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March 1988

N. 44

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March 1988

Abstract

We study the R & D performance of Cournot oligopolists. To this end we model a one-shot noncooperative game in which firms invest in R & D, with the aim of being first in an uncertain competition for a patentable cost-reducing innovation. The incentives to innovate are market profits and not exogenously given prizes as in most of the earlier literature. Thus, incentives depend on the number of firms. We show that (i) increasing rivalry may increase or decrease the individual R & D expenditure, (ii) there may be underinvestment with respect to the socially optimal level. Moreover, we identify the various effects which are responsible for the difference between our results and some conclusions of closely related contributions.

1. INTRODUCTION

A classical problem in industrial economics is whether a more competitive market structure leads to a faster pace of innovations. Schumpeter [1947] discussed this issue contrasting a monopoly and a perfectly competitive market. Obviously, this comparison is far from exhaustive. One can study an oligopolistic market with n firms, assuming that n is a proxy of the degree of rivalry. The problem then becomes: how does an increase in n affect firms' investment in Research and Development (R & D)? A related issue is whether the market equilibrium aggregate level of R & D is greater or lower than the socially optimal level. For, the private benefit from an innovation may be lower than the social benefit (1), but in the market equilibrium there may be a "duplication of efforts" because only the firm innovating first benefits from the discovery.

Most of the recent game theoretic literature on R & D relies upon the seminal contributions of Loury [1979] and Lee and Wilde [1980](2). These models provide an elegant framework to analyse the R & D performance of competing firms and the consequent effects on social welfare.

Loury [1979] models a one-shot non cooperative game in which n identical players invest in R & D with the aim of innovating first. The probability of success by firm i is an exponential function of $h(x_i)$ – i's hazard rate – given no success to date. The firm which innovates first is awarded an exogenously given prize. The losers of the R & D race get nothing and therefore suffer a loss given by the R & D cost x_i. There is perfect and infinitely long patent protection, and no further innovation is anticipated. The R & D cost for firm i is a lump-sum x_i

at time t = 0.

Among the results, Loury shows that an increase in the number of firm reduces the equilibrium individual R & D effort. Lee and Wilde [1980] reformulate Loury's model assuming that the R & D expenditure is a flow cost that firm i pays at rate x_i until some player succeeds. This difference in the specification of R & D costs turns out to be responsible for the different impact of an increase in the number of firms on the equilibrium individual R & D effort. That is, while such effect is negative in Loury's model, it is positive in Lee and Wilde's model (3).

As far as social welfare is concerned, both models yield identical conclusions, i.e., given the finite number of firms, in the industry equilibrium each firm invests more in R & D than is socially optimal (see Loury 1979, Proposition 5, and Lee and Wilde 1980, sect. 3). It should be noticed that both Loury and Lee and Wilde assume that the social benefit from the innovation is equal to the prize obtained by the winning firm. Then, the socially optimal level of R & D is obtained maximising the difference between the discounted value of the social benefit flow and the total R & D costs.

Although in the literature referred to above great emphasis has been put on the relationship between the intensity of rivalry and the R & D performance, the relationship between the structure of the product market and incentives to invest in R & D has been almost completely overlooked. Indeed, neither Loury [1979] nor Lee and Wilde [1980] explicitely model the product market. In their models, the prize is exogenously given and is independent of the number of firms, the losers get nothing, and it is not taken account of the possibility that before the innovation firms make positive profits.

This very particular specification of incentives and payoffs can be rationalised in at least three different ways. The first one is to assume that the n firms compete in prices in a homogenous product market, so that a Bertrand equilibrium results. Then, before the innovation -- when they share the same technology -- all firms make zero profits, and after the innovation the winner, which has reduced his own cost, will be the only active firm. Second, one can think of an initial position where all firms have such high costs that it is not profitable for them to enter the market (as in the case when the innovation creates a new product); thus, only the innovator will be active in the post-innovation equilibrium. Finally, a third interpretation of Loury's and Lee and Wilde's models is that of firms (laboratories) as producers of know-how which is valuable only to the first succeeding in discovery, which can patent the innovation and sell it to the final producers.

However, we think that other specifications of the incentives should be allowed: for instance, one may imagine that firms are quantity-setting Cournot players in the product market. The interest of modelling the R & D performance of Cournot oligopolists can be motivated by noticing that:

"The Cournot situation is of particular interest because price, quantity, and profits <u>are strictly functions of n</u>, the number of firms in the industry, and the results [...] have some very interesting implications for the more general problem of market structure and research inclination" [Horowitz 1963, p. 128, emphasis added].

It seems therefore that a Cournot oligopoly provides a richer setting to investigate the relationship between the intensity of rivalry (as measured by the number of firms) and the R & D performance (4).

The type of competition in the product market is also relevant to social welfare. In order to perform a welfare analysis of the industry equilibrium, Loury [1979] and Lee and Wilde [1980] postulate the equality between social and private returns from R & D. Thus, they can capture only the "duplication of effort" effect, and overinvestment with respect to the social optimal level of R & D results. However, when the product market is a Cournot oligopoly, the private benefit from the innovation is given by the gain in terms of future profits, while the social benefit also includes the change in consumers' surplus. In a more general model, therefore, Loury and Lee and Wilde's conclusions cannot be taken for granted.

In what follows we shall present a model in which, while retaining the basic framework of the aforementioned models - actually we shall adopt the R & D cost specification of Lee and Wilde - it can be shown that some of their conclusions no longer hold true. More precisely, in a simple model where firms make positive profits in the pre-innovation Cournot market equilibrium and in the post-innovation Cournot equilibrium all firms still have positive (although different) market share, simple comparative statics shows that: (a) an increase in the number of firms may result in a decrease in the equilibrium R & D effort of each firm (i.e., the opposite of Lee and Wilde's result); (b) in equilibrium there may be underinvestment with respect to properly defined social optimum levels.

The rest of the paper is organized as follows. We present the model in section 2, where we show that Lee and Wilde's result ($\delta x/\delta n > 0$ in equilibrium) still holds when the post-innovation market profits of the winner as well as those of the losers do not depend of the number of firms. When they do, generally speaking the effect of an increase in n is ambiguous. To obtain more definite results, we specialise the model in section 3. In section 4 we show that an increase in the degree of rivalry can reduce the equilibrium R & D effort rate. Section 5 contains the welfare analysis of the market equilibrium. Finally, some concluding remarks and suggestions for extending the model are presented in section 6.

2. BACKGROUND

Lee and Wilde consider a variant of Loury's model with non contractual R & D costs. They model a one-shot game where the payoff function of firm i (i=1,2,...,n) is the present value of expected profits, net of R & D costs, i.e.

$$V_i = \int_0^\infty \exp[-(\Sigma_j h(x_j)+r)t]\,[h(x_i)W - x_i]\,dt - F$$

$$= \frac{Wh(x_i) - x_i}{a + h(x_i) + r} - F \qquad (1)$$

where W is the prize accruing to the innovating firm, which is assumed to be a fixed sum (the same for all firms), x_i is i's R & D expenditure, $h(x_i)$ is i's instantaneous probability of innovating, $a = \Sigma_{j \neq i} h(x_j)$ is the instantaneous probability that one of the (n - 1) rivals of firm i innovates, r is the discount rate, and F is a fixed R & D cost. For sake of simplicity, the hazard function $h(x_i)$ is assumed to be strictly concave and to satisfy the following conditions

$$h(0) = 0$$

$$\lim_{x_i \to \infty} h'(x_i) = 0$$

$$\lim_{x_i \to 0} h'(x_i) = \infty$$

These conditions guarantee that the maximisation problem will always yield an interior solution, and that the second order condition is satisfied [5].

The first order condition for a maximum for firm i $(i=1,2,\ldots,n)$ is

$$(a + r)[Wh'(x_i) - 1] - [h(x_i) - x_i h'(x_i)] = 0 \qquad (2)$$

In a symmetric equilibrium, (2) becomes

$$[(n-1)h(x) + r][Wh'(x) - 1] - [h(x) - xh'(x)] = 0 \qquad (3)$$

Let $x^\wedge$ denote the solution to (3), i.e. the equilibrium R & D effort. Implicitly differentiating $x^\wedge$ with respect to n one gets

$$\frac{\delta x}{\delta n} = - \frac{h(x)[Wh'(x) - 1]}{\delta f/\delta x} \qquad (4)$$

where f denotes the L.H.S. of equation (3). A "stability condition" of the model is that in equilibrium a marginal increase in R & D investment by any single firm causes the investment of each other firm to fall by a smaller amount. It can be shown [6] that this condition is equivalent to $\delta f/\delta x < 0$. Then, the sign of $\delta x^\wedge/\delta n$ is given by the sign of the term inside square brackets in (4). By the first order condition (2), [Wh'(x) - 1] has the same sign as [h(x) - xh'(x)], which is positive by the concavity of $h(x_i)$. Hence $\delta x\hat{}/\delta n$ is positive.

Retaining all the other features of Lee and Wilde's model, Stewart [1983] drops the assumption that the winner takes all. She introduces a "share parameter" σ, such that the prize to the winner is σW, and each loser gets an equal share $(1-\sigma)/(n-1)$ of the total benefit W, where $(1/n) < \sigma \leq 1$. The objective function of firm i then becomes

$$V_i = \frac{h(x_i)\sigma W + a\dfrac{(1-\sigma)}{(n-1)}W - x_i}{r + a + h(x_i)} - F \qquad (5)$$

Proceeding as above, one gets the symmetric equilibrium condition:

$$[r + (n-1)h(x)][h'(x)\sigma W - 1] - h'(x)h(x)(1-\sigma)W - h(x) + xh'(x) = 0 \qquad (6)$$

Implicitly differentiating x^ with respect to n one obtains:

$$\frac{\delta x}{\delta n} = -\frac{h(x)[\sigma Wh'(x) - 1]}{\delta f/\delta x} > 0 \qquad (7)$$

Thus, Lee and Wilde's result still holds (7). In a more structured model, however, the payoff functions can be more complicated than those assumed by Lee and Wilde [1980] and Stewart [1983].

Consider a symmetric Cournot oligopoly with n active firms. The firms are competing in the product market, and are also competing for a cost reducing innovation. As far as the technological competition is concerned, we model a one-shot game, i.e. only one innovation is in prospect and it gives the winner the exclusive right to use forever a more productive technology. Assume that all firms have the same cost before the innovation and the pre-innovation equilibrium in the product market is symmetric. The payoff of firm i (i=1,2,...,n) in the R & D game is given by the present value of future profits net of R & D costs:

$$V_i = \int_0^{\infty} \exp[(-\Sigma_j h(x_j) - r)t][h(x_i)\pi_W*/r + a\pi_L*/r + \pi_i - x_i]dt - F$$

$$= \frac{h(x_i)\pi_W*/r + a\ \pi_L*/r + \pi_i - x_i}{r + a + h(x_i)} - F \qquad (8)$$

where π_W* is the flow of profits accruing forever to the firm which innovates first, i.e. the winner of the R & D race, π_L* is the flow of profits accruing forever to the losers, and π_i is the current profit of firm i.

The first order condition for a maximum for firm i now becomes

$$[r + a][h'(x_i)\pi_W*/r - 1] - h'(x_i)a\pi_L*/r - h'(x_i)\pi_i +$$

$$- h(x_i) + x_i h'(x_i) = 0 \qquad (9)$$

which in a symmetric equilibrium reduces (dropping the subscript i) to:

$$[r + (n-1)h(x)][h'(x)\pi_W*/r - 1] - h'(x)(n-1)h(x)\pi_L*/r +$$

$$- h'(x)\ \pi - [h((x) - xh'(x)] = 0 \qquad (10)$$

Equation (10) determines the equilibrium value of the R & D expenditure $x^\wedge$. The two terms which appear in (10) and did not appear in the first order condition of the Lee and Wilde model are $h'(x)(n-1)h(x)\pi_L^*/r$ and $h'(x)\pi$. They are both negative and therefore reduce the incentive to invest in R & D. The former is the effect due to the fact that in this formulation even the losers make positive profits in the post-innovation equilibrium. The latter is the "replacement effect" (see Fudenberg and Tirole, 1986, p. 32): the presence of current profits induces firms to delay the expected date of innovation.

Notice that in a Cournot equilibrium, π_W^*, π_L^* and π_i are all decreasing functions of n, the number of firms. Implicitly differentiating $x^\wedge$ with respect to n we get

$$\frac{\delta x^\wedge}{\delta n} \quad \alpha \quad h(x^\wedge)[h'(x^\wedge)\pi_W^*/r - 1] - h(x^\wedge)h'(x^\wedge)\pi_L^*/r + h'(x^\wedge)[h(x^\wedge)$$

$$(n-1)(1/r)(\delta\pi_W^*/\delta n - \delta\pi_L^*/\delta n) + \delta\pi_W^*/\delta n] - h'(x^\wedge)\ \delta\pi/\delta n \qquad (11)$$

where α means "has the same sign as". There are four terms on the R.H.S. of (11). The first one is the "Lee and Wilde effect", and, by the first order condition (9), it is always positive. The second term reduces the Lee and Wilde effect, taking into account that now even losers make profits in the post-innovation equilibrium. However, if this were the only difference with respect to the Lee and Wilde model, by the first order condition (6) it would still follow that $\delta x^\wedge/\delta n > 0$. The third term captures the change in the incentive to innovate due to a change in the number of firms. The sign of this term is ambiguous, because both π_W^* and π_L^* are decreasing function of n, and generally speaking one cannot rank the derivatives of π_W* and π_L* with respect to n without further specifications of costs and market demand. Finally, the fourth term captures the change occurring in the replacement effect due to a change in the number of firms, and it is always positive.

Clearly, if the third term vanishes, then the sign of the derivative (11) will unambiguously be positive. Thus, we have

Proposition 1. If the profits of the winner and of the losers after the innovation are independent of n, and if the "stability condition" ($\delta f/\delta x < 0$) holds, then an increase in the number of firms increases the equilibrium R & D effort rate of each firm.

A "drastic" innovation gives the winner monopoly power; in other words, the post-innovation monopoly price is lower than losers' production cost. Since post-innovation profits then do not depend on the number of firms, from Proposition 1 it follows:

Corollary. If the innovation is drastic, then an increase in the number of firms increases the equilibrium R & D effort rate of each firm.

In Stewart's model, the profit of the winner (given by $r\sigma W$) is independent of n, but the profits of the losers - i.e., $r(1-\sigma)W/(n-1)$ - are decreasing in n. However, in her model (11) reduces to

$$\frac{\delta x\hat{}}{\delta n} \propto h(x)[h'(x)\pi_W*/r - 1] - h(x)h'(x)\pi_L*/r +$$

$$- h'(x)h(x)(n-1)(1/r)\ \delta\pi_L*/\delta n \qquad (12)$$

and $h(x)h'(x)\pi_L*/r = -h'(x)h(x)(n-1)(1/r)\ \delta\pi_L*/\delta n$, so that we are left only with the Lee and Wilde effect. This explains the positive sign of the derivative.

Unfortunately, in more general cases very little can be said about the sign of the derivative (11). However, one may conjecture that the neat result of Lee and Wilde (i.e., $\delta x^\wedge/\delta n > 0$) may fail to hold in some circumstances. To obtain more definite results, we now turn to a particular specification of the model.

3. A SIMPLE MODEL

In this section we specialise the model set forth in section 2, assuming a linear market demand function, constant marginal (and average) production cost, and a specific, well behaved hazard function.

There are n identical firms, each producing a homogenous good whose market demand function is

$$p = a - Q \qquad (13)$$

where p denotes the price and Q is total output. We assume that firms compete in output levels in the product market and a Cournot equilibrium is established. Before the innovation, all firms produce at constant marginal and average cost c, $0 < c < a$. Then, in the Cournot equilibrium

$$q_i = \frac{s}{n+1} \qquad i=1,2,\ldots,n \qquad (14)$$

$$Q = \frac{ns}{n+1} \qquad (15)$$

$$p = \frac{nc + a}{n + 1} \qquad (16)$$

where q_i is the output of firm i, and $s = (a - c)$ can be interpreted as a measure of the size of the market. In equilibrium each firm earns profits per unit of time π_i given by

$$\pi_i = \frac{s^2}{(n + 1)^2} \qquad i=1,2,...,n \qquad (17)$$

Firm i invests in R & D at rate x_i, where x_i is the flow cost that firm i pays until someone succeeds. The probability of firm i being first to innovate at time t, given no success to date, is

$$h(x_i) = 2\,\mu\,\sqrt{x_i} \qquad (18)$$

where $\mu > 0$ is a parameter measuring the efficiency of R & D expenditure.

Firms compete for an innovation which gives the winner the exclusive right to produce at cost $c* < c$ forever. Since the case of a drastic innovation is already covered by the Corollary to Proposition 1, we assume that the innovation is non drastic; in our model this means that $c* > 2c - a$, or

$$s > d \qquad (19)$$

where $d = (c - c*)$ denotes the cost improvement. This assumption implies that the losers of the R & D race, while continuing to produce at cost c, will remain active in the post-innovation Cournot equilibrium, though their market shares will shrink. More precisely, in the post-innovation Cournot equilibrium we have

$$q_W{*} = \frac{s + nd}{n + 1} \qquad (20)$$

$$q_L{*} = \frac{s - d}{n + 1} \qquad (21)$$

where W denotes the winner of the R & D game, L denotes the losers, and a star denotes post-innovation variables. From (20) and (21) it follows that

$$Q* = \frac{ns + d}{n + 1} \qquad (22)$$

$$p* = \frac{a + nc - d}{n + 1} \qquad (23)$$

$$\pi_W{*} = \frac{(s + nd)^2}{(n + 1)^2} \qquad (24)$$

$$\pi_L{*} = \frac{(s-d)^2}{(n + 1)^2} \qquad (25)$$

Firms maximize expected discounted profits; firm i's payoff in the R & D game then becomes:

$$V_i = \frac{2\mu\sqrt{x_i}\,\pi_W*/r + 2\mu\Sigma_{j\neq i}\sqrt{x_j}\ \pi_L*/r + \pi_i - x_i}{r + 2\ \mu\ \Sigma_j\ \sqrt{x_j}} \qquad (26)$$

We now study the symmetric equilibrium of the R & D game. Given our well behaved hazard function, the maximisation problem faced by firm i ($i=1,2,\ldots,n$) always yields an interior solution, which must satisfy the first order condition

$$\mu(\pi_W* - \pi_i)/\sqrt{x_i} + 2\mu^2\Sigma_{j\neq i}\sqrt{x_j}(\pi_W* - \pi_L*)/r\sqrt{x_i} - r + \mu\sqrt{x_i} +$$
$$- 2\mu\ \Sigma_j\ \sqrt{x_j} = 0 \qquad (27)$$

In a symmetric equilibrium, $x_i = x$ ($i=1,2,\ldots,n$), and therefore (27) reduces to

$$-\theta(2n-1)x + [2\theta^2(n-1)(\pi_W* - \pi_L*) - 1]\sqrt{x} + \theta(\pi_W* - \pi_i) = 0 \qquad (28)$$

Equation (28) determines x as a function of n, s, d, and θ, where $\theta = \mu/r$. Let x^ be the strictly positive solution of equation (28). In the next section a comparative statics analysis will be performed.

4. COMPARATIVE STATICS

Using equation (28), we can easily derive the effects on the equilibrium R & D effort x^ of a change in the parameters. For sake of completeness, we establish the following rather intuitive results.

Proposition 2. An increase in the efficiency of R & D technology and/or a decrease in the discount rate will increase the equilibrium effort rate.

Proof. Implicitly differentiating expression (28) we get

$$\frac{\delta x}{\delta \theta} = - \frac{-(2n-1)x + 4\theta(n-1)(\pi_W* - \pi_L*)\sqrt{x} + (\pi_W* - \pi_i)}{\dfrac{\delta f(x,n,s,d,\theta)}{\delta x}} \tag{29}$$

where $f(x,n,s,d,\theta)$ is the L.H.S. of equation (28). Notice that the stability condition $\delta f/\delta x < 0$ always holds in our model, so that the denominator of (29) must be negative. Hence, the sign of the derivative is given by the sign of the numerator. Using (28), the numerator of (29) reduces to

$$[2\theta(n-1)(\pi_W* - \pi_L*) + 1/\theta]\sqrt{x} > 0 \; \blacksquare$$

Proposition 3. An increase in the cost improvement d will increase the equilibrium R & D effort.

Proof. Differentiating (28) we get

$$\frac{\delta x}{\delta d} = - \frac{2\theta^2(n-1)\dfrac{\delta}{\delta d}(\pi_W* - \pi_L*) + \theta\dfrac{\delta}{\delta d}(\pi_W* - \pi_i)}{\dfrac{\delta f(x,n,s,d,\theta)}{\delta x}} \tag{30}$$

By the stability condition, the denominator of (30) is negative. Hence, the sign of (30) will be positive provided both $(\pi_W* - \pi_L*)$ and $(\pi_W* - \pi_i)$

are increasing functions of d. Using (17), (24) and (25) it follows easily that

$$\frac{\delta}{\delta d}(\pi_W^* - \pi_L^*) = \frac{2d(n^2-1) + 2s(n+1)}{(n+1)^2} > 0$$

and

$$\frac{\delta}{\delta d}(\pi_W^* - \pi_i) = \frac{2n(nd + s)}{(n+1)^2} > 0. \quad ■$$

Dasgupta (1988) reports that he is not aware of any theoretical construct in which an increase in the demand for the product does not stimulate R & D activities of the firms producing it. Our model is no exception:

Proposition 4. An increase in the size of the market s will increase the equilibrium R & D effort x^.

Proof. Proceed as in the proof of Proposition 3, noting that

$$\frac{\delta}{\delta s}(\pi_W^* - \pi_L^*) = \frac{2(n+1)d}{(n+1)^2} > 0$$

and

$$\frac{\delta}{\delta s}(\pi_W^* - \pi_i) = \frac{2nd}{(n+1)^2} > 0. \quad ■$$

We now turn to the central issue of this section, namely the relationship between the intensity of rivalry and the individual R & D performance. As we have seen, in Lee and Wilde [1980] an increase in n always increases x^. Proposition 5 shows that in our model this conclusion may be reversed.

Proposition 5. If $n > s/(s-d)$, then for θ low enough an increase in the number of firms n will reduce the equilibrium R & D effort.

Proof. Differentiating (28) we get

$$\frac{\delta x}{\delta n} = - \frac{-2\theta x + 2\theta^2 \sqrt{x}\,\frac{\delta}{\delta n}[(n-1)(\pi_W^* - \pi_L^*)] + \theta\,\frac{\delta}{\delta n}(\pi_W^* - \pi_1)}{\frac{\delta f(x,n,s,d,\theta)}{\delta x}} \tag{31}$$

From (28) it is apparent that as θ goes to 0, $x(\theta)$ goes to 0 with the same speed as θ^2. It follows that as θ goes to 0, the sign of the derivative (31) is determined by the sign of the third term of the numerator. Since

$$\frac{\delta}{\delta n}(\pi_W^* - \pi_1) = \frac{2d[s - n(s-d)]}{(n+1)^3}$$

it follows that (31) will be negative for $n > s/(s-d)$.∎

5. WELFARE ANALYSIS

In this section we compare the market equilibrium with the social optimum. Specifically, we study whether the equilibrium aggregate R & D effort, $nx^{\wedge}$, is greater or lower than the socially optimal level. We distinguish three different notions of social optimum, according to whether perfect competition can be enforced (i.e., firms can be induced to behave as price taker) in the product market or not, and whether there is instantaneous dissemination of technological knowledge or not. We continue to assume that there are n distinct firms.

The first best social optimum is obtained when there is perfect competition in the product market, and instantaneous dissemination of technological knowledge. The number of firms is irrelevant as the product market is concerned, given perfect competition and constant marginal costs; however, it matters as far as the R & D technology is concerned, for the hazard function displays decreasing returns. If perfect competition prevails, firms' profits always equal zero, and social welfare coincides with consumers' surplus.

The second and third best solutions are achieved when the R & D effort of the n firms can somehow be controlled , but the outcome of the competition in the product market cannot be influenced. In other words, we assume that in the second and third best world a Cournot equilibrium prevails in the product market. In this scenario, social welfare is given by the sum of aggregate profits and consumers' surplus. In the second best world there is instantaneous dissemination of technological knowledge, whereas in the third best world there is perfect patent protection.

These distinctions are not considered in earlier contributions. Since Loury [1979] and Lee and Wilde [1980] do not specify the type of competition and the structure of the product market, in their setting it seems plausible to identify social and private returns from the innovation. Thus, in the case of blockaded entry, the only effect they can discuss is the "duplication of effort", due to the fact that firms do not take account of the parallel nature of their activities. Two additional effects turn out to be important in our richer setting. The first one is the "efficiency effect" (cf. Fudenberg and Tirole, 1986, p. 32) due to the diversity of incentives of private firms and a social planner (8). The second one is the "replacement effect" arising because social welfare is positive before the innovation, so that there is an incentive to delay the expected date of innovation.

In a framework as general as the one presented in section 2, the net balance of the duplication, efficiency and replacement effects is ambiguous. Again, to obtain more definite results we tackle the issue in the simplified model of section 3.

The objective function of the social planner is given by U, the expected value of the discounted flow of consumers and producers' surpluses, W, over an infinite time horizon:

$$U_z = \int_0^{\infty} \exp[- H(x)t - rt]\,[H(x)W_z^*/r + W_z - nx]\,dt$$

$$= \frac{H(x)W_z^*/r + W_z - nx}{r + H(x)} \qquad (32)$$

where $H(x) = n\,h(x)$ is the aggregate hazard function and it is assumed that the social discount rate equals the market interest rate; $Z = F,S,T$ denotes the first, second, third best, respectively. Notice that, given that there are decreasing returns in the R & D technology, it is efficient to spread total effort uniformly across firms. The socially optimal level of R & D effort, denoted by x^Z, is then the strictly positive solution of the following first order condition

$$\theta n x + \sqrt{x} - \theta(W_Z^* - W_Z) = 0 \qquad (33)$$

To begin with, we consider the first best social optimum. Before the innovation, consumers' surplus (which equals social welfare) is given by

$$W_F = \tfrac{1}{2} s^2 \qquad (34)$$

while after the innovation it is

$$W_F^* = \tfrac{1}{2} (s+d)^2 \qquad (35)$$

Both Loury [1979] and Lee and Wilde [1980] prove that the socially optimal level of investment in R & D is lower than the equilibrium investment of the R & D game, i.e. that firms overinvest in R & D compared to the social optimum. We show that this conclusion can be reversed in our model.

Proposition 6. For sufficiently low values of θ, the equilibrium R & D effort is lower than the first best social optimum, i.e., $\hat{x} < x^F$.

Proof. Follows from Proposition 7 below and the fact that $x^F > x^S$. ∎

In the second best world, the R & D effort of the n firms can be controlled, but a Cournot equilibrium prevails in the product market. However, there is no patent protection. In the pre-innovation equilibrium, social welfare (which equals the sum of profits and consumers' surplus) is:

$$W_S = \frac{n\, s^2}{(n+1)^2} + \frac{n^2\, s^2}{2(n+1)^2}$$

$$= \frac{n(n+2)s^2}{2(n+1)^2} \qquad (36)$$

In the post-innovation equilibrium, social welfare is

$$W_S* = \frac{n(n+2)(s+d)^2}{2(n+1)^2} \qquad (37)$$

Since $(W_S* - W_S) < (W_F* - W_F)$, it follows easily that $x^F > x^S$. Furthermore, x^S may be greater than x^, as shown by the following Proposition.

Proposition 7. For sufficiently low values of θ, the equilibrium R & D effort is lower than the second best social optimum, i.e., x^ < x^S.

Proof. From (33) it follows that

$$\sqrt{x^s} = \frac{-1 + [1 + 2\,\theta^2\, n\,(d^2 + 2sd)]^{1/2}}{2\theta n} \qquad (38)$$

On the other hand, from (28), neglecting the terms of order θ^2 we get:

$$\sqrt{x\hat{}} = \frac{-1 + \left[1 + \frac{4\theta^2}{(n+1)^2}[n(s+nd)^2 - ns^2 + (n-1)(d^2 - 2sd)]\right]^{1/2}}{2\theta(2n-1)} \qquad (39)$$

Since the denominator of (39) is greater than the denominator of (38), in order to show that x^s is greater than $x\hat{}$ it suffices to show that the term under square root in (38) is greater than the corresponding term in (39). Remembering that $s > d$, a little algebra shows that this is indeed the case. ∎

We now turn to the third best world in which the social planner can control the R & D effort of the n firms, but a Cournot equilibrium prevails in the product market and there is perfect patent protection. Obviously, $W_T = W_S$; in the post-innovation equilibrium, on the other hand, social welfare is

$$W_T* = \frac{(s+nd)^2 + (n-1)(s-d)^2}{(n+1)^2} + \frac{(ns+d)^2}{2(n+1)^2}$$

$$= \frac{(ns+d)^2 + 2(nd+s)^2 + 2(n-1)(s-d)^2}{2(n+1)^2} \qquad (40)$$

Again, we wish to compare the socially optimal level of R & D in this third best scenario, x^T, with the equilibrium of the R & D game obtained in section 3.

Proposition 8. If $s/d < n^2/(n-1)^2$, then for sufficiently low values of θ, the equilibrium R & D effort is lower than the third best social optimum, i.e. $\hat{x} < x^T$.

Proof. From (33) it follows

$$\sqrt{x^S} = \frac{-1+\left[1+\frac{2\theta^2}{(n+1)^2}n[(ns+d)^2+2(nd+s)^2+2(n-1)(s-d)^2-n(n+2)s^2]\right]^{\frac{1}{2}}}{2\theta n} \qquad (41)$$

On the other hand, from (28), neglecting the terms of order θ^2 we get (39). Since the denominator of (39) is greater than the denominator of (41), a sufficient condition for x^T to be greater than $\hat{x}$ is that the term under square root in (41) be greater than the corresponding term in (39). Remembering that $s > d$, a little algebra shows that this is indeed the case provided that $s/d < n^2/(n-1)^2$. ∎

6. CONCLUDING REMARKS

In this paper we have analyzed the relationship between market structure and incentives to innovate in a model in which firms are quantity setting oligopolists in the product market. This implies that when the number of firms varies, current as well as future market profits (and hence incentives to innovate) vary. Therefore, the effect of a change in n on the equilibrium R & D effort turns out to be much more complex than in models where the expected returns from R & D investment are exogenously given. Specifically, we have shown that in a simple model with linear market demand function and constant marginal cost, if the discount rate is sufficiently high and /or the productivity of R & D investment is sufficiently low, and if n is large enough, then an increase in n leads to a reduction in the individual effort rate. This is the opposite of Lee and Wilde's standard result with non contractual R & D costs.

Furthermore, our model permits a comprehensive comparison between the R & D performance of a private industry and that of a socially managed one. To this purpose, we have identified three notions of social optimum and we have shown that, under some circumstances, private incentives are lower than the social ones, and underinvestment in R & D results.

Our model could be extended in several ways. First, since the Lee and Wilde model has been used as the constituent game of more sophisticated analyses (9), these could be reformulated as well along the lines of our model. Second, it would be interesting to assess the effects of asymmetries between firms (e.g., different initial production costs) on the properties of the R & D equilibrium. Third, we have confined our attention to the case of an oligopoly with blockaded entry. Taking account of the fixed R & D cost, one could determine the number of firms endogenously, and study whether entry occurs at the socially optimal level (10). Finally, non quantity (e.g., price) competition in the product market could be considered as well. All these issues seem worth addressing.

FOOTNOTES

(1) Cf., e.g., Arrow [1962] and Dasgupta and Stiglitz [1980 a]. It can be noticed, however, that private returns to investment in R & D can exceed social returns, as argued by Kamien and Schwartz [1982], p. 187-9.

(2) Another important contribution is Dasgupta and Stiglitz [1980 a]. They adopt the same R & D cost specification as Loury and confirm most of his results. However, they assume (p. 17) that each firm ignores the impact of its investment on the aggregate probability of discovery.

(3) "The intuition behind these conclusions is simple. In the Dasgupta and Stiglitz ([1980 a]) and Loury model, an increase in the number of firms reduces the expected benefit to investment..., leaving expected costs unchanged. The firm responds by reducing investment. In the Lee and Wild model, both expected benefits and expected costs are reduced by the addition of another firm ... and the net effect is to enhance incentives to invest". Cf. Reinganum [1984], p. 62.

(4) An early attempt to analyse, in a deterministic setting, the effect of increasing rivalry on the pace of technological research in a Cournot oligopoly is Scherer [1967].

(5) These hypotheses are slightly more restrictive than those of Lee and Wilde, but do not affect the basic conclusions.

(6) From expression (3) we get

$$\delta f/\delta x = (n-1)h'(x)[Wh'(x) - 1] + [(n-1)h(x) + r]\, Wh''(x) + xh''(x)$$

On the other hand, Lee and Wilde's stability condition (see p. 432) is

$$1 - (\delta x/\delta a)(n-1)h'(x) > 0$$

which reduces to

$$Wh''(x)[(n-1)h(x) + r] + xh''(x) < -(n-1)h'(x)[Wh'(x) - 1]$$

which implies $\delta f/\delta x < 0$.

(7) It might appear that this result contradicts Stewart's finding that an increase in the number of firms decreases equilibrium R & D effort if σ is set at a critical value $\sigma*$ (p. 689). Notice, however, that $\sigma*$ is a decreasing function of n (cf. equation (10) on p. 686), so that when n is increased while keeping σ equal to $\sigma*$, σ must decrease. This obviously has a negative impact on the incentive to innovate, which is responsible for the negative sign of the derivative found by Stewart.

(8) Dasgupta and Stglitz [1980 a] show that the incentive to invest in R & D of a social planner is greater than that of a competitive firm, which in turn is not lower than that of a monopolist. As far as a Cournot oligopoly is concerned, the conclusion is less clear cut. In our simple model, some algebra shows that the efficiency effect is always greater for the social planner (in all the first, second and third best worlds) than for a private firm. In other words,

$$(W_Z* - W_Z) > (\pi_W* - \pi_L*) \qquad \text{for } Z = F, S, T.$$

(9) The one-shot game of Lee and Wilde has been extended in two directions: the analysis of a multi-stage race for a single prize (e.g., Harris and Vickers [1987]) and the analysis of a sequence of one-stage technological competitions (e.g., Reinganum [1985]).

(10) This problem has been tackled, *inter alia*, by Loury [1979], Dasgupta and Stiglitz [1980 b], and Tandon [1984].

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