\alpha,\beta) = 0$ for $\alpha > \beta$; hence, the high cost firm bid is zero. By virtue of (H1) we can write

$$(6) \qquad \Upsilon(s,s+k) - \Upsilon(s+1,s+k-1) = \Omega(s+1,s+k-1) - \Omega(s,s+k)$$

$$= \sum_{t=0}^{s-1} \Pi(s-t,s+k) \quad - \sum_{t=0}^{s-1} \Pi(s+1-t,s+k-1)$$

which is positive because of the properties of Π. Thus, (5) holds and L wins also race at t=s. This completes the proof. ∎

Looking at the economic interpretation of this result, it supports the intuition behind Vickers' (1986) result (B). The presence of Bertrand competition in the product market, entailing zero profit for the high cost firm, gives the leader a greater incentive to win each race than tha follower.

Proposition 2. The following

(H2) $\quad \sigma(\alpha-1,\beta) > \sigma(\alpha,\beta-1) \qquad \alpha > \beta,$

is a sufficient condition for CU.

Proof. This proof too proceeds by backwards induction. Consider the last race: H has $c_H = k$ and L has, say, $c_L = 2$, for some $k > 2$. H wins this race if

$$(7) \qquad \Upsilon(k-1,2) > \Upsilon(k,1)$$

which reduces, being this the last race, to

$$(8) \qquad \sigma(k-1,2) > \sigma(k,1)$$

which holds by (H2). Thus H wins race at t=1.

Assume now that for races at stages $t=2,3,\ldots s-1 < T$, H wins; i.e.,

$$(9) \qquad \Upsilon(t+k-1,t+1) > \Upsilon(t+k,t) \qquad k > 2$$

Consider now race at t=s. We must show that

$$(10) \qquad \Upsilon(s+k-1,s+1) > \Upsilon(s+k,s)$$

We have

$$(11) \qquad \Omega(s+1,s+k-1) = \Pi(s+1,s+k-1) + \Omega(s+1,s+k-2)$$

and

$$(12) \qquad \Omega(s+k-1,s+1) = \Pi(s+k-1,s+1) + \Omega(s+k-2,s+1) - \left[\Omega(s,s+k-1) - \Omega(s+1, s+k-2)\right]$$

Notice that (12) is H's payoff in the subgame beginning immediately after the race at t=s if H wins this race. The three terms on the right hand side (RHS) have the following meaning:

$\pi(s+k-1,s+1)$ is H' payoff in the period after race s;

$\Omega(s+k-2, s+1)$ is H's payoff in the subgame immediately after race (s-1), which he wins, by assumption (9);

$\Omega(s,s+k-1) - \Omega(s+1,s+k-2)$ is the amount that H bids in race (s-1), i.e., how much L would have been prepared to pay for patent assigned at t=s-1.

Then, adding (11) and (12) we get

$$(13) \quad \Upsilon(s+1,s+k-1) = \sigma(s+1,s+k-1) + \Upsilon(s+1,s+k-2) - \Omega(s,s+k-1) + \\ + \Omega(s+1,s+k-2)$$

Moreover,

$$(14) \quad \Omega(s,s+k) = \pi(s,s+k) + \Omega(s,s+k-1)$$

and

$$(15) \quad \Omega(s+k,s) = \pi(s+k,s) + \Omega(s+k-1,s) - \Omega(s-1,s+k) + \Omega(s,s+k-1)$$

Adding (14) and (15) we get

$$(16) \quad \Upsilon(s,s+k) = \sigma(s,s+k) + \Upsilon(s,s+k-1) - \Omega(s-1,s+k) + \Omega(s,s+k-1)$$

i.e., just like (13). Subtracting (16) from (13) we obtain

$$(17) \quad \Upsilon(s+1,s+k-1) - \Upsilon(s,s+k) = \left[\sigma(s+1,s+k-1) - \sigma(s,s+k)\right] + \\ + \left[\Upsilon(s+1,s+k-2) - \Upsilon(s,s+k-1)\right] - \Omega(s,s+k-1) \\ + \Omega(s+1, s+k-2) + \Omega(s-1,s+k) - \Omega(s,s+k-1)$$

Looking at (17), the term in the first bracket on RHS is positive by

(H2). The term in the second bracket on the RHS is positive by (9). In order to prove the proposition we might show that the remaining expression on the RHS of (17) is positive, i.e., that

$$(18) \quad -\Omega(s,s+k-1) + \Omega(s+1,s+k-2) + \Omega(s-1,s+k) - \Omega(s,s+k-1) > 0$$

After some manipulations (shown in Appendix A) we see that (18) holds if

$$(19) \quad \Omega(s+k-2,s+1) < \Omega(s-1,s+k)$$

which can be rewritten as

$$(20) \quad \left[\sum_{t=2}^{K-1} \Pi(s+k-t,s+1) - \sum_{t=2}^{K} \Pi(s,s+k-t) \right] + \left[\sum_{t=3}^{K-1} \Pi(s+1,s+k-t) - \sum_{t=0}^{K-1} \Pi(s-1,s+k-t) \right] < 0$$

Now we will use the fact that (19)→(18)→(10). The expression in the second bracket of (20) is certainly non positive by the decreasingness of Π in its first argument; moreover, it is also patent that the expression in the first bracket of (20), which we can state as

$$\left[\sum_{t=2}^{K-1} \Pi(s+k-t,s+1) - \sum_{t=2}^{K-1} \Pi(s,s+k-t) \right] - \Pi(s,s)$$

is negative by the properties of Π, and $\Pi(s,s) \geq 0$ by hypothesis. Thus, (19) holds and then (10) holds. This completes the proof. ■

Condition (H2) guarantees that the currently high cost firm wins all races. Of course, with T races, A catches B up if, for some $i \in [1,T]$, $\Delta_i = a-b$, where a and b are again the two cost levels of A and B respectively, before the first race occurs. What happens when A catches B up? At that point both firms have cost levels b. Then, if there are other races to be p yed, the name of the winner of the race allowing one to gain the cost level b-1 might be decided by chance. Let us consider first the case when A (the previous high cost firm) wins. Then the situation after this race is: $c_A = b-1$, $c_B = b$, namely, A overtakes B. What happens at the next race? The incentives to win next race are

$$h = \Omega(b-1,b-1) - \Omega(b,b-2)$$

and

$$\ell = \Omega(b-2,b) - \Omega(b-1,b-1)$$

and B wins if $\gamma(b-1,b-1) > \gamma(b,b-2)$. If this race is the last one, the last inequality reduces to $\sigma(b-1,b-1) > \sigma(b,b-2)$, which holds by (H2). If that race is not the last one, one can invoke the inductive argument used in the proof of Proposition 2 and show that the high cost firm (namely B) wins the subsequent race. Once B catches A up the same reasoning applies.

If B wins the race occurring when firms share the same cost level, we have this situation; $c_A = b$, $c_B = b-1$. Condition (H2) allows one to prove, again by backwards induction, that A wins the subsequent race. Thus, when players have the same cost, whoever wins a race loses the subsequent one. This circumstance is easily explained: since incentives are equal, net payoffs are zero once firms are level. So, it is as if the game is over.

If for some i, $\Delta_i > a-b$, there is overtaking, i.e., the high cost firm continues to win and becomes the low cost firm. If overtaking occurs, say, at race s, race s+1 will be won by the other firm, and the evolution of market leadership will be characterised by Action-Reaction as described above after CU has occurred.

As far as the interpretation of Proposition 2 is concerned, it is worth noting that it is not the extension of Vickers' result (A) to the incremental innovations case, but it is a slightly different result.

Indeed it is not correct to speak of Action-Reaction when we deal with incremental innovations (in the sense made precise above). Action-Reaction properly occurs only once the follower has caught the leader up. After that event, whoever wins a race loses the subsequent one and the technological leadership continues to switch from one firm to the rival.

2.3

Appendix A

In this Appendix we will show that (19) implies (18) above, as it was argued in the proof of Proposition 2. Starting from (18), we add and subtract $\Omega(s+k-2,s+1)$ and $\Omega(s,s+k-1)$. So we get

$$(21)\quad \Upsilon(s+1,s+k-2) - \Upsilon(s,s+k-1) + \Omega(s-1,s+k) - \Omega(s+k-2,s+1) - \\ - \Omega(s,s+k-1) + \Omega(s+k-1,s) > 0$$

Now, add and subtract $\Omega(s+k-1,s)$ and rearrange the terms

$$(22)\quad \left[\Upsilon(s+1,s+k-2) - \Upsilon(s,s+k-1)\right] + \left[\Upsilon(s+k-1,s) - \Upsilon(s,s+k-1)\right] + \\ + \Omega(s-1,s+k) - \Omega(s,s+k-1) + \Omega(s+k-1,s) - \\ - \Omega(s+k-2,s+1) > 0$$

The expression in the first bracket is positive by (9), and the expression in the second bracket obviously vanishes. Let us detect the third bracket. Adding and subtracting $\Omega(s,s+k-1)$ and $\Omega(s+k-1,s)$ we see that the expression in the third bracket of (22) is positive if

$$(23)\quad 2\Omega(s+k-1,s) > \Omega(s+k-2,s+1) - \Omega(s-1,s+k)$$

Since the LHS is positive, (23) certainly holds if the RHS is negative. This is precisely (19). So, (19)$\rightarrow$(18).

3. A SEQUENCE OF TECHNOLOGICAL COMPETITIONS

3.1 The setting

In the model I am going to present I shall consider a sequence of technological competitions. This model borrows heavily from Loury (1979) and Vickers (1986); thus, it seems fruitful to sketch briefly the main features of Loury's (shared also by Dasgupta and Stiglitz (1980)) model, whereas Vickers'model has already been discussed in previous Sections.

Loury's model is a one-shot noncooperative game in which n symmetrically placed firms invest in R&D with the aim of innovating first. R&D costs are contractual, i.e., fixed, and the expense in R&D is then a lump-sum decided at the beginning of the competition. In Dasgupta (1982) terminology, each competition is a tournament, that is the winner gets the entire value of the innovation (the prize; for instance, a patent) and losers get nothing.[3] More precisely, losers incur negative profits given by R&D expense. There are no spillover effect in the innovative activity. There are two forms of uncertainty (see Kamien and Schwartz (1982) on this distinction): technological uncertainty and market uncertainty. The former is captured through an exponential distribution function of the so-called waiting time for innovation (see Appendix B for the derivation and the main properties of this distribution function); the latter is captured in the structure of the payoffs. There is discounting. The firms evaluate the prize equally; then, being symmetrically placed, they have the same incentive to win the competition. Furthermore, at equilibrium within Loury's model each firm is equally likely to win the competition.

In what follows I shall combine features of both Loury's and Vickers' models. To be more precise, I consider two firms competing in a finite sequence of technological competitions. Each competition is modelled like a Loury competition (contractual costs, market and technological uncertainty), but I rule out discounting and the infinite time horizon. Moreover, I focus on the asymmetric situation which arises when firms have different costs each period and hence different incentives to win each competition. During each period between two races there is perfect patent protection (i.e., no imitation) and there is no spillover effect in the R&D activity.

3.2 The model

There are two firms labelled A and B which invest in R&D at the beginning of each competition (race). There are T ($<\infty$) races, one each period, which is convenient to number backwards: t= T,T-1,....., 2,1.

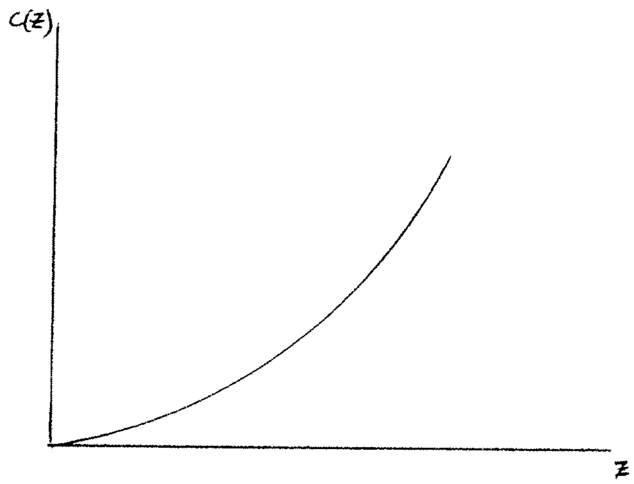
The "leader" is the firm with the lowest unit cost. Let $c_A = a$ and $c_B = b$ be the initial indices of cost of firms A and B, respectively ($b > a > 0$). Let x_t and y_t be the efforts in R&D af A and B, respectively, $t \in [1,T]$. Of course, $x_t, y_t \geq 0$.

The cost of effort is c(z), z=x,y. It is assumed that

c(z) is strictly increasing and twice differentiable;

$$c(0) = 0,\ c'(0) = 0,\ c'' > 0,\ \lim_{z \to \infty} c'(z) \to \infty$$

The figure below shows a cost curve which satisfies these conditions



For any t, if A innovates first then he receives a prize V_t^+ and B receives W_t^- . When B innovates first then he receives a prize W_t^+ and A receives V_t^-.

Let V_t and W_t be the incentives (or prize evaluations) to win race t (t=1,2,...,T) of A and B, respectively. Using Harris and Vickers (1986) notation we can write

$$V_t = V_t^+ - V_t^-$$

or alternatively, in Vickers (1986) notation,

$$V_t = \Omega(t,b) - \Omega(a,t)$$

Notice that this form of the incentives assumes that the winner gets a technological lead (as in Vickers'model). Analogously, for firm B

$$W_t = W_t^+ - W_t^-$$

or

$$W_t = \Omega(t,a) - \Omega(b,t)$$

where t (<a) is the cost level reached by the winner of the t-th competition.

It can be shown (see Appendix B) that the probability that A wins race t (i.e., innovates first in the t-th competition) is $x_t / (x_t + y_t)$ and B wins with complementary probability $y_t / (x_t + y_t)$. The payoff of A when indices of cost are a and b and firms are going to compete for an innovation allowing the winner to get cost t is

$$\Omega_A(a,b) = \pi(a,b) + \frac{x_t V_t^+}{x_t + y_t} + \frac{y_t V_t^-}{x_t + y_t} - c(x_t)$$

which can be rewritten as

$$\Omega_A(a,b) = \pi(a,b) + V_t^- + \frac{x_t V_t}{x_t+y_t} - c(x_t)$$

Analogously for B we have

$$\Omega_B(b,a) = \pi(b,a) + \frac{y_t W_t}{x_t+y_t} + \frac{x_t Wt}{x_t+y_t} - c(y_t)$$

$$= \pi(b,a) + W_t^- + \frac{y_t W_t}{x_t+y_t} - c(y_t)$$

It is assumed that

$V_t^+ > V_t^-$, $W_t^+ > W_t^-$ $\forall\, t \in [1,T]$; i.e., both players (firms) prefer to be first;

π is strictly increasing in its second argument and strictly decreasing in its first argument. This means that the profit of each firm increases with the rival's cost and decreases with its own cost.

Deleting subscripts, first order conditions are

$$\partial\Omega_A/\partial x = yV/(x+y)^2 - c'(x) = 0$$

$$\partial\Omega_B/\partial y = xW/(x+y)^2 - c'(y) = 0$$

Second order conditions for a maximum are satisfied;

$$\partial^2\Omega/\partial x^2 = -2yV/(x+y)^3 - c''(x) < 0$$

$$\partial^2 \Omega / \partial y^2 = -2xW/(x+y)^3 - c''(y) < 0$$

Moreover, $\partial \Omega_A / \partial x > 0$ at x=0 provided that $y>0$, and $\partial \Omega_B / \partial y > 0$ at y=0 provided that $x>0$. Therefore, the first order conditions describe global maxima. Hence in equilibrium we have

$$x^* = (Vy^*/c'(x^*))^{1/2} - y^*$$

and

$$y^* = (Wx^*/c'(y^*))^{1/2} - x^*$$

where x^* and y^* denote the equilibrium values of the effort rates. Under the above assumptions on V, W, c and Π, it is possible to show that an equilibrium exists in which x^* and y^* are both strictly positive. The argument follows the same strategy of Harris and Vickers (1986) in their existence theorem which they prove in the non-contractual case. The equilibrium in our context displays the following important feature

Property 1: At equilibrium

$$x^* > y^* \Leftrightarrow V > W \Leftrightarrow V^+ + W^- > V^- + W^+$$

Proof. From the two first order conditions we have

$$V/W = x^*c'(x^*)/y^*c'(y^*)$$

■

This means that the firm with the greater incentive invests more and then has a greater probability of innovating first.

In what follows I shall find a sufficient condition for a property analogous to what Vickers (1986) has called Increasing Dominance. However, given the stochastic framework above, I must redefine such a pattern of market evolution. I define

PID (Probabilistic Increasing Dominance): the firm with the lowest cost has a greater probability of winning race t than the rival; i.e., if $a < b$,

$$\forall t \in [1,T] \ , \ x_t > y_t .$$

Given Property 1, this is equivalent to

$$\forall t \in [1,T] \ , \ V_t = V_t^+ - V_t^- > W_t^+ - W_t^- = W_t$$

As a first step I shall confine myself to the two-period version of the model, i.e., T=2 and t=2,1. However, once one tries to solve the model, i.e., comparing V_1 with W_1 and V_2 with W_2,[4] one notices that V_1 and W_1 have relatively simple algebraic espressions, but this is not unfortunately the case for V_2 and W_2. Since the comparison between V_2 and W_2 turns out to be puzzling without any further specification of the ingredients of V_2 and W_2 (i.e., of c and Π), I shall prove my main results using simple functional forms for the cost and the profit functions. To be more precise, I use the following functions

$$c(z) = z^2/2 \qquad (z=x,y) \text{ and}$$

$$\Pi(\alpha,\beta) = \beta - \alpha + \delta$$

where $\delta = \max_{\alpha,\beta} |\beta - \alpha|$

It is easy to ascertain that this form of c(z) satisfies the properties we required to the cost function. Concerning the profit function, first of all it is noteworthy that with this specification each competition among firms is a constant-sum noncooperative game.[5] Indeed,

$$\Pi(\alpha,\beta) + \Pi(\beta,\alpha) = 2\delta$$

Thus, δ does not depend on α and β and it may be thought of as an index of market profitability (the extent of the "cake"). Since the case $\alpha = \beta$ is ruled out by the twofold circumstance that firms have different initial costs and the innovations are such that the winner gains (or mantains) the leadership (no catching up), δ is strictly positive. Furthermore, this specification of the profit function satisfies both properties we assumed above, i.e., profit is

(i) non-negative;

(ii) strictly increasing in its second argument and strictly decreasing in its first argument.

3.3 Results

The first result is the following

Proposition 3. The following

(H3) $\alpha > \beta \Rightarrow \Pi(\alpha, \beta) = 0$

is a sufficient conditon for PID.[6]

Proof. Before the beginning of the competition allowing the winner to get c=2, firms have cost levels $c_A = 3$ and $c_B = k$ for some $k > 3$. The tree of the game is depicted in Figure 1.

(Figure 1 here)

In order to distinguish the efforts in R&D for the last race in the two intermediate nodes (2,k) and (3,2) - i.e., when $(c_A = c_L = 2, c_B = c_H = K)$

and ($c_A=c_H=3, c_B=c_L=2$) - I have denoted $\hat{x}_1, \hat{y}_1$ the efforts of A and B, respectively, once A has won the previous race (t=2) and x_1, y_1 their efforts once B has won the previous race. Corresponding to the nodes other than the initial one, I have specified in Figure 1 the payoffs of the winner and the loser. According to (H3), the loser of a race gets nothing (at least) until next race.

We must show that the low cost firm has a greater probability of winning each race than the high cost firm. The proof proceeds by backwards induction. Let us consider the last race; i.e., t=1. Firms have cost levels $c_A=2$ and $c_B=k$ or some $k>2$. Then, the incentives in the last race are

$$V_1 = \Pi(1,k) > \Pi(1,2) = W_1$$

because of the assumption that Π is strictly increasing with respect to its second argument. Thus A, the low cost firm, has a greater probability of winning the last race because

$$V_1 > W_1 \Longleftrightarrow x_1 > y_1 \Longleftrightarrow x_1/(x_1+y_1) > y_1/(x_1+y_1)$$

Let us consider now t=2, the first (or last but one) race. Firms have cost levels $c_A=3$ and $c_B = k$ for some $k>3$.
We have

$$V_2 = \Omega(2,k) - \Omega(3,2)$$

$$= \Pi(2,k) + \frac{\hat{x}_1\Pi(1,k)}{\hat{x}_1+\hat{y}_1} - c(\hat{x}_1) - \frac{x_1\Pi(1,2)}{x_1+y_1} + c(x_1)$$

$$W_2 = \Omega(2,3) - \Omega(k,2)$$

$$= \Pi(2,3) + \frac{y_1\Pi(1,3)}{x_1+y_1} - c(y_1) - \frac{\hat{y}_1\Pi(1,2)}{\hat{x}_1+\hat{y}_1} + c(\hat{y}_1)$$

Remembering our choice of π and c, deleting subscript 1, we can write

$$V_2 = (k-2+\delta)+\hat{x}(k-1+\delta)/(\hat{x}+\hat{y})-\hat{x}^2/2-x(1+\delta)/(x+y)+x^2/2$$

$$W_2 = (1+\delta)+y(2+\delta)/(x+y)-y^2/2-\hat{y}(\delta+1)/(\hat{x}+\hat{y})+\hat{y}^2/2$$

Defining

$$\hat{z} = \sqrt{\frac{\delta+1}{k-1+\delta}}$$

$$\hat{s} = k-1+\delta$$

$$z = \sqrt{\frac{\delta+2}{\delta+1}}$$

from the first order conditions of page 27 we may calculate the equilibrium values of $\hat{x}$, x, $\hat{y}$, y:

$$\hat{x}^* = \sqrt{\hat{s}\hat{z}/(1+\hat{z})} \quad ; \quad \hat{y}^* = \hat{z}\sqrt{\hat{s}\hat{z}/(1+\hat{z})}$$

$$x^* = \sqrt{(\delta+1)z/(1+z)} \quad ; \quad y^* = z\sqrt{(\delta+1)z/(1+z)}$$

and the corresponding probabilities are

$$\hat{x}/(\hat{x}+\hat{y}) = 1/(1+\hat{z}) \quad ; \quad \hat{y}/(\hat{x}+\hat{y}) = \hat{z}/(1+\hat{z})$$

$$x/(x+y) = 1/(1+z) \quad ; \quad y/(x+y) = z/(1+z)$$

Hence we can write

$$V_2 = (k-2+\delta)+(k-1+\delta)/(1+\hat{z})-\hat{s}\hat{z}/2(1+\hat{z})^2-(1+\delta)/(1+z)+z(\delta-1)/2(z+1)^2$$

$$W_2 = (1+\delta)+z(2+\delta)/(1+z)-z^3(\delta-1)/2(z+1)^2-\hat{z}(1+\delta)/(1+\hat{z})+s\hat{z}^3/2(1+\hat{z})^2$$

Lengthy and tedious calculations show that $V_2 > W_2$. This means that A, the low cost firm, has a greater probability of winning also race t=2, and it completes the proof. ■

Our next result is contained in Proposition 4. It gives a sufficient condition in the two period case for what I label PNID (Probabilistic Non-Increasing Dominance). PNID means that the high cost firm at each stage has a probability of winning at least as great as the probability of the low cost firm. PNID is a weaker result than AR (Action-Reaction).

I shall confine again myself to the two-period case.

Proposition 4. The following

$$\text{(H4)} \quad \sigma(t,t+1) \geq \sigma(t,t+k) \qquad k>1$$

is a sufficient condition for PNID.

Proof. Considering again the extensive form of the game we have

(Figure 2 here)

where the notation has the same meaning as in the proof of Proposition 3. We show that $W_1 \geq V_1$ and $W_2 > V_2$.

Let us consider the last race; i.e., t=1. Firms have cost levels $c_A=2$ and $c_B=k$ for some $k>2$. The incentives to win the last race are

$$V_1 = \pi(1,k) - \pi(2,1)$$

$$W_1 = \pi(1,2) - \pi(k,1)$$

and $W_1 \geq V_1$ by (H4). Thus the high cost firm (B) has a probability at least as great as firm A of winning the last race because

$$W_1 \geq V_1 \longleftrightarrow y_1 \geq x_1 \longleftrightarrow y_1/(x_1+y_1) \geq x_1/(x_1+y_1)$$

Let us consider now t=2. Firms have costs $c_A=3$ and $c_B=k$ for some $k>3$. Deleting subscript 1, the incentives to win this race are

$$V_2 = \pi(2,k) + \hat{x}\pi(1,k)/(\hat{x}+\hat{y}) + \hat{y}\pi(2,1)/(\hat{x}+\hat{y}) - c(\hat{x})$$
$$- \pi(3,2) - x\pi(1,2)/(x+y) - y\pi(3,1)/(x+y) + c(x)$$

for the low cost firm (A), and

$$W_2 = \pi(2,3) + y\pi(1,3)/(x+y) + x\pi(2,1)/(x+y) - c(y)$$
$$- \pi(k,2) - \hat{y}\pi(1,2)/(\hat{x}+\hat{y}) - \hat{x}\pi(k,1)/(\hat{x}+\hat{y}) + c(\hat{y})$$

for the high cost firm (B).

Since A and B have the same chance of winning the last race (because they have the same incentive, i.e., $V_1=W_1$), we see that $W_2 > V_2$ if

$$\left[(2,3) + \frac{1}{2}(1,3)\right] - 2c > \left[(2,k) + \frac{1}{2}(1,k)\right] - 2\hat{c}$$

where $c= c(x_1) = c(y_1)$ and $\hat{c} = c(\hat{x}_1) = c(\hat{y}_1)$.

Since the expression in square bracket on the LHS is greater than the expression in square bracket on the RHS by (H4), the above inequality holds if $c < \hat{c}$. This turns out to be the case, as one can easily check using the usual first order conditions of page 27 and the chosen functional forms of the profit and the cost functions. More precisely, $\hat{c} > c$ and then W_2 is strictly grater than V_2. Thus, B has a greater probability of winning the first race than A, and this completes the proof. ∎

It is worth noting that this result is weaker than that contained in Proposition 3. In fact, we have proved that the high cost firm has always a probability at least as great as the low cost firm of winning the competition. More precisely, B has the same probability as A of winning the last race and a greater probability of winning the first race.

3.4 Concluding remarks and further extensions

The main conclusions which can be drawn from this work can be summarized as follows.

(a) The two regimes labelled Increasing Dominance (ID) and Action-Reaction (AR) may be derived from the same conditions (hypotheses) irrespective of the nature of the innovations. Namely, either the innovations are drastic or incremental. In this sense, Propositions 1 and 2 may complement the results of Vickers (1986), although it is not really AR in the context of my Proposition 2.

(b) Relaxing the bidding game-like form of the patent race and adopting a Loury-like competition, it is still possible - see Proposition 3 - to show that the stochastic version of ID (PID) derives from the same assumption (sufficient condition) as in the bidding game case. A result similar to (but weaker than) AR, christened PNID (Probabilistic Non-Increasing Dominance), has been proved in Proposition 4.

It seems worth exploring the relationship between market structure and technological competition in presence of a sequence of innovative opportunities in other directions. In particular:

(i) Adopting a Lee and Wilde form of competition, i.e., non-contractual R&D costs.

(ii) Considering a sequence of Loury and/or Lee and Wilde competitions for incremental innovations.

(iii) Allowing for an infinite time horizon; i.e., an infinite sequence of innovations ($T=\infty$). This extension could be pursued in the simple context of the last model, where the specific functional form of the profit function converted the original game in a constant-sum game. On this topic - the value of a constant-sum two person game with an infinite number of stages - I shall try to apply the approach of Zamir (1973), whereas the infinite horizon version in a less restrictive perspective (i.e., non constant games) could be explored along the lines of Fudenberg and Levine (1983) and Harris (1985).

(iv) Considering a sequence of product-innovations. This case has been scrutinized by Beath *et al.* (1985), but it seems worth looking more closely at the relationship between product - and process-innovations with respect to the market leadership when a sequence of innovations are in prospect.

I hope to tackle these extensions in further research.

3.5 Appendix B

In this Appendix the derivation and the main properties of the exponential distribution function are shown and commented.

Suppose that F is the distribution function of the waiting time to the occurrence of an innovation. As the waiting time must be positive, let we assume

$$F(0)=0$$

Furthermore suppose that

$$F(t) < 1 \quad \forall t \quad \text{and}$$

$$(*) \quad (1-F(t_1,t_2))/(1-F(t_1)) = 1 - F(t_2) \qquad t_1, t_2 \geq 0$$

The RHS of (*) is the probability that the waiting time exceeds t_2; by the definition of conditional probability, the LHS is the probability that the waiting time exceeds (t_1+t_2), given that it exceeds t_1. In other words, (*) captures the lack of memory (or Markov property or aftereffect) of the waiting time mechanism.

If after a time interval of t_1 the innovation has not yet occurred, the waiting time still remaining is conditionally distributed just as the entire waiting time from the beginning. The condition (*) completely determines the form of F. Let us substitute the distribution function of t with its tail:

$$U(t) = 1 - F(t)$$

Then (*) is now

$$U(t_1 + t_2) = U(t_1)U(t_2)$$

which is a form of Cauchy's equation. Since U is bounded it may be proved (e.g., Billingsley (1979), p. 168) that

$$U(t) = \exp(-\alpha t) \qquad \text{for some } \alpha$$

Furthermore, since

$$\lim_{t \to \infty} U(t) = 0$$

α must be positive. Thus condition (*) implies that F has the following exponential form

$$F(t) = 0 \text{ if } t \leq 0; \qquad F(t) = 1 - \exp(-\alpha t) \quad \text{if} \quad t \geq 0$$

It is worth noting that the lack of memory (*) above carries over to the exponential distribution (which can be derived as the limit of geometric distributions) and to no other distributions.

The exponential distribution has mean

$$E(t) = \int_0^\infty (1 - F(t)\, dt = 1/\alpha$$

which is interpreted as expected waiting time for the innovation.

NOTES

1) Reinganum (1982) model belongs to this group although in her game firms do not choose a number but a time-dipendent path of R&D expenditure.

2) Of course, this is not to say that models with a single race do not capture any form of interaction; for instance, Harris and Vickers (1985) (1986) models are about a single race, but they are games of real interaction.

3) See Stewart (1983) for a generalization of Loury (1979) and Lee and Wilde (1980) models according to which there is a "share parameter" describing the manner in which profits are shared among rivals once one firm innovates.

4) Indeed we have

$$V_2 = \Omega(2,k) - \Omega(3,2) = \pi(2,k) + \frac{\hat{x}_1 \pi(1,k)}{\hat{x}_1 + \hat{y}_1} + \frac{\hat{y}_1 \pi(2,1)}{\hat{x}_1 + \hat{y}_1} - c(\hat{x}_1)$$
$$-\pi(3,2) - \frac{x_1 \pi(1,2)}{x_1 + y_1} - \frac{y_1 \pi(3,1)}{x_1 + y_1} + c(x_1)$$

$$W_2 = \Omega(2,3) - \Omega(k,2) = \pi(2,3) + \frac{y_1 \pi(1,3)}{x_1 + y_1} + \frac{x_1 \pi(2,1)}{x_1 + y_1} - c(y_1)$$
$$-\pi(k,2) - \frac{\hat{y}_1 \pi(1,2)}{\hat{x}_1 + \hat{y}_1} - \frac{\hat{x}_1 \pi(k,1)}{\hat{x}_1 + \hat{y}_1} + c(\hat{y}_1)$$

where the key symbols have been defined in the proof of Proposition 3.

5) Being this a constant-sum game, any equilibrium (which we have

seen to exist) is Pareto efficient. Moreover, if the game were reformulated in a proper and more rigorous way, one could check that the noncooperative equilibrium which emerges is also a perfect equilibrium.

6) Notice that (H3) might seem inconsistent with the assumption on Π, i.e., strict increasingness in the first aurgument. Indeed, under (H3), if L wins reducing furtherly his cost level, H's profit should become negative, violating the assumed non-negativity of Π. Actually, there is not any inconsistency because H would choose to not produce. This amounts to interpreting players in the race like _active_ as well as _potential_ firms.

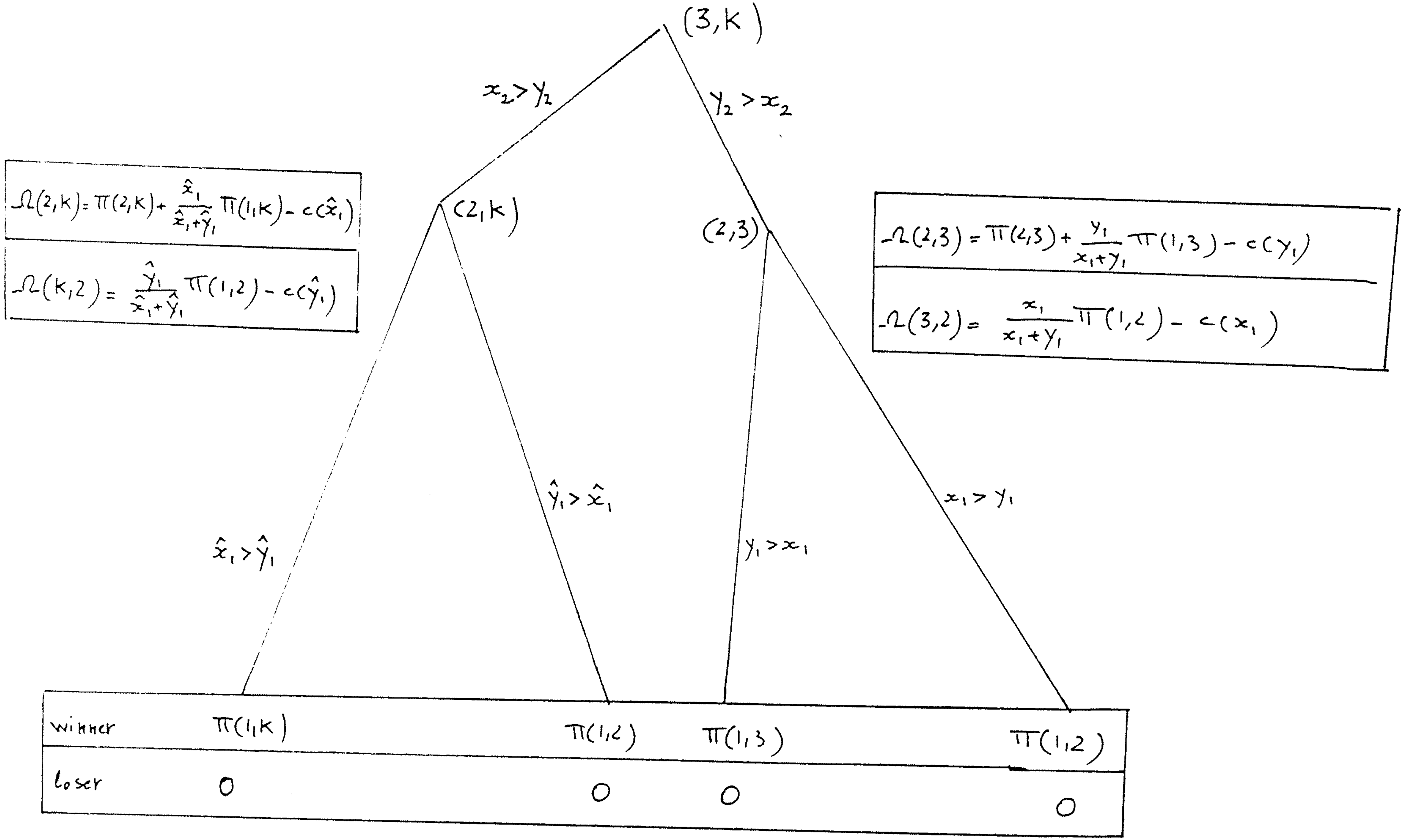

(3,K)
$x_2 > y_2$
$y_2 > x_2$
(2,K)
(2,3)
$\Omega(2,K) = \pi(2,K) + \frac{\hat{x}_1}{\hat{x}_1+\hat{y}_1}\pi(1,K) - c(\hat{x}_1)$
$\Omega(K,2) = \frac{\hat{y}_1}{\hat{x}_1+\hat{y}_1}\pi(1,2) - c(\hat{y}_1)$
$\Omega(2,3) = \pi(2,3) + \frac{y_1}{x_1+y_1}\pi(1,3) - c(y_1)$
$\Omega(3,2) = \frac{x_1}{x_1+y_1}\pi(1,2) - c(x_1)$
$\hat{x}_1 > \hat{y}_1$
$\hat{y}_1 > \hat{x}_1$
$y_1 > x_1$
$x_1 > y_1$
winner
loser
$\pi(1,K)$
$\pi(1,2)$
$\pi(1,3)$
$\pi(1,2)$
0
0
0
0

FIGURE 1

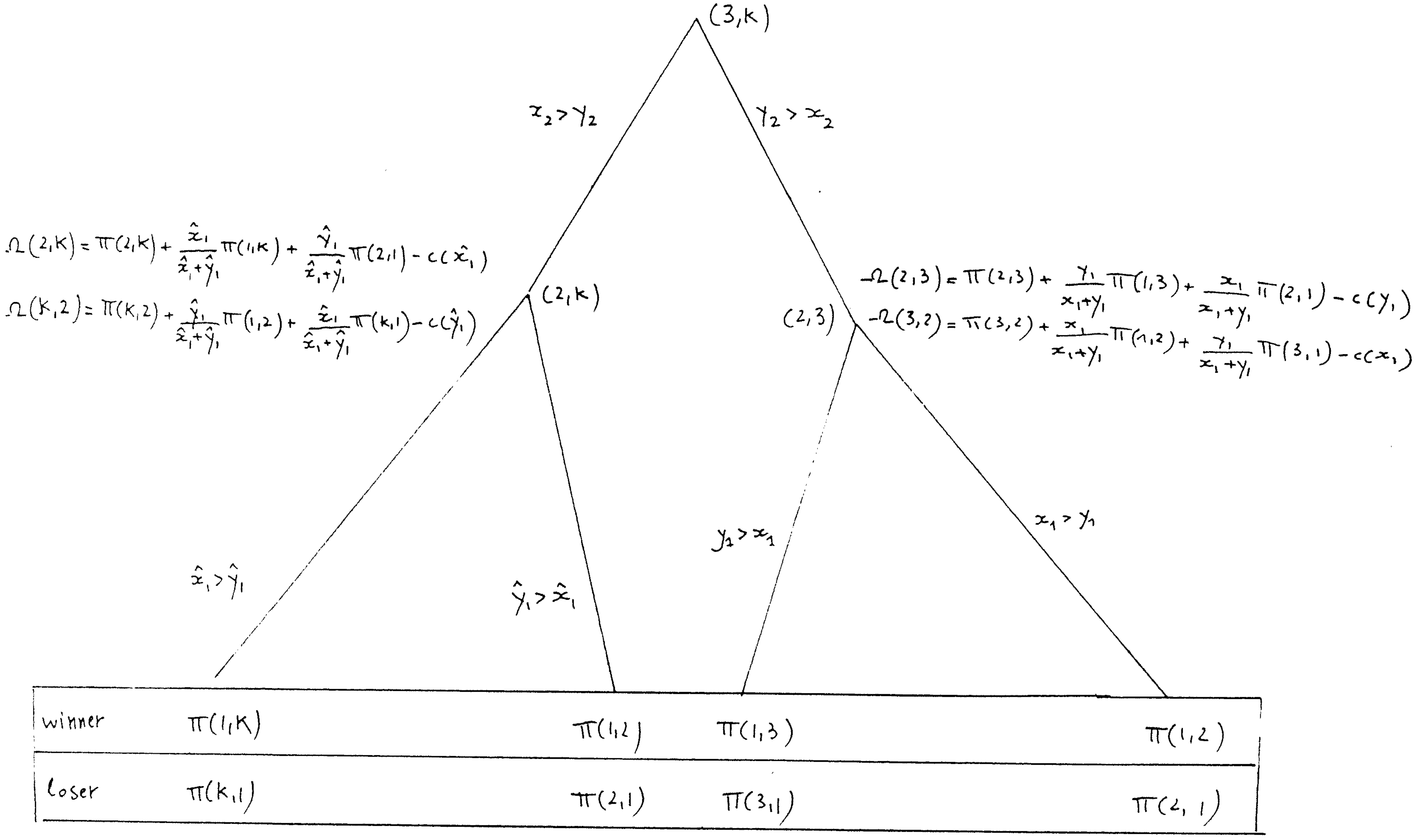

(3,K)
$x_2 > y_2$
$y_2 > x_2$
$\Omega(2,K) = \pi(2,K) + \frac{\hat{x}_1}{\hat{x}_1+\hat{y}_1}\pi(1,K) + \frac{\hat{y}_1}{\hat{x}_1+\hat{y}_1}\pi(2,1) - c(\hat{x}_1)$
$\Omega(K,2) = \pi(K,2) + \frac{\hat{y}_1}{\hat{x}_1+\hat{y}_1}\pi(1,2) + \frac{\hat{x}_1}{\hat{x}_1+\hat{y}_1}\pi(K,1) - c(\hat{y}_1)$
(2,K)
(2,3)
$\Omega(2,3) = \pi(2,3) + \frac{y_1}{x_1+y_1}\pi(1,3) + \frac{x_1}{x_1+y_1}\pi(2,1) - c(y_1)$
$\Omega(3,2) = \pi(3,2) + \frac{x_1}{x_1+y_1}\pi(1,2) + \frac{y_1}{x_1+y_1}\pi(3,1) - c(x_1)$
$\hat{x}_1 > \hat{y}_1$
$\hat{y}_1 > \hat{x}_1$
$y_1 > x_1$
$x_1 > y_1$
winner
$\pi(1,K)$
$\pi(1,2)$
$\pi(1,3)$
$\pi(1,2)$
loser
$\pi(K,1)$
$\pi(2,1)$
$\pi(3,1)$
$\pi(2,1)$

FIGURE 2

REFERENCES

Beath, J.A., et al (1985), "Product Innovation and Industry Evolution", DP no85/170, University of Bristol.

Billingsley, P. (1979), Probability and Measure, John Wiley & Sons.

Dasgupta, P. (1982), "The Theory of Technological Competition", London School of Economics ICERD Discussion Paper.

Dasgupta, P., and Stiglitz, J. (1980) "Uncertainty, Industrial Structure and the Speed of R and D", Bell Journal of Economics, 11, 1-28.

Fudenberg, D. et al (1983), "Preemption, Leapfrogging and Competition in Patent Races", European Economic Review, 22, 3-31.

Fudenberg, D., and Levine, D. (1983), "Subgame-Perfect Equilibria of Finite- and Infinite-Horizon Games", Journal of Economic Theory, 31,251-68.

Futia, C. (1980), "Schumpeterian Competition", Quarterly Journal of Economics, 94,675-95.

Gilbert, R., and Newbery, D. (1982), "Preemptive Patenting and the Persistence of Monopoly", American Economic Review, 72,514-26.

Harris, C. (1985), "A Characterization of the Perfect Equilibria of Infinite Horizon Games", Journal of Economic Theory, 37, 99-125.

Harris, C., and Vickers, J. (1985), "Perfect Equilibrium in a Model of a Race", Review of Economic Studies, 52,193-209.

---------------------- (1986), "Racing with Uncertainty", Review of Economic Studies, forthcoming.

Kamien, M., and Schwarz, N. (1982), Market Structure and Innovation, Cambridge University Press.

Lee, T. and Wilde, L. (1980), "Market Structure and Innovation: A Reformulation", Quarterly Journal of Economics,94, 429-36.

Loury, G. (1979), "Market Structure and Innovation", Quarterly Journal of Economics, 93,395-410.

Reinganum, J. (1982), "A Dynamic Game of R and D: Patent Protection and Competitive Behaviour", Econometrica, 50,671-88.

----------------------(1983), "Uncertain Innovation and the Persistence of Monopoly", American Economic Review, 73,741-8.

----------------------(1985), "Innovation and Industry Evolution", Quarterly Journal of Economics, 99,81-99.

Stewart, M.B. (1983), "Non cooperative Oligopoly and Preemptive Innovation without Winner-take-all", Quarterly Journal of Economics, 97,681-94.

Vickers, J. (1985), Patent Races and Market Structure, Unpublished D. Phil. Thesis, Oxford.

---------------- (1986), "The Evolution of Market Structure When There Is a Sequence of Innovations", Journal of Industrial Economics, 35,1-12.

Zamir, S. (1973), "On the Notion of Value for Games with Infinitely Many Stages", Annuals of Statistics, 1,791-6.