FINANCIAL PANIC, RATIONAL EXPECTATIONS (EQUILIBRIUM) AND CHAOTIC DYNAMICS: POWER AND LIMITS OF FINANCIAL MARKETS STATIBILIZATION UNDER STOCHASTIC MONEY INTEREST RATE AND THE ROLE OF INTERNATIONAL ASYMETRIES (*)

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1. INTRODUCTION

This paper analyses the concept of Rational Expectations Equilibrium (henceforth REE) in a simple dynamic model of (equilibrium) assets price determination and the important implications for the relative concept of informational efficiency. By contrast to the recent critique to this concept by Grossman and Stiglitz (1976) and (1980),focusing mainly on the incentive to spend resources on information costlessly available,we shall concentrate the attention on the prior problem concerning the REE existence in a dynamic framework,irrespective of the information structure assumed for the market process. The notion of informationally efficient capital markets is very stringent,because,at each instant,it requires that market equilibrium price "fully reflects all available information in the market and communicates it noiselessly to the market participants who evaluate this information correctly"[1].
Against this concept,we set up a simple mean-variance (optimizing) model of financial assets demand under Rational Expectations and non-constant rate of interest borne by money. The main task accomplished by the paper is to check under what conditions the REE actually exists,either when information is symmetrically or asymmetrically spread in the market.

Irrespective of the possible informational asymmetries as well as dividends policy,we shall see that specification of rate of interest policy rules turn out to be crucial for REE existence,ruling out speculative (cyclical) bubbles (in expectations),which would lead to agents'loss of confidence in the information conveied by (equilibrium) price and then to a financial panic.

This result provides a rationale for Government (and Central Bank) intervention to restore "orderly financial markets",involving a suitable level of variance for the stochastic process underlying the realizations of the rate of interest; as well as the maintenance of the investors' confidence in the market,as far as the simple monetary policy rules admitted in this paper are concerned.

Moreover,we shall give a brief account of various links relating these results to the existing (outstanding) literature,trying to unify the REE existence problem with the emergence of speculative (cyclical) bubbles (in expectations) leading to financial panic in assets market. This latter is therefore captured by the REE destruction due to the raising of speculative (cyclical) bubbles, degenerating in chaotic behaviour concernig the expected assets price.

The relative novelty of this economic application mainly refers to that unification, however, we provide a fresh exploration of the consequences of non linear dynamics involving rational expectations and the related nature of speculative (cyclical) bubbles. In this context, the standard saddlepoint approach turns out to be not able to give sufficient conditions for guaranteeing a unique REE. This will shown to be true, because transversality conditions implied by Hamiltonian dynamics do not rule out, as it is known,(2) periodic cycles or chaotic behaviour.

Hopefully, this paper tries to provides some new insights when the Rational Expectations Hypothesis (henceforth REH) is fully embodied in a perfectly competitive non (log)-linear model, without surrogating it, invoking an approximation argument, with perfect foresight or autoregressive expectations.(3) The typical volatility characterizing assets markets could find here a natural interpretation in terms of speculative (cyclical) bubbles (in expectations), suggesting in turn that the customary Efficient Market Hypothesis, regarding efficient asset prices as 'jump' variables that incorporate almost istantaneously news about current and future events, might be incompatible with market equilibrium under rational expectations and optimizing behaviour held by investors.

2. REE: A DIGRESSION.

The way in which the economic model is conveying and processing, as well as aggregating, information, compatibly with agents' maximizing behaviour, is quite often summarized with the notion of REE, usually referred to the equilibrium reached by means of the price system. The essence of the REE is that agents have some economic model (implemented in their own computer) relating somehow the exogenous (stochastic) variables to the endogenous variables which ought to be forecasted[4]. Actually,it is not a totally new notion in economics; Hicks (1939)[5],for instance,suggested the term perfect foresight for the si= tuation where producers anticipate a price and then engage in production deci= sion with the property that the actual price coming about is just the anticipa= ted price. The key point which Hicks emphasized is that expectations held by firms about endogenous future variables actually help determine the true values of the future endogenous variables. Thus,perfect foresight might be thought of as an equilibrium concept rather than a mere condition of individual rationality.

In a seminal paper,Muth (1961) presented a sort of generalization of Hicks' per= fect foresight idea for a (simple) stochastic economy,showing that the information requirements under REH are no greater than in models with rational distributed lags,like the adaptive expectations case under optimal weights selection. This latter should assume that agents know the underlying stochastic process genera= ting prices,whereas in a REE they need only to know the stochastic process underlying the equilibrium price,not necessarily the structural form of the economy,which is relevant just indirectly to explain how equilibrium comes about. The notion of REE plays a remarkable role in the study of resource allocation in stochastic economy,where Walrasian equilibrium can lead to a very poor allocation[6],since traders are just told to undertake their action exclusively based upon their own private information,ignoring the information content of the price system. Obviously,a difference can arise between a standard Walrasian allocation and a REE,as far as information is not uniformly spread in the economy. On the other hand,the Walrasian equili= brium concept does not allow to capture the old idea in economics that price contain information. Perhaps,the clearest expression of this idea appears in Hayek (1945)[7] "We must look at the price system as (...) mechanism for com= municating information if we want to understand its real function (...). The most significant fact about this system is the economy of knowledge with which it operates or how little the individual participants need to know in order to be able to take the right action (...) by a kind of symbol,only

the most essential information is passed on (...)" .
Under certain conditions[8],it can be shown that the price system has the remarkable property that Hayek suggested;in an economy where traders have diverse information,the REE allocation is <u>as if</u> each trader has all the economy's information[9]. In technical terms,the REE price is a sufficient statistics for all of the economy's information[10]. One can say that allocations are as they were generated by a Walrasian equilibrium for an artificial economy where each trader has all of the economy's information,as far as competitive prices are the best information aggregator. Hence,REE cannot be Pareto dominated by a fully informed (central) planner.[11]
The terminology often used in the literature [12] when prices convey all the information needed at REE is 'full communications equilibrium',recalling the idea of traders pooling -perhaps with the aid of a central planner- all the information signals before trading. A quite recent result,due to Jordan (1983),tells us that a REE may not be 'fully revealing' -i.e. not a full communications equilibrium- if,loosely speaking,the dimension of the price space is higher than the dimension of the signal space. This comes about,because the map $P:S \rightarrow S^n$,associating each vector of signals $s \in S$ -where S is the signal space- to the (equilibrium) price vector p^* in the n-dimensional unit simplex,S^n,is not invertible,when $DIM(S^n) > DIM(S)$. Nevertheless,it can be shown that under certain conditions REE certainly exists,provided there are a finite number of different possible signals[13]. On the other hand,Allen (1982) shows that,if the dimension of the signal space,S,is lower (or equal) than (to) the dimension of the price space a fully revealing REE generically exists. The full communications property now holds,as far as traders can learn the all economy information simply by inverting the map $P(S)$[14],which is always possible in this case,up to a set of (Lebesgue)measure zero. The implication of this latter fact is that,if $P(S)$ were not invertible,we can approximate as close as we like this 'bad behaved' economy with one having a fully revealing equilibrium price.

At this stage,it seems worthwhile to stress that the possible non existence of a REE found in this paper for assets market does not depend upon the relative dimensionality of price and signal space,which is set,by construction,to be identical. This result suggests that Allen's and Jordan's theorems do not carry over in a "dynamic framework"characterizing the assets market equili= brium price. Essentially,this is due to the presence of a forward looking behaviour inducing a dynamic structure in expectations becoming intrinsically non linear,because not only the first moment is entering in the model,but also the variance. Moreover,this results restricts the general validity of the 'Efficient Market Hypothesis'theorem for assets market,due to Fama (1970) and (1979),even in the 'sophisticated' version proposed by Hellwig (1982). In fact,if one is not restricting himself to the special case of constant interest rate - as Hellwig (1982) is doing - but we allow for a more flexible monetary policy rule,then the variance underlying the realizations of the interest rate stochastic process becomes crucial for thr REE existence. Although it is not perfectly clear how to restablish informational efficiency under asymmetric information from what we have found,it seems reasonably evident,under symmetric information,that,regulating the interest rate stocha= stic property,one can achieve financial markets stabilization .
It is noteworthy that Grandmont (1984) has studied an analogous kind of stabilization for goods market. However,there is a substantial difference with the present way of modeling expectations,although Grandmont does not seem aware of this distinction,because (15) is claiming that an autoregressive price expectations rule of the type:

$$P^e_{t+1} = \Psi (P_t , P_{t-1} , \dots , P_{t-\tau}) \qquad \forall t = 0, 1, 2, \dots \quad \tau > 0 \tag{1}$$

where Ψ is a known function,is a suitable approximation of the REH. This is not actually true,unless all other information different from prices is redundant,which is not generally the case. Hence,one can creep into the doubt that the effectiveness of monetary policy in Grandmont's model is coming from not a fully rational expectations held by traders.

On the other hand, there is a sense in which the present model is related to the Grandmont's one. In this latter model monetary policies should be designed to force the economy to settle at only one of the many long run (periodic) equilibria. In the present model the Government (or Central Bank) can force the existence of the REE, sticking to a certain interval of variability for the second moment of interest rate stochastic process that speculative (cyclical) bubbles (in expectations) are thrown away from the market.

Finally, to get a flavour of how the non-existence theorem for REE works in an assets market framework, by contrast to the standard pure exchange (stochastic) competitive model, one can compare a traditional goods market equilibrium price determination where:

$$\tilde{P}_t = D\,(Q_t\,,\,\tilde{\eta}_t)$$
$$Q_t = H\,(E\tilde{P}_t) \qquad (2)$$

$\tilde{P}_t$ = Price

E(.)=Expectation operator taken with respect to the distribution of $\{\tilde{\eta}_t\}$
H(.)= Supply , D(.)=Demand , Q_t=Quantity, $\{\tilde{\eta}_t\}$ =stochastic factors affecting demand.

with the assets market equilibium price. The existence of REE for (2) requires that Q_t=H(.), market equilibrium, so that actual and expected price are constrained to fulfil the equations:

$$\tilde{P}_t = D\,(H(E\tilde{P}_t)\,,\,\tilde{\eta}_t) \qquad (3)$$

$$p^* = E\,(D(H(p^*)\,,\,\tilde{\eta}_t)) \qquad (4)$$

where $p^*=Ep_t$ is just the only (real)number solving the equation (4). Suitable assumptions on D(.) and H(.) can guarantee that p* is actually unique, so that conditional expectation is well-defined. As far as the assets market is concerned, there cannot be general presumption, as we shall see, that equation of type (4) admits just a solution (in the real field), unless restrictive conditions are imposed on parameters entering in the model.

3. A DYNAMIC ASSET PRICE EQUILIBRIUM MODEL UNDER RATIONAL EXPECTATIONS AND RISK AVERSION.

3.1

I shall study a capital market with one risky asset and one riskless asset in an infinite sequence of periods $T=\{1,2,\dots\}$. We are going to consider a fairly general model which could be related to the outstanding literature on REE and on the Efficient Market Hypothesis(16).

An investor, buying one unit of risky asset, traded at price p_t in period t, will receive a dividend D_{t+1} in period t+1 and, moreover, he can resell the asset at the new ex-dividend price p_{t+1}. One unit then yields the gross return $H_{t+1}=D_{t+1}+p_{t+1}$ in period t+1. The alternative riskless asset, say money, has a constant price, say 1, for all $t\in T$ and a net rate of return r_t, which is known at t. Thus, the purchase of one unit of riskless asset in period t yields a certain gross return $1+r_t$.

The sequence $\{p_t,r_t,D_t\}_{t=0}^{\infty}$ of prices, rates of interest and dividends is the realization of a stochastic process $\{\tilde{p}_t,\tilde{r}_t,\tilde{D}_t\}_{t=0}^{\infty}$ on an underlying probability space $(\Omega,\mathcal{F},\nu)$. The dividend process $\{\tilde{D}_t\}_{t=0}^{\infty}$ and the rate of interest process $\{\tilde{r}_t\}_{t=0}^{\infty}$ are assumed to be exogenous. The price process $\{\tilde{p}_t\}_{t=0}^{\infty}$ is exogenous as far as the individual investor is concerned, but it will be determined endogenously in market equilibrium.

The set of all agents is represented by a measure space $(A,\mathcal{A},\mu)$. In each period t, a given agent $a\in A$, choosing his holding of risky asset y_t^a, can get a wealth at t+1, W_{t+1}^a, drawn from the random variable:

$$\tilde{W}_{t+1}^a=\left(\tilde{H}_{t+1}-p_t\,(1+r_t)\right)y_t^a+W_t^a\,(1+r_t) \tag{5}$$

$$\forall a\in A,\ \forall t\in T$$

Assuming that he is maximizing a utility function of the type:

$$U_a(\tilde{W}^a)\equiv-\operatorname{Exp}\left(-b(a)\,\tilde{W}^a\right) \tag{6}$$

$$b(a)>0 \qquad \forall a\in A$$

implying a constant (absolute) risk aversion coefficient b(a) (higher values of b(a) mean greater risk aversion),the holding y_t^a is chosen so as:

$$\underset{y_t^a}{MAX}\ \underset{t}{E}^a\left(-EXP\left(-b(a)\left(\left(\tilde{H}_{t+1}-P_t(1+r_t)\right)y_t^a+W_t^a(1+r_t)\right)\right)\right) \qquad (7)$$

$$\forall a \in A,\ \forall t \in T$$

provided the Expected Utility theorem for the choice under uncertainty holds.[17]
Moreover,we shall assume that the sequence $\{\tilde{D}_t,\tilde{p}_t\}_{t=0}^{\infty}$ is generated by normal random variables,so that (7) can be written as

$$\underset{y_t^a}{MAX} - EXP\left(-b(a)\left(\underset{t}{E}^a\tilde{W}_{t+1}^a - \frac{1}{2}b(a)\underset{t}{VAR}^a\tilde{W}_{t+1}^a\right)\right) \qquad (8)$$

$$\forall a \in A,\ \forall t \in T$$

since $\tilde{W}_{t+1}^a$ is now a normally distributed random variable,therefore (7)becomes the moment generating function of the random variable W_{t+1}^a.
Substituting (5) into (8) and differentiating (8) with respect to y_t^a ,one gets the demand for risky asset for each agent,setting the derivative equal to zero

$$y_t^a = \frac{\underset{t}{E}^a\tilde{H}_{t+1} - (1+r_t)P_t}{b(a)\ \underset{t}{VAR}^a\tilde{H}_{t+1}} \qquad \forall t \in T,\ \forall a \in A \qquad (9)$$

where $\underset{t}{E}^a\tilde{H}_{t+1} \equiv E(\tilde{H}_{t+1}\,|\,\Omega_t^a)$, $\underset{t}{VAR}^a\tilde{H}_{t+1} = E((\tilde{H}_{t+1}-\underset{t}{E}^a\tilde{H}_{t+1})^2\,|\,\Omega_t^a)$
are the conditional mean and variance of $\tilde{H}_{t+1}$ held by agent a.
The corresponding aggregate demand is given by the mean of individual demand

$$\int_A y_t^a\, d\mu(a)$$

with respect to the measure μ. The market clears whenever aggregate demand for the risky asset is just equal to the aggregate supply,which is taken to be an exogenous constant,y. Therefore we get from (9) the equilibrium condition:

$$P_t = \rho_t\left(\left(\int_A \frac{\underset{t}{E}^a\tilde{H}_{t+1}}{b(a)\ \underset{t}{VAR}^a\tilde{H}_{t+1}}\,d\mu(a) - y\right) \Big/ \left(\int_A \frac{1}{b(a)\ \underset{t}{VAR}^a\tilde{H}_{t+1}}\,d\mu(a)\right)\right) \qquad \forall t \in T \qquad (10)$$

To make (10) operational, one must specify the process $\{\tilde{D}_t, \tilde{\rho}_t\}_{t=0}^{\infty}$, where $\rho_t = 1/1+r_t$ is the rate of discount. Throughout the paper we shall assume it consists of random walks with constant anticipated component, namely:

$$\tilde{D}_t = d + \tilde{\epsilon}_t \qquad \tilde{\epsilon}_t \sim I.I.N.(0, b_\epsilon^2), \quad d \in [0, +\infty)$$
$$\tilde{\rho}_t = \delta + \tilde{\beta}_t \qquad \tilde{\beta}_t \sim I.I.D.(0, b_\beta^2, \dots), \quad \delta \in (0,1] \tag{11}$$

Moreover, to make sure that the integrals in (10) are well defined, we impose the following condition:

A_1: The function $b: A \rightarrow R_{++}$ is measurable. The harmonic mean, $\hat{b} := 1/\int_A (1/b)\, d\mu$, exists and is strictly positive.

3.2

We shall restrict ourselves in this section to the special simple case, where all agents share the same information about $\{\tilde{D}_t, \tilde{\rho}_t\}_{t=0}^{\infty}$, Ω_t, including all realizations up to period t . Obviously, each agent is aware of (11), that is to say monetary policy is supposed to enforce a rate of interest path, which has a constant anticipated component over time and a transitory component indipendently and identically distributed with known and constant mean and variance. The same hypothesis applies for the dividends policy except the further assumption of normality for the transitory component.

Making full use of (11) we can now rewrite (10) as:

$$P_t = \rho_t \left(E_t \tilde{P}_{t+1} + d - \hat{b}\, y \left(VAR_t\, \tilde{P}_{t+1} + b_\epsilon^2 \right) \right) \qquad \forall t \in T \tag{12}$$

which is the equation driving the asset price over time. It is worthwhile to notice that, aside of minor changes, (12) is similar to the equation studied by Begg (1984) and Van der Ploeg (1935). Despite that, we should give some details in addressing the solution for two main reasons. First, we wish to point out how the arising of speculative bubbles can be related to the non existence of a REE, or, in other words, that (12) needs not to admit a solution. Secondly, the nature of the (potential) REE seems to be essential in order to justify the Begg's (and Van der Ploeg's) method of solution.

A way to approach (12), looking for a REE, is by means of the undetermined coefficients method, namely to restrict the attention to linear REE of the type:

$$p_t = \pi_1 \rho_t \qquad \forall t \in T, \quad \pi_1 \in \mathbb{R} \tag{13}$$

Plugging (13) into (12) we get the relation: (18)

$$\pi_1 = \frac{(\delta - 1) \pm ((\delta-1)^2 + 4\hat{b}y\sigma_\beta^2(d - \hat{b}y\sigma_\varepsilon^2))^{\frac{1}{2}}}{2\hat{b}y\sigma_\beta^2} \tag{14}$$

so that the expected value of p_{t+1} conditional to Ω_t is therefore given by:

$$\underset{t}{E}\tilde{p}_{t+1} \equiv E(\tilde{p}_{t+1}|\Omega_t) = \begin{cases} \dfrac{(\delta-1)+((\delta-1)^2+4\hat{b}y\sigma_\beta^2(d-\hat{b}y\sigma_\varepsilon^2))^{\frac{1}{2}}}{(2\hat{b}y\sigma_\beta^2/\delta)} \\ \dfrac{(\delta-1)-((\delta-1)^2+4\hat{b}y\sigma_\beta^2(d-\hat{b}y\sigma_\varepsilon^2))^{\frac{1}{2}}}{(2\hat{b}y\sigma_\beta^2/\delta)} \end{cases} \tag{15}$$

which is not in general a well defined map from the set of information, Ω_t to the real line, as it is required by the definition of conditional expectation (19). As a matter of fact, it is a two-values correspondance mapping on the imaginary axis as long as:

$$4\hat{b}\sigma_\beta^2 y(\hat{b}y\sigma_\varepsilon^2 - d) > (\delta - 1)^2 \tag{16}$$

so that in this case there cannot be any equilibrium not only because the expectation is not well defined, but also for the fact that imaginary values for expected and actual asset price do not make any sense.

In principle, only when

$$(\delta - 1)^2 = 4\hat{b}\sigma_\beta^2 y(\hat{b}y\sigma_\varepsilon^2 - d) \tag{17}$$

(15) becomes a well defined map, but the REE coming out, namely:

is economically unfeasible, since the asset prices must be negative, because $\delta \in (0,1]$.

The third possibility is given by the inequality

$$(\delta - 1)^2 > 4 \hat{b}\, b_\beta^2\, y (\hat{b}\, y\, b_\varepsilon^2 - d) \qquad (19)$$

implying two real values in (15), one positive and the other one negative, which still makes the REE in trouble. A sensible way out might be to suppose that agents discard in (15) the negative root for π_1, because it would imply a negative actual market price. In other words, the correspondance would be transformed in a function on economic grounds. However, there is no a priori reason why (19) needs to hold, unless a proper Government (or Central Bank) intervention establishes a suf= ficient low variance for the rate of interest or adequate measure to guarantee enough confidence in the market, stimulating a suitable propension to bear market risk, i.e. low values of $\hat{b}$. The same effect is associated to a low variance in the dividends process or to high expected dividends. If we were considered the bond market case, where there is no uncertainty about the asset's yield - as Begg (1984) did- then b_ε^2 is vanishing, (provided a constant coupon is payed out) hence (19) always holds.

Thus, despite Begg's opinion, a REE equilibrium always exists as far as bond markets are concerned, provided one accepts the argument put forward before. One the other hand, we must be careful about this conclusion, because, as we will se later on, this existence relies strongly upon certain transversality condi= tions, varying according to certain parameters appearing in (15).

This consideration introduces the need of a detailed analysis of the expectations dynamics, in order to specify those transversality conditions.

To do that one has to form conditional expectations at (t-1) into (12) to get:

$$\underset{t-1}{E} \tilde{P}_t = \delta \left(\underset{t-1}{E} \tilde{P}_{t+1} + d - \hat{b}\, y \left(\underset{t-1}{VAR} \tilde{P}_{t+1} + b_\varepsilon^2 \right) \right) \qquad (20)$$

$$\forall t \in T$$

We shall prove that the view at (t-1) about the mean and the variance of $\tilde{p}_{t+1}$ is identical to the view at t;this is the case not only because $\tilde{\rho}_t$ is I.I.D., so that the agents learn nothing from the realizations $\{\rho_t\}_{t=0}^{\infty}$ -as Begg seems to believe as well as Van der Ploeg -but also as far as a particular dynamic structure,like (12),is driving the asset price. Simple calculations from (13) yield in fact:

$$\underset{t}{E}\,\tilde{p}_{t+1} = \underset{t-1}{E}\,\tilde{p}_{t+1} = \delta \pi_1 \;;\quad \underset{t-1}{VAR}\,\tilde{p}_{t+1} = \underset{t}{VAR}\,\tilde{p}_{t+1} = \pi_1^2 b_\beta^2 \qquad \forall t \in T \tag{21}$$

as long as the REE exists.

On the other hand,if the asset price in (12) were depending also upon the past rate of interest,like for instance:

$$p_t = \rho_t \left(\underset{t}{E}\,\tilde{p}_{t+1} + d - \hat{b}y \left(\underset{t}{VAR}\,\tilde{p}_{t+1} + b_\epsilon^2 \right) \right) + c\,\rho_{t-1} \qquad c \in \mathbb{R}_+ ,\ \forall t \in T \tag{22}$$

thus the REE has the form: (20)

$$p_t = \pi_1 \rho_t + \pi_2 \rho_{t-1} + \pi_3 \rho_t^2 \qquad \forall t \in T \tag{23}$$

where $\{\pi_i\}_{i=1}^{3}$ are given by the equations:

$$\pi_2 = \pi_3 = c$$

$$\hat{b}y\, b_\beta^2\, \pi_1^2 + (1-\delta)\pi_1 + (d - \hat{b}y\, b_\epsilon^2) + c\left(c\, b_\beta^2\, \hat{b}y\, (2\delta + b_\beta^2) - (\delta^2 + b_\beta^2) \right] = 0 \tag{24}$$

so that now the view of p_{t+1} at (t-1) differs form that one at t;in fact from (23) we get:

$$\underset{t}{E}\,\tilde{p}_{t+1} = \delta \pi_1 + \pi_2 \rho_t + \pi_3 (\delta^2 + b_\beta^2) \;;\quad \underset{t-1}{E}\,\tilde{p}_{t+1} = \delta(\pi_1 + \pi_2) + \pi_3 (\delta^2 + b_\beta^2) \tag{25}$$

$$\underset{t}{VAR}\,\tilde{p}_{t+1} = \pi_1^2 b_\beta^2 + \pi_3^2 b_\beta^2 (2\delta + b_\beta^2) \;;\quad \underset{t-1}{VAR}\,\tilde{p}_{t+1} = b_\beta^2 \left(\pi_1^2 + \pi_2^2 + \pi_3^2 (2\delta + b_\beta^2) \right)$$

Property (21) plays a crucial role in solving (20), since by subtracting it from

(12) and squaring both side,one can take conditional expectation,with respect to Ω_{t-1} ,so that we end up with an expression for the conditional variance:

$$\underset{t-1}{E}\left(\tilde{P}_t - \underset{t-1}{E}\tilde{P}_t\right)^2 \equiv \underset{t-1}{VAR}\,\tilde{P}_t = \left(b_\beta^2 / \delta^2\right)\left(\underset{t-1}{E}\tilde{P}_{t+1}\right)^2 \qquad \forall t \in T \tag{25}$$

which allows us to rewrite (20) as:

$$\underset{t-1}{E}\tilde{P}_t = \delta\left(\underset{t-1}{E}\tilde{P}_{t+1} + d - \hat{b}y\left(\left(\underset{t-1}{E}\tilde{P}_{t+1}\right)^2\left(b_\beta^2/\delta^2\right) + b_\epsilon^2\right)\right) \qquad \forall t \in T \tag{26}$$

(26) must hold for all future periods t+j not merely for t,because $_t$ is I.I.D. and its conditional expectation and unconditional are identical,hence we have:

$$X_J = \delta\left(X_{J+1} + d - \hat{b}y\left(\left(X_{J+1}^2\left(b_\beta^2/\delta^2\right) + b_\epsilon^2\right)\right)\right) \equiv g(x_{J+1}) \tag{27}$$

$$X_J \equiv \underset{t-1}{E} P_{t+J} \qquad \forall J = 0,1,2,\ldots \;, \; \forall t \in T$$

3.3

In order to shed some light on the solution of the forward looking difference equation (27),we will deal with the topologically conjugate equation:

$$Z_{\tau+1} = M Z_\tau \left(1 - Z_\tau\right) \equiv f(z_\tau);\; M \equiv \delta(1 - 2N), \tag{28}$$

$$Z_\tau \equiv \left(X_{J+\hat{\tau}-\tau}\left(\hat{b}y\, b_\beta^2 / \delta^2\right) - N / 1 - 2N\right) \qquad \forall \tau = 0,1,2,\ldots,\hat{\tau} \quad \forall J = 0,1,2,\ldots$$

where N must satisfy the polynomial equation:

$$N^2\delta + (1-\delta)N - \delta\left(d - \hat{b}y\, b_\epsilon^2\right)\hat{b}y\left(b_\beta^2/\delta^2\right) = 0 \tag{29}$$

and $\hat{\tau}$ is some terminal date that in what follows must be assumed equal to + ∞. The direction of the time is reversed in (28),with respect to the forward looking equation (27),because the transformation makes more natural the application of some standard theorems for non linear difference equations.

Comparing (30) with (15) ,we get non surprisingly the equality:

$$\underset{t}{E}\,\tilde{P}_{t+1} = \begin{cases} X^2 \Rightarrow Z^2 = 1-(1/M) \\ X^1 \Rightarrow Z^1 = 0 \end{cases} \tag{31}$$

so that the stationary points of g(x) coincide with the expected price in REE; these points are in turn biunivocally related to the stationary points of f(z).

The key point to understand at this stage is that,since by construction

$$\lim_{\tau \to +\infty} z_\tau = \lim_{\tau \to +\infty} f^{\tau}(\hat{z}_0) = \lim_{\tau \to \hat{\tau}} X_{\hat{\tau}-\tau} = X_0 = \underset{t-1}{E}\,\tilde{P}_t \quad , \hat{\tau} = \infty \tag{32}$$

then the asymptotic behaviour of $\{z_\tau\}$ is crucial for the existence of the REE; in other words,if the limit in (32) does not exist (finite),the REE breaks down. We shall see that this depends upon the terminal condition, $\hat{z}_0$, chosen as well as the parameter M characterizing f(z).

To start off the analysis we have to assume that (19) is holding,otherwise there cannot be any REE,eihter under (16),because meaningless,or (17), because economically unfeasible.

First we notice that the first fixed point z^1 is always not reachable by almost all sequences $\{z_\tau\}_{\tau=0}^{\infty}$,since we have:

$$f'(Z^1) = M = 1 + \left((\delta-1)^2 + 4\hat{b}yb_\beta^2(d-\hat{b}yb_\epsilon^2)\right)^{\frac{1}{2}} > 1 \tag{33}$$

unless the terminal condition is chosen in such a way that:

$$\hat{Z}_0 = Z^1 = 0 \Rightarrow X_J \equiv \underset{t-1}{E}\,\tilde{P}_{t+J} = \hat{X} = X^1 = \frac{(\delta-1) - \left((\delta-1)^2 + 4\hat{b}yb_\beta^2(d-\hat{b}yb_\epsilon^2)\right)^{\frac{1}{2}}}{(2\hat{b}yb_\beta^2/\delta)} \quad \forall J = 0,1,\ldots \tag{34}$$

But,as we have pointed out before,this choice would imply a negative expected and actual price,so that the agents would probably wish to rule out this possibility.Obviously,this argument is satisfactory as long as we believe in the confidence in the market for the agents.

To make the solution of (27) fully determined, so to have a fully specified path for all expected future prices at any date t, we ought to fix a terminal condition, which has the form in terms of x or z:

$$Z_0 = \hat{Z}_0 \quad , \quad \lim_{J\to\infty} X_J = \hat{X} \quad , \quad Z_0 = \hat{X}\,(\hat{b}y\,b_\beta^2/\delta^2) - N/1-2N \qquad (29)$$

In linear rational expectations models the set of admissable terminal conditions is customarily chosen so that the solution lies on a stable manifold, according to the conventional saddlepoint approach. This (arbitrary) choice can sometimes be rationalized in terms of the transversality conditions characterizing an optimal plan in models with infinite-lived households(21). Essentially, it requires that non-predetermined variables -like assets price here involved- behave as if they were co-state variables of Hamiltonian dynamics (i.e. shadow prices). Even though in many models this justification is totally ad hoc, it is enough for having a well defined REE, as far as a linear dynamics in expectations is concerned.

We shall see that, although we are assuming expected price not growing "too fast", namely of exponential order lower than the rate of pure time preference held by the agents, a REE needs not to be well defined, unless the choice were restricted to a set with only one element.

To make (28) operational, one must choose one value of N from (29); it is fairly natural to stick with:

$$N = \frac{(\delta-1) - \left((\delta-1)^2 + 4\hat{b}y\,b_\beta^2\,(d-\hat{b}y\,b_\epsilon^2)\right)^{\frac{1}{2}}}{2\hat{b}y\,b_\beta^2} \qquad (30)$$

because it allows the following relationship in terms of fixed points for f(z) and g(x) defined in (28) and (27):

$$z^1 = 0 \implies x^1 = \frac{(\delta-1) - \left((\delta-1)^2 + 4\hat{b}y\,b_\beta^2\,(d-\hat{b}y\,b_\epsilon^2)\right)^{\frac{1}{2}}}{(2\hat{b}y\,b_\beta^2/\delta)} \qquad (30)$$

$$z^2 = 1 - \frac{1}{M} \implies x^2 = \frac{(\delta-1) + \left((\delta-1)^2 + 4\hat{b}y\,b_\beta^2\,(d-\hat{b}y\,b_\epsilon^2)\right)^{\frac{1}{2}}}{(2\hat{b}y\,b_\beta^2/\delta)}$$

$$z^i = f(z^i)\,,\quad x^i = g(x^i) \qquad i=1,2$$

On the other hand one can show, plotting the parable defind by $f(z)$ in a phase dyagram, that the economically feasible REE defined by x^2, i.e. z^2 is always reachable by the sequence $\{z_\tau\}_{\tau=0}^{\infty}$ whatever terminal condition $\hat{z}_0$ is chosen, because it is a locally stable stationary point, since we have:

$$f'(z^2) = -M - 2 \equiv 1 - ((\delta-1)^2 + 4\hat{b}y b_\beta^2 (d - \hat{b}y b_\epsilon^2))^{\frac{1}{2}} \in (-1, 1) \tag{35}$$

provided that:

$$((\delta-1)^2 + 4\hat{b}y b_\beta^2 (d - \hat{b}y b_\epsilon^2))^{\frac{1}{2}} \in (0, 2) \tag{36}$$

Furthermore one can prove that z^2 is a stable node for

$$((\delta-1)^2 + 4\hat{b} b_\beta^2 y (d - \hat{b}y b_\epsilon^2))^{\frac{1}{2}} (0, 1) \tag{37}$$

or a stable spiral if

$$((\delta-1)^2 + 4\hat{b} b_\beta^2 y (d - \hat{b}y b_\epsilon^2))^{\frac{1}{2}} \in (1, 2) \tag{38}$$

These two cases correspond respectively to monotonic convergence and over or under-shooting of the sequence $\left\{ \underset{t-1}{E} P_{t+\hat{\tau}-\tau} \right\}_{\tau=0}^{\hat{\tau}=+\infty}$ towards the REE

$$\underset{t-1}{E} P_t = x^2 = (\delta-1) + ((\delta-1)^2 + 4\hat{b}y b_\beta^2 (d - \hat{b}y b_\epsilon^2))^{\frac{1}{2}} / (2\hat{b}y b_\beta^2/\delta) \tag{39}$$

Whenever

$$((\delta-1)^2 + 4\hat{b}y b_\beta^2 (d - \hat{b}y b_\epsilon^2))^{\frac{1}{2}} \geq 2 \tag{40}$$

the sequence $\{z_\tau\}_{\tau=0}^{\infty}$ never converges, unless the terminal condition is chosen in such a way to have:

$$\hat{z}_0 = z^2 = 1 - (1/M) \Longrightarrow X_J \equiv \underset{t-1}{E} P_{t+J} = \hat{X} = x^2 = \frac{(\delta-1) + ((\delta-1)^2 + 4\hat{b}y b_\beta^2 (d - \hat{b}y b_\epsilon^2))^{\frac{1}{2}}}{(2\hat{b}y b_\beta^2/\delta)} \quad \forall_{J=0,1,2,\ldots} \tag{41}$$

Therefore the REE does not exists as long as (40) does not hold.

This result throws new light on the condition (19) for the existence of a REE, which turns out to be far more restrictive than before from a practical viewpoint; in fact the discriminant defined in (14) needs to be not only positive, but has in practice an upper bound, namely 4. The terminology "in practice" here means that, whenever (40) holds, the only admissable transversality condition for the existence of the REE becomes (41), so that even a very small perturbation, due for instance to a not perfect knowledge of the parameters appearing in the model by the agents, can destroy the potential REE.
Summarizing, we might say that, if (40) is holding, then the REE is not robust under small (and unperceived) perturbations in the parameters. On the contrary, as long as (36) is fulfilled, the REE is very robust, because any terminal condition that one might have chosen leads to a well defined REE.
To complete the framework, we shall give some indication about the expectations path under (40); well known results on chaotic dynamics tell us the following classification[22]

$\{E_{t-1} P_{t+\hat{\tau}-\tau}\}_{\tau=0}^{\hat{\tau}=+\infty}$	values of $((\delta-1)^2+4\hat{b}yb_\beta^2(d-\hat{b}yb_\epsilon^2))^{\frac{1}{2}}$
2-period cycle	(2, 2.449)
4-period cycle	(2.449, 2.544)
8-period cycle	(2.544, 2.564)
Even k-period cycle, k $\geq$ 16	(2.564, 2.570)
All even-period cycles	(2.570, 2.679)
Odd-period cycles	(2.679, 2.828)
Chaotic behaviour	(2.828, 3)
$-\infty$	(3, $+\infty$)

this table is essential based on the relationship:

$$M = 1 + ((\delta-1)^2 + 4\hat{b}yb_\beta^2(d-\hat{b}yb_\epsilon^2))^{\frac{1}{2}} \tag{42}$$

therefore the classificatory parameter M,characterizing $f(z)$,has been used as reference to evaluate the numbers in the table .

As it is well known[23],the emergence of chaos implies a very strong sensitivity of the expected price path from the terminal condition,so that even an infinitesimal deviation from $\tilde{z}_0 = z^2_*$ can lead to a very complicated trajectory for $\left\{E_{t-1} P_{t+\hat{\tau}-\tau}\right\}_{\tau=0}^{\hat{\tau}=\infty}$,destroying the REE,since it is certainly not convergent to a point (in the backward direction). Hence,erratic path occurring for expectations would demand extraordinary analytical power of the investors in the assets market to locate the terminal condition in the unique point allowing the existence of the REE. Any,even little,change of regime in the market could be accompanied by the REE breakdown,because agents might not be able to detect istantaneously the switch.

To conclude this section,we ought to say something about the results found by Begg (1984) and Van der Ploeg (1985) for bonds market case. Clearly this paper is inspired by those works,although there are some fairly important differences. Essentially Begg's conclusions coincide with what we have found in this section,because,holding $\sigma_\varepsilon^2=0$ in (35),we get the expression (30) in Begg (1984) and,roughly speaking,our conditions (36) and (40) can be locally summarized by the figures 3 and 4 that he puts forward.[24] The same,on the other hand,does not apply to Van der Ploeg (1985),who first suggests the analytical treatment in terms of chaotic dynamics for the equation (26),but it does not seem aware of the fact that these dynamics contradict the notion of REE. As we have seen,non convergent sequences of $\{z_\tau\}_{\tau=0}^{\infty}$ are incompatible with the existence of the REE,hence it is substan= tially wrong to say that "(...) there exists,(...),a unique non-convergent and forward-looking rational expectations path in the shape of a limit cycle or erratic behaviour"[25]. The implication of this incompatibility is that the Efficient Market argument ,regarding asset prices as jump variables that efficiently incorporate 'news' about current and future events,might contradict market equilibrium.

Next section will be devoted to a more in-depth exploration of this issue.

3.4

So far we have dealt with the case of symmetric information ,that is to say each agent in the market is conditioning the expected price upon the same set of information. We shall show that the non-existence problem for the REE carries over also when there are informational asymmetries in the market. Consequently,the standard notion of informationally efficient capital markets where equilibrium price is supposed to fully reflect all available information in the market and communicate it noiselessly to the market participants who evaluate this information correctly,breaks down,because,under rational expecta= tions and stochastic rate of interest,there might not be equilibrium price in the market.(26) It is noteworthy to point out that this result requires as necessary and sufficient condition a rate of interest being a realization of an underlying stochastic process. In fact,as Hellwig (1982) has shown,if the rate of interest is constant the (linear) REE is well defined,moreover,in contrast with the Grossman and Stiglitz (1976) and (1980) critique,the market can ap= proximate full informational efficiency arbitrarily closely and yet the re= turn to being informed remains bounded away from zero. Nevertheless,the Hellwig's result does not appear to be robust,relying on a constant (and deterministic) rate of interest,even though it has overcome the main Grossman and Stiglitz argument,according to which informational efficiency destroys any incentive to spend resource on information,so that there would not be equili= brium in both markets,goods and information,simultaneously.

The asymmetry in the information structure will be modeled in the simplest possible way. Some agents in the market are assumed to be informed in period t of the realization of the dividends transitory component at the end of pe= riod t;formally,we may write down

$$\epsilon_{t+1} \in \Omega_t^{a_1} \qquad \forall a_1 \in A_1 \;,\; A_1 \subset A \;,\; \forall t \in T \qquad (43)$$

where A_1 should indicate the group of the so called informed agents, whereas with A_2 is the label of the complement of A_1, namely the agents who know, as before, just all realizations up to period t of all variables in the model. (42) implies in conjunction with (11) that informed agents actually are aware of the dividend that will be payed at the end of period t when they formulate their demand of assets. in the light of this argument, the expected gross return and its variance conditional to the information set held by the informed agents are given by:

$$\underset{t}{E}^{a_1} \tilde{H}_{t+1} = d + \epsilon_{t+1} + \underset{t}{E} \tilde{P}_{t+1} \, ; \, \underset{t}{VAR}^{a_1} \tilde{H}_{t+1} = \underset{t}{VAR} \tilde{P}_{t+1} \qquad \forall a^1 \in A_1 \quad \forall t \in T \tag{44}$$

whereas for those uninformed we have:

$$\underset{t}{E}^{a_2} H_{t+1} = d + \underset{t}{E} \tilde{P}_{t+1} \quad ; \quad \underset{t}{VAR}^{a_2} \tilde{H}_{t+1} = \sigma_{\epsilon}^2 + \underset{t}{VAR} \tilde{P}_{t+1} \qquad \forall a_2 \in A_2 \quad \forall t \in T \tag{45}$$

we have intentionally dropped the superscript a_1 and a_2 in the expectation operator for price, because, under normality assumption as well as independency for the process $\{\tilde{\epsilon}_t\}_{t\in T}$, it turns out that(27)

$$\underset{t}{E}^{a_1} \tilde{P}_{t+1} = \underset{t}{E} \tilde{P}_{t+1} + R^2 E \tilde{e}^2_{P_{t+1}} \left(\epsilon_{t+1} - \underset{t}{E} \tilde{\epsilon}_{t+1} \right) = \underset{t}{E} \tilde{P}_{t+1} \equiv \underset{t}{E}^{a_2} \tilde{P}_{t+1} \tag{46}$$

$$\underset{t}{VAR}^{a_1} \tilde{P}_{t+1} = \underset{t}{VAR} \tilde{P}_{t+1} (1 - R^2) = \underset{t}{VAR} \tilde{P}_{t+1} \equiv \underset{t}{VAR}^{a_2} \tilde{P}_{t+1} \qquad \forall a_i \in A_i \quad i = 1,2 \quad \forall t \in T$$

For the sake of simplicity, we shall assume that the "size" of the informed agents set is exogenously determined and costant over time, although the return of being informed it is not. This latter could be easily checked out, plugging (44) and (45) into (9) and evaluating the related indirect utility function (8). It would turn out that there would be always an incentive to gather information, since the gain from being informed is strictly positive. This would also prove that the presence of incentive to become informed rules out the non-existence problem caused by the "Grossman and Stiglitz paradox".

Substituting (44) and (45) into (10), we get the dynamic equation driving the assets price under asymmetric information:

$$P_t = \rho_t \left(\left(\hat{b}_1 \frac{d + \epsilon_{t+1} + \underset{t}{E} \tilde{P}_{t+1}}{\underset{t}{VAR}\, \tilde{P}_{t+1}} + \hat{b}_2 \frac{d + \underset{t}{E} \tilde{P}_{t+1}}{\underset{t}{VAR}\, \tilde{P}_{t+1} + b_\epsilon^2} - y \right) \right/ \tag{47}$$
$$\left/ \frac{\hat{b}_1}{\underset{t}{VAR}\, \tilde{P}_{t+1}} + \frac{\hat{b}_2}{\underset{t}{VAR}\, \tilde{P}_{t+1} + b_\epsilon^2} \right) \; ; \int_{A_1} (1/b(a))\, d\mu(a) \equiv \hat{b}_1 \, , \; \int_{A_2} (1/b(a))\, d\mu(a) \equiv \hat{b}_2$$

To solve (47), we look for a REE of the following linear type:

$$P_t = \rho_t \left(\pi_1 + \pi_2 \epsilon_{t+1} \right) \qquad \forall t \in T \tag{48}$$

Plugging (48) in(o (47), the undetermined coefficients method tells us that the following system must be fulfilled:(28)

$$\begin{aligned} Z_1 &= d \hat{b}_1 b_\epsilon^2 + v \left[d \hat{b} - y (v + b_\epsilon^2) \right] \\ Z_2 &= \hat{b}_1 (v + b_\epsilon^2) \end{aligned} \tag{49}$$

where:

$$\begin{aligned} Z_1 &\equiv (1-\delta)\left(\hat{b} v + \hat{b}_1 b_\epsilon^2 \right) \pi_1 \\ Z_2 &\equiv \left(\hat{b} v + \hat{b}_1 b_\epsilon^2 \right) \pi_2 \\ v &\equiv \pi_1^2 b_\beta^2 + \pi_2^2 b_\epsilon^2 (\delta^2 + b_\beta^2) \\ \hat{b} &\equiv \hat{b}_1 + \hat{b}_2 \end{aligned} \tag{50}$$

It turns out that the following polynomial equation in λ:

$$\begin{aligned} &\lambda^4 y^2 b_\beta^2 - \lambda^3 \left(\hat{b}^2 (1-\delta)^2 + 2 y b_\beta^2 h_0 \right) + \lambda^2 \left(h_1 + h_0^2 b_\beta^2 - \right. \\ &\left. - 2 y d \hat{b}_1 b_\beta^2 b_\epsilon^2 - 2 \hat{b}_1 b\, b_\epsilon^2 (1-\delta)^2 \right) + \lambda \left(2 b_\epsilon^2 h_1 + 2 b_\beta^2 b_\epsilon^2 d \hat{b}_1 h_0 - \right. \\ &\left. - (1-\delta)^2 (\hat{b}_1 b_\epsilon^2)^2 \right) + h_1 (b_\epsilon^2)^2 + b_\beta^2 (d \hat{b}_1 b_\epsilon^2)^2 = 0 \\ &h_0 \equiv d \hat{b} - y b_\epsilon^2 \quad ; \quad h_1 \equiv b_\epsilon^2 \hat{b}_1^2 (1-\delta)^2 (\delta^2 + b_\beta^2) > 0 \end{aligned} \tag{51}$$

with: $$\lambda \equiv (z_2 / \hat{b}_1) - b_\epsilon^2 \tag{52}$$

can determine the unknown coefficients π_1 and π_2. In fact, calling $\bar{\lambda}$ a root (51), we get from (49) and (50) the equality:

$$\pi_1 = d\,\hat{b}_1 b_\epsilon^2 + \bar{\lambda}\left(d\,\hat{b} - y(\bar{\lambda} + b_\epsilon^2)\right) \Big/ (1-\delta)(\hat{b}\bar{\lambda} + \hat{b}_1 b_\epsilon^2) \tag{53}$$

$$\pi_2 = \hat{b}_1 (\bar{\lambda} + b_\epsilon^2) \Big/ \hat{b}\bar{\lambda} + \hat{b}_1 b_\epsilon^2$$

so that the nature of the (potential) REE depends crucially upon the roots of (51), being, in principle, 4. Moreover, one can notice that the relative weights $\hat{b}_1$ and $\hat{b}_2$ of informed and uninformed agents significantly affect the (potential) REE.

Since (51) is a polynomial of fourth degree, it has always, if any, two real roots, say $\bar{\lambda}_1$ and $\bar{\lambda}_2$, so that the expected price looks like:

$$\underset{t}{E}\,\tilde{P}_{t+1} = \begin{cases} \delta\left(d\,\hat{b}_1 b_\epsilon^2 + \bar{\lambda}_1\left(d\,\hat{b} - y(\bar{\lambda}_1 + b_\epsilon^2)\right)\right) \Big/ (1-\delta)(\hat{b}\bar{\lambda}_1 + \hat{b}_1 b_\epsilon^2) \\ \delta\left(d\,\hat{b}_1 b_\epsilon^2 + \bar{\lambda}_2\left(d\,\hat{b} - y(\bar{\lambda}_2 + b_\epsilon^2)\right)\right) \Big/ (1-\delta)(\hat{b}\bar{\lambda}_2 + \hat{b}_1 b_\epsilon^2) \end{cases} \tag{54}$$

obtained plugging (53) into (48) and conditioning to Ω_t.

(54) does not define, as in (15), a well defined map from the set of information to the real line, unless $\bar{\lambda}_1$ and $\bar{\lambda}_2$ were coincident roots. Therefore, up to a set of (Lebesgue) measure zero, the REE does not exist. As a matter of fact, now there is no sensible (economic) reason for discarding <u>a priori</u> (and in general) one of the branches of (54), because, by contrast with (14), the sign of (43) is basically undetermined, as far as the realizations of $\{\tilde{\epsilon}_t\}_{t \in T}$ can have either positive or negative sign.

We should now address the special case, analysed by Hellwig (1982) in a bit more complicated framework, where the rate of interest is supposed to be a fixed constant. In our terminology this implies the assumption:

$$\rho_t = \rho \qquad \forall t \in T \tag{55}$$

Inserting (55) into (47), the related REE of linear type is found in the form:

$$P_t = \alpha_0 + \alpha_1 \epsilon_{t+1} \qquad \forall t \in T \tag{56}$$

giving rise to the system of equations:

$$\alpha_0 = \rho \left(d + \alpha_0 - (y/\hat{b})\, \alpha_1^2 b_\epsilon^2 \right)$$

$$\alpha_1 = 1 \,/\, 1 + (\hat{b}_2/\hat{b}_1)(\alpha_1^2 b_\epsilon^2 \,/\, \alpha_1^2 b_\epsilon^2 + b_\epsilon^2) = 1/1 + (\hat{b}_2/\hat{b}_1)(\alpha_1^2/1+\alpha_1^2 \tag{57}$$

whose solution, as we show in the Appendix, is unique for real values of α_1 and α_2 [29]. Hence, the REE is now well defined.

One must however resist to the temptation of concluding that now as well by reducing b_β^2, namely the interest rate variability, we can come "close" to the REE existence, in the light of what comes about under (55), when the variance is vanishing. This would not be a priori legitimate, because, once it has been assumed:

$$\beta_t = 0 \qquad \forall t \in T \tag{58}$$

one has to prove that:

$$\lim_{b_\beta^2 \to 0} \pi_2 (b_\beta^2) = \frac{\alpha_1}{\delta} \quad , \quad \lim_{b_\beta^2 \to 0} \pi_1 (b_\beta^2) = \frac{\alpha_0}{\delta} \tag{59}$$

which is not absolutely obvious, since the nature of the function $\bar{\lambda}(b_\beta^2)$ is basically unknown on the grounds of (51). Moreover, there is no a priori reason to discard bifurcations possibilities, being able to make invalid e= ven the existence of such a limit. In conclusion, there is no evident and sensi= ble economic way to restore informational efficiency under non constant rate of interest.

4. CONCLUDING REMARKS

In this paper I have analysed the existence of the REE for a mean-variance (optimazing)model of financial markets and the related notion of informational efficiency. The specification of a suitable interest rate policy rule turns out to be crucial for thr existence of a robust REE,although this needs not to be true under asymmetric information. In fact,under the simple monetary policy rules admitted in this paper,there is no evident mechanism to restore informational efficiency in the market, as far as the REE does not exist.
A clear-cut rationale for Government (or Central Bank)intervention is therefore provided throughout the paper by restoring investors'confidence in the market and stimulating a suitable propension to bear market risk . The intervention must be designed,as far as the REE is concerned,to avoid speculative (cyclical)bubbles,which make in trouble the (potential) REE. However this type of intervention cannot be easily extended to the case of asymmetric of information,where,apparently,there is no sensible way to escape from the non-existence problem. Nevertheless the preceding result is still higly suggestive,because it sheds some light on the insufficiencies of the standard saddlepoint approach under rational expectations. This latter turns out to not be able to give sufficient conditions for having the REE existence,as far as non linear dynamics (in expectations) are concerned . This is coming about simply because Hamiltonian dynamics imposed to prices path do not rule out periodic cycles or chaotic behaviour, whose emergence is sufficient to destroy the (potential) REE.
Therefore,our results might be useful in explaining the seeming contradiction between tests of Market Efficiency on the one hand and tests of variance bounds on the other for asset prices in the standard approach. This latter seems empirically violated - according to LeRoy (1984) and Shiller (1981)- because, we believe, it rules a priori out(cyclical) bubbles in expectations by vertue of a transversality condition,which is working insofar as the variance of investment return is ignored as well as higher moments.

5. APPENDIX

5.1

We solve (12) under the linear assumption (13), which gives us the following expressions for conditional mean and variance:

$$\underset{t}{E}\,\tilde{P}_{t+1} = \delta\pi_1 \quad , \quad \underset{t}{VAR}\,\tilde{P}_{t+1} = \pi_1^2 b_\beta^2 \qquad \forall t \in T \tag{1a}$$

Plugging (13) and (1a) into (12) we get the identity:

$$\pi_1 \rho_t = \rho_t \left(\delta\pi_1 + d - \hat{b}y \left(\pi_1^2 b_\beta^2 + b_\varepsilon^2 \right) \right) \qquad \forall t \in T \tag{2a}$$

impliying the polynomial equation:

$$\pi_1^2 \hat{b}y\, b_\beta^2 - (\delta - 1)\pi_1 - (d - \hat{b}y\, b_\varepsilon^2) = 0 \tag{3a}$$

whose solution is given by(14).

Q.E.D.

5.2

To solve (22), we evaluate from (23) the expressions for conditional mean and variance:

$$\begin{aligned} \underset{t}{E}\,\tilde{P}_{t+1} &= \delta\pi_1 + \pi_2 \rho_t + \pi_3 (\delta^2 + b_\beta^2) \\ \underset{t}{VAR}\,\tilde{P}_{t+1} &= \pi_1^2 b_\beta^2 + \pi_3^2 b_\beta^2 (2\delta + b_\beta^2) \end{aligned} \qquad \forall t \in T \tag{4a}$$

so that plugging (4a) and (23) into (22), the following identity must hold:

$$\begin{aligned} \pi_1 \rho_t + \pi_2 \rho_{t-1} + \pi_3 \rho_t^2 = {} & \rho_t \big(\delta\pi_1 + \pi_3 (\delta^2 + b_\beta^2) - \\ & - \hat{b}y \left(\pi_1^2 b_\beta^2 + \pi_3^2 b_\beta^2 (2\delta + b_\beta^2) \right) \big) + c\rho_{t-1} + \pi_2 \rho_t^2 \end{aligned} \tag{5a}$$

equating the coefficients according to the date attached to the rate of discount,we end up with the equations (24).

Q.E.D.

5.3

To prove the second equalty into (46),it is sufficient to notice that the correlation coefficient R^2 is vanishing,since $\{\tilde{\epsilon}_t\}_{t\in T}$ is independently distributed by hypothesis,so that one can write:

$$R^2 = \frac{E(\tilde{e}_{P_{t+1}}\cdot\tilde{e}_{\epsilon_{t+1}})}{E\tilde{e}^2_{P_{t+1}}\,E\tilde{e}^2_{\epsilon_{t+1}}} = \frac{E((\tilde{P}_{t+1}-\underset{t}{E}\tilde{P}_{t+1})\tilde{\epsilon}_{t+1})}{E\tilde{e}^2_{P_{t+1}}\,E\tilde{\epsilon}^2_{t+1}} = \qquad (6a)$$

$$=(E\,\epsilon_{t+1})(E(\tilde{P}_{t+1}-\underset{t}{E}\tilde{P}_{t+1}))\,/\,((E\tilde{e}^2_{P_{t+1}})(b_\epsilon^2)) = 0$$

where:

$$\tilde{e}_{P_{t+1}} \equiv \tilde{P}_{t+1} - \underset{t}{E}\tilde{P}_{t+1}\,,\quad \tilde{e}_{\epsilon_{t+1}} \equiv \tilde{\epsilon}_{t+1} - \underset{t}{E}\tilde{\epsilon}_{t+1} \qquad (7a)$$

Q.E.D.

5.4

As usual,in order to solve (47),we should evaluate conditional mean and variance from the assumed linear type REE,i.e. (48):

$$\begin{aligned}\underset{t}{E}\tilde{P}_{t+1} &= \delta\pi_1\\ \underset{t}{VAR}\tilde{P}_{t+1} &= \pi_1^2 b_\beta^2 + \pi_2^2 b_\epsilon^2(\delta^2+b_\beta^2)\end{aligned} \qquad (8a)$$

Thus,plugging (8a) and (48) into (47),one can get the system of equations

$$\begin{aligned}\pi_1 = \Big(&\hat{b}_1(d+\delta\pi_1)(\pi_1^2 b_\beta^2+\pi_2^2 b_\epsilon^2(\delta^2+b_\beta^2))^{-1}+\\ &+\hat{b}_2(d+\delta\pi_1)(\pi_1^2 b_\beta^2+b_\epsilon^2(\pi_2^2(\delta^2+b_\beta^2)+1))^{-1}-y\Big)\Big(\hat{b}_1(\pi_1^2 b_\beta^2+\\ &+b_\epsilon^2\pi_2^2(\delta^2+b_\beta^2))^{-1}+\hat{b}_2(\pi_1^2 b_\beta^2+b_\epsilon^2(\pi_2^2(\delta^2+b_\beta^2)+1))^{-1}\Big)^{-1}\\ \pi_2 = \;&\hat{b}_1(\pi_1^2 b_\beta^2+\pi_2^2 b_\epsilon^2(\delta^2+b_\beta^2))^{-1}\Big(\hat{b}_1(\pi_1^2 b_\beta^2+\pi_2^2 b_\epsilon^2(\delta^2+b_\beta^2))^{-1}+\\ &+\hat{b}_2(\pi_1^2 b_\beta^2+b_\epsilon^2(\pi_2^2(\delta^2+b_\beta^2)+1))^{-1}\Big)^{-1}\end{aligned} \qquad (9a)$$

yielding the system (49),once it has been made use on the definitions (50).

Now we plug the first two definitions in (50) into the third one, so that one gets

$$\mathcal{V}=\left(\frac{z_1}{(1-\delta)(\hat{b}v+\hat{b}_1 b_\epsilon^2)}\right)^2 b_\beta^2+\left(\frac{z_2}{\hat{b}v+\hat{b}_1 b_\epsilon^2}\right)^2 b_\epsilon^2(\delta^2+b_\beta^2) \tag{10a}$$

moreover, (49) can be insert into (10a) giving us

$$v=\left(d\hat{b}_1 b_\epsilon^2+v\left(d\hat{b}-y(v+b_\epsilon^2)\right)\right)\Big/\left((1-\delta)(\hat{b}v+\hat{b}_1 b_\epsilon^2)\right)^2 b_\beta^2+ \tag{11a}$$
$$+\left(\hat{b}_1(v+b_\epsilon^2)\Big/\hat{b}v+\hat{b}_1 b_\epsilon^2\right)^2 b_\epsilon^2(\delta^2+b_\beta^2)\equiv l(v)$$

therefore solving (11a) is equivalent to find out fixed points for $l(v)$. Suitable rearrangements within (11a) yield the polynomial equation (51), once the variable v is replaced by the dummy variable λ when we look for $l(.)$'s fixed points on the real and imaginary axis.

Q.E.D.

5.5

To show that (57) has a unique (real) solution, we rewrite the second equation as:

$$q(\alpha_1)\equiv \alpha_1-\left(1\Big/1+(\hat{b}_2/\hat{b}_1)(\alpha_1^2/1+\alpha_1^2)\right)=0 \tag{12a}$$

it is easy to see that $q(\alpha_1)<0$ for $\alpha_1\leqslant 0$, $q(\alpha_1)>0$ for $\alpha_1\geqslant 1$. Moreover, $q(\alpha_1)$ is continuous and strictly increasing for positive α_1, since:

$$\frac{d\,q(\alpha_1)}{d\alpha_1}=1+(\alpha_1-q(\alpha_1))^2(\hat{b}_2/\hat{b}_1)\left(1/\;1+\alpha_1^2\right)^2>0 \tag{13a}$$

therefore, there exists a unique α_1^*, such that $q(\alpha_1^*)=0$ and $\alpha_1^*\in(0,1)$. Substituting for α_1^* into the first equation of (57), one gets

$$\alpha_0^*=\frac{\rho}{1-\rho}\left(d-(y/\hat{b})(\alpha_1^*)^2 b_\epsilon^2\right) \tag{14a}$$

Q.E.D.

NOTES

(1) See Fama (1970) and (1979). Unfortunately, the meaning of the term "fully reflect" has proved somewhat elusive (see Rubinstein (1975)).

(2) See Benhabib and Nishimura (1979) and Montrucchio (1984).

(3) Benhabib and Day (1982) and Grandmont (1984) provide overlapping generations models that generate competitive business cycles relying essentially on this argument, as far as expectations are concerned.

(4) This is obviously a very loose definition; for a more comprehensive analysis of this concept we refer to Frydman and Phelps (eds.) (1983).

(5) p. 132.

(6) See Grossman (1981) for a proof of this statement.

(7) p. 527. This point has been also brought out by Arrow (1978).

(8) See Grossman (1981), Allen (1982) and (1983), and Jordan (1983) for such con= ditions and related results.

(9) Obviously, we are referring to an artificial economy where every piece of information is common knowledge in the Aumann's (1976) sense.

(10) For a rigourous definition of sufficient statistics see Grossman (1978); roughly speaking, one might say that all the economy's information is made redundant by the price system.

(11) This extension of the standard theorems of welfare economics requires additively separable utility and a set of complete markets to hold. The former condition is certainly just sufficient, whereas the latter might be also necessary (see Radner (1979)).

(12) A good survey is provided by Jordan and Radner (1982) as well as by Radner (1982).

(13) Some counterexamples show that this restriction turns out to be crucial for the existence proof; see Kreps (1977), Green (1977), Jordan and Radner (1979) and (1982), and Radner (1982) for a broader perspective.

(14) Lucas (1972) first (perhaps) used this innocuous-looking technique, giving a tremendous insight on the notion of information conveied by prices.

(15) See Grandmont (1984) p.21.

(16) See, for instance, Fama (1970), Grossman and Stiglitz (1976), Grossman (1976) and (1978), Bray (1981) and (1985), Futia (1981), Hellwig (1982) and Stiglitz (1982).

(17) For a useful introduction to the theory of choice under uncertainty leading to the Expected Utility Theorem see the survey by Schoemaker (1982). However, the standard reference is Von Neumann and Morgenstern (1944).

(18) See section 5.1 in Appendix.

(19) Any textbook of (advanced) probability theory provides a proof of this statement (for example, Tucker (1967), p. 211, th.1).

(20) See section 5.2 in Appendix.

(21) See Brock (1975).

(22) Many authors have dealt with non linear difference equations of type (28), for instance, Hoppensteadt and Hyman (1977), Collet and Eckmann (1980) and Grandmont (1983). From this latter we draw the tools to construct the table.

(23) This is a remarkable property of chaotic dynamics to be strongly influenced by initial (or terminal, in dealing with forward-looking dynamics) conditions (see, for example, Li and Yorke (1975)).

(24) p.56.

(25) Van der Ploeg (1985), p.2.

(26) The non-existence of the REE is not, per se, a new result (see Jordan and Radner (1979) and (1982) as well as Radner (1982) for a microeconomic example). What appears to be new here is the generality of this possibility in the model we have explored. Actually, the same conclusion should be applied to the macro-model explored in Violi (1984).

(27) See section 5.3 in Appendix for a proof. We draw the reader's attention on the fact that the first equality in (46) is a standard result for conditional mean and variance (see, for example, Anderson (1958)).

(28) See section 5.4 in Appendix.

(29) See section 5.5.

REFERENCES

B. Allen (1982) "Strict Rational Expectations Equilibria with Diffuseness", Journal of Economic Theory, 27, p.20-46.

B. Allen (1983) "Expectations Equilibria with Dispersed Information: Existence with Approximate Rationality in a Model with a Continuum of Agents and Finitely Many States of the World", Review of Economic Studies, 50, p. 267-285.

T.W. Anderson (1958) "An Introduction to Multivariate Statistical Analysis", Wiley, New York.

K.J. Arrow (1978) "The Future and the Present in Economic Life", Economic Inquiry, 16, p. 157-169.

R. Aumann (1976) "Agreeing to Disagree", Annals of Statistics, 4, p.1236-39.

D.K.H. Begg (1984) "Rational Expectations and Bond pricing: Modelling the Term Structure with and without Certainty Equivalence", Economic Journal, suppl. to vol. 94, p.45-58.

J. Benhabib and K. Nishimura (1979) "The Hopf Bifurcation and the Existence and Stability of Closed Orbits in Multisector Models of Optimal Economic Growth", Journal of Economic Theory, 21, p.421-444.

J. Benhabib and R.H. Day (1982) "A Characterization of Erratic Dynamics in the Overlapping Generations model", Journal of Dynamics and Control, 4, p.37-55.

M.M. Bray (1981) "Futures Trading, Rational Expectations, and the Efficient Market Hypothesis", Econometrica, 49, p. 575-596.

M.M. Bray (1985) "Rational Expectations, Information and Asset Markets: an Intro= duction", Oxford Economic Papers, 37, P.161-195.

W.A. Brock (1975) "A Simple Perfect Foresight Monetary Model", Journal of Monetary Economics, 1, p.133-150.

P. Collet and J.P. Eckmann (1980) "Iterated Maps on the Interval as Dynamical Systems", Birkhauser, Boston.

E.F. Fama (1970) "Efficient Capital Markets: a Review of Theory and Empirical Work", Journal of Finance, 25, p.383-417.

E.F. Fama (1979) "Foundations of Finance", Blackwell, Oxford.

R. Frydman and E.S. Phelps (eds.) (1983) "Individual Forecasting and Aggregate Outcomes", Cambridge University Press, Cambridge.

C.A. Futia (1981) "Rational Expectations in Stationary Linear Models", Econometrica, 49, p.171-192.

J.M. Grandmont (1983) "Periodic and Aperiodic Behaviour in Discrete One-Dimensional Dynamical Systems", CEPREMAP D.P. No.8317, Paris.

J.M. Grandmont (1984) "On endogenous Competitive Business Cycles", IMSS No.438 Technical Paper, Stanford University, Stanford.

J.R. Green (1977) "The Non-Existence of Informational Equilibria", Review of Economic Studies, 44, p.451-463.

S.J. Grossman (1976) "On the Efficiency of Competitive Stock Markets where Traders Have Diverse Information", Journal of Finance, 31, p.573-585.

S.J. Grossman (1978) "Further Results on the Informational Efficiency of Competitive Stock Markets", Journal of Economic Theory, 18, p.81-101.

S.J. Grossman (1981) "An Introduction to the Theory of Rational Expectations under Asymmetric Information", Review of Economic Studies, 48, p.541-560

S.J. Grossman and J.E. Stiglitz (1976) "Information and Competitive Price Systems", American Economic Review (Pap.and Proc.), 66, p.246-253.

S.J. Grossman and J.E. Stiglitz (1980) "On the Impossibility of Informationally Efficient Markets", American Economic Review, 70, p.393-408.

F.H. Hayek (1945) "The Use of Knowledge in Society", American Economic Review, 35, p.519-530.

M.F. Hellwig (1982) "Rational Expectations Equilibrium with Conditioning on Past Prices: a Mean-Variance Example", Journal of Economic Theory, 26, p.279-312.

J.R. Hicks (1939) "Value and Capital", Oxford University Press, London.

F. Hoppensteadt and J. Hyman (1977) "Periodic Solutions of a Logistic Difference Equation", Siam Journal of Applied Mathematics, 32, p. 73-81.

J.S. Jordan (1983) "On the Efficient Market Hypothesis",Econometrica,51,p.1325-1343

J.S. Jordan and R Radner (1979) "The Non-Existence of Rational Expectations Equilibria :a Robust Example",Department of Economics,University of Minnesota,Minneapolis .

J.S. Jordan and R. Radner (1982) "Rational Expectations in Microeconomic Models:an Overview",Journal of Economic Theory,26,p.201-223.

D. Kreps (1977) "A Note on Fulfilled Expectations Equilibria",Journal of Economic Theory,14,p.32-43.

S.F. LeRoy (1984) "Efficiency and the Variability of Asset Prices", American Economic Review, Pap. and Proc. p. 183-87.

T. Li and J.A. Yorke (1975) "Period Three Implies Chaos",American Mathematical Monthly,82,p.985-992.

R.E. Lucas (1972) "Expectations and Neutrality of Money",Journal of Economic Theory, 4,p.103-124.

L. Montrucchio (1984) "Optimal Decisions over Time and Strange Attractors",Internal Report No.9,Department of Mathematics,Turin.

J.F. Muth (1961) "Rational Expectations and the Teory of Price Movements",Econometrica,29,p.315-335.

J. Von Neumann and O. Morgenstern (1944) "Theory of Games and Economic Behavior", Princeton University Press, Princeton.

R. Radner (1979) "Rational Expectations Equilibrium Generic Existence and the Information Revealed by Prices",Econometrica,47,p.655-678.

R. Radner (1982) "Equilibrium under Uncertainty" in Handbook of Mathematical Economics,vol.II,K.J. Arrow and M.D. Intriligator (eds.),North Holland,Amsterdam.

M.E. Rubinstein (1975) "Securities Market Efficiency in an Arrow-Debreu Economy", American Economic Review, 65, p. 812-24.

P.S.H. Schoemaker (1982) "The Expected Utility Model:Its Variants,Purposes,Evidence and Limitations",Journal of Economic Literature,20,p.529-63

J.E. Stiglitz (1982) "Information and Capital Markets",in Financial Economics: Essays in Honor of Paul Cootner,W.F. Sharpe and C.M. Cootner (eds.),Prentice-Hall

R.J. Shiller (1981) "Do Stock prices move too much to be Justified by Subsequent Changes in Dividends", American Economic Review, 71, p. 421-36.

H.G. Tucker (1967) "A Graduate Course in Probability",Academic Press,New York.

F. Van der Ploeg (1985) "Rational Expectations,Risk and Chaos in Financial Markets", Mimeo presented to the A.U.T.E./R.E.S. Conference,March, Keble College,Oxford.

R. Violi (1984) "Modelli Economici Sequenziali e Subottimalità Predittiva per le Aspettative Razionali",Rivista Internazionale di Scienze Sociali,2-3,p.416-443.