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High Frequency vs. Daily Resolution: the Economic Value of Forecasting Volatility Models
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High Frequency vs. Daily Resolution: the Economic Value of Forecasting Volatility Models*

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Abstract

Forecasting volatility models typically rely on either daily or high frequency (HF) data and the choice between these two categories is not obvious. In particular, the latter allows to treat volatility as observable but they suffer from many limitations. HF data feature microstructure problem, such as the discreteness of the data, the properties of the trading mechanism and the existence of bid-ask spread. Moreover, these data are not always available and, even if they are, the asset's liquidity may be not sufficient to allow for frequent transactions. This paper considers different variants of these two family forecasting-volatility models, comparing their performance (in terms of Value at Risk, VaR) under the assumptions of jumps in prices and leverage effects for volatility. Findings suggest that daily-data models are preferred to HF-data models at 5% and 1% VaR level. Specifically, independently from the data frequency, allowing for jumps in price (or providing fat-tails) and leverage effects translates in more accurate VaR measure.

JEL-Classification: C58 C53 C22 C01 C13

Keywords: GARCH, DCS, jumps, leverage effect, high frequency data, realized variation, range estimator, VaR

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1 Introduction

Modeling and forecasting volatility of asset returns are crucial for many applications, such as asset pricing model, risk management theory and portfolio allocation decisions. An earlier literature, including Engle (1982) and Bollerslev (1986) among others, has developed models of asset volatility dynamics in discrete time, known as heteroscedastic volatility models, i.e. ARCH-GARCH. Thanks to the availability of high frequency (HF) data, a new strand of literature has originated a new class of models based on the Realized Volatility (RV) estimator, therefore introducing a non-parametric measure of return volatility (see Andersen et al., 001a, Barndorff-Nielsen, 2002 and Andersen et al., 2012). As thw main innovation, RV models provides an *ex-post* observation of volatility, at odds with the standard ARCH-GARCH approach, that treats volatility as a latent variable. Although forecasting-volatility models based on HF data are getting more and more popular in the literature, the choice between HF-data and daily-data models is yet not obvious, in particular from an applied standpoint. In particular, the former still suffer from various limitations, that can be addressed only at the cost of a heavy manipulation of the original data.

One of the main issues is the presence of the market microstructure noise, which prevents from getting a perfect estimate (at the limit) of the returns' variance (see Hansen and Lunde, 2006 and Aït-Sahalia et al., 2005, 2011). The market microstructure noise may originate from different sources, including the discreteness of the data, the properties of the trading mechanisms and the existence of a bid-ask spread. Regardless of the source, when return from assets are measured based on their transaction prices over very tiny time intervals, these measures are likely to be heavily affected by the noise and therefore brings little information on the volatility of the price process. Since the level of volatility is proportional to the time interval between two successive observations, as the time interval increases, the incidence of the noise remains constant, whereas the information about the "true" value of the volatility increases. Therefore, there is a trade-off between high frequency and accuracy, which has led authors to identify an optimal sampling frequency of 5 minutes[1].

[1]Since the best remedy for market microstructure noise depends on the properties of the noise, if data sampled at higher frequency, e.g. tick-by-tick, are used the noise term needs to be modeled and, as far as I know, there is no unified framework about how to deal with it. Aït-Sahalia et al. (2005) define a new estimator, Two Scales Realized Volatility (TSRV), which takes advantages of the rich information of tick-by-tick data and corrects the

HF data also features another inconvenient: they are not always available and, even if they are, the asset may be not liquid enough to be frequently traded. On the contrary, daily data are relatively simple to record and collect and are commonly easy-to-get.

This paper sheds light on the choice between HF-data and daily data models, by assessing the economic value of the two family models, based on a comparison of their performance in forecasting asset volatility. Following the risk management perspective, I use value at risk (VaR) as the econometric metric of volatility forecastability, as suggested by Christoffersen and Diebold (2000).

VaR is defined as the quantile of the conditional portfolio distribution, and is therefore quite intuitive as a measure: indeed, it is the most popular quantitative measure of the market risk associated with a portfolio of assets, and is generally adopted by banks and required by regulators all over the world[2].

In running the comparison between HF-data and daily data models, this paper introduces two key assumptions. Firstly, the data generating process for asset prices features discontinuities in its trajectories, *jumps*[3]. Secondly, volatility (i.e. the standard deviation of asset return) reacts differently to changes in asset return which have the same magnitude, but different sign, *leverage effect*. These two assumptions represent the main novelty of this paper since none of the previous studies on the economic value of different forecasting-volatility models has investigated the matter under both jumps in price and leverage effect combined

effects of microstructure noise on volatility estimation. The authors, instead of sampling over a longer time horizon and discarding observations, make use of all data and model the noise as an "observation error". But the microstructure noise modeling goes beyond the scope of this work.

[2]Banks often construct VaR from *historical simulation* (HS-VaR): VaR is the percentile of the portfolio distribution obtained using historical asset prices and today weights. This procedure is characterized by a slow reaction to market conditions and for the inability to derive the term structure of VaR. The VaR term structure explains how risk measures vary across different investment horizons. In HS-VaR, for example, if T-day 1% VaR is calculated, the 1-day 1% VaR is simply scaled by $\sqrt{T}$. This relation is valid only if daily returns are i.i.d. realizations of a Normal distribution. We know that is not the case since returns present leptokurtosis and asymmetry. The main limit of HS-VaR is the substitution of the conditional return distribution with the unconditional counterpart. Risk Metrics and GARCH models represent improvements over HS-VaR measure. Both of them provide an explicit assumption about the DGP and the conditional variance but they have also important differences. In addition to the estimation method: GARCH conditional volatility is estimated by maximizing the log-likelihood function while the parameters used in Risk Metrics are chosen in an ad hoc fashion, they differ for the possibility to account for the term structure of VaR. This is because GARCH process allows for mean reversion in volatility while Risk Metrics does not, reproducing a flat term structure for VaR.

[3]A continuous price process is a restrictive assumption since it is not possible to distinguish between the dynamic originated from the two sources of variability, i.e. continuous and discontinuous movements with consequences on the return generating process

together. Giot and Laurent (2004) compare the performance of a daily ARCH-type model with the performance of a model based on the daily RV in a VaR framework. The authors find that VaR specification based on RV does not really improve the performance of a VaR model estimated using daily returns. This paper underlines an important issue: in economics applications, it is important to recognize and take into account the key features of the empirical data in order to choose a valid data generating process. Clements et al. (2008) evaluate quantile forecasts focusing exclusively on models based on RV in order to understand if the results presented for stock returns can be carried over exchange rates. According to the results in Clements et al. (2008) the distributional assumption for expected future returns is needed for computing quantile, irrespective of the frequency of data used. Brownlees and Gallo (2010) forecast VaR using different volatility measures based on ultra-high-frequency data using a two-step VaR prediction procedure. They find that using ultra-high-frequency observations, VaR predictive ability is considerably improved upon relative to a baseline GARCH but not so relative to the range. The reason is related to the microstructure noise issue which arises when ultra high-frequency data are used. Indeed I want to contribute to the existing literature focusing on the measurement and the efficient use of the information embedded in HF data with respect to the information content of daily observations. Assuming both jumps and leverage effects in the returns dynamics for both data categories, I provide a more balanced comparison than in the previous work.

In the choice of the model to use for the comparison, I consider the GARJI model of Maheu and McCurdy (2004), as the baseline for the daily data models. The latter is a mixed-GARCH jump model which allows for asymmetric responses to past innovations in asset returns: the news impact (resulting in jump innovations) may have a feedback effect on the expected volatility, in addition to the feedback effect associated with the normal error term. For the case of HF data, I consider models in which Realized Volatility (RV) is decomposed into continuous and discontinuous volatility components. The continuous component is captured by means of the bi-power variation (BV), introduced by Barndorff-Nielsen and Shephard (2004), whereas the discontinuous component (JV) is obtained as the difference between RV and BV at given point in time[4]. In Andersen et al. (2007), JV is obtained considering only jumps that are found to be significative, and neglecting the others[5]. Corsi et al. (2010) consider instead all jumps, stressing the importance to correct the positive bias in BV due to jumps classified as consecutive. In this paper, I consider both these approaches and make a comparison among them, finding evidence in favor of jump identification strategy of Corsi et al. (2010) when the leverage effect is introduced. To account for the leverage effect, I introduce in this class of models the heterogeneous structure proposed by Corsi and Renó (2009).

Throughout this paper, the GARJI-VaR measures are obtained by following Chiu et al. (2005), that is, by adjusting for skewness and fat tails in the specification of the conditional distribution of returns[6]. The HF-VaR measures, instead, are computed by assuming a conditional Gaussian distribution for asset returns: as shown in Andersen et al. (2010), returns standardized for the square root of RV are indeed approximatively Normal[7].

In order to assess the model's capability to forecast future volatility, I implement a backtesting procedure based on both the Christoffersen (1998) test and the Kupiec (1995) test. In addition to comparing the economic value of daily data and HF-data models, the analysis performed in this paper sheds light on three other issues. The first is represented by the economic value per se, i.e. out of the comparison, of the class of forecasting volatility models adopting HF-data. This is done by considering different specifications of this family models. I first run a comparison among them (based on their forecasting performances); then, I compare some of them with their variant, obtained by using the Range estimator (RA) of Parkinson (1980). The choice of this particular benchmark is motivated by the fact that the RA estimator is likely to deliver a measure of volatility which lies in the middle of the mea-

[4]As shown in Andersen et al. (2002), Andersen et al. (2007), RV is a consistent estimator for the quadratic variation, whereas BV represents a consistent estimator of the continuous volatility component, i.e. the so-called integrated volatility, in the presence of jumping prices.

[5]The authors with significant jumps refer to large value of $RV_t - BV_t$ while small positive values are treated both as part of continuous sample path variation or as measurement errors.

[6]The computation of VaR measure requires, in addition to the conditional volatility dynamics, the specification of the conditional distribution of returns.VaR is a conditional risk measure so an assumption on the conditional distribution of returns is needed. Conditional normality is an acceptable assumption (returns standardized by their conditional volatility could be approximately Gaussian even if the unconditional returns are not Gaussian) only if the volatility model is able to fatten conditionally Gaussian tails enough to match the unconditional distribution. If this is not the case another conditional distributional assumption is necessary.

[7]This result is confirmed by the standardized returns of the sample used in this paper. See Section 2.

sure obtained from HF estimators and that obtained from daily data models[8]. My findings suggest that HF-data models which explicitly provide both jumps and leverage factors stand out from the others in term of forecasting capability.

The second by-product of my analysis is a quantitative assessment of the importance of the explicit jump component in the conditional distribution of asset returns [9]. This point is addressed in both the family models considered in this paper. Hence, I first compare the forecasting volatility performances of each HF-data model with and without a decomposition of the RV into the continuous and the discontinuous component. Then, I run a similar analysis for the case of the daily data models, considering the GARCH-t model, as well as the Beta-t model[10] proposed by Harvey and Luati (2014). According to my analysis, introducing an explicit, persistent jump component in the conditional return dynamics (together with an asymmetric response to bad and good news into conditional volatility dynamics) may help to forecast the ex-post volatility dynamics and obtain more accurate VaR measures, at least at the VaR level required by Basel accords (1%). For HF-data models, accounting for jumping prices does not seem to improve significantly the accuracy of the estimates.

The last issue of my analysis is related to the importance of leverage effect in forecasting volatility. The findings in this paper recommend the explicit introduction of a factor that generates the asymmetric volatility response to price movements in the forecasting model.

The rest of the paper is organized as follows. Section 2 summarizes the volatility measures and the forecasting models based on both HF and daily data. Section 3 and Section 4 show, respectively, the backtesting methods used to evaluate forecasting models accuracy and the empirical results. Section 5 concludes.

[8] The RA estimator exploits information on the highest and the lowest price recorded in a given day for a particular asset. In this respect, it requires information on the intra-day activity (going beyond the simple closing price of the asset), but without relying on further information, that might be not readily available).

[9] The presence of a jump component is justified both at theoretical and empirical level. From a theoretical perspective, an explicit discontinuous volatility-component allows to have information on the market response to outside news, which is key for many applications. From an empirical standpoint, instead, it is very difficult to distinguish power-type tails from exponential-type tails, given that is not clear to what extent the return distribution is heavily tailed. In this regard, the jump component of a jump-diffusion model may be interpreted as the market response to outside news: when good or bad news arrive at a given point in time, the asset price changes according to the jump size (and the jump sign) and an extreme sources of variation is added to the idiosyncratic component.

[10] Beta-t model, belongs to the general class of Dynamic Conditional Score (DCS) model. They are also known as Generalized Autoregressive Score (GAS) model proposed by Creal et al. (2013).

2 Volatility Measures and Forecasts

2.1 Estimates of volatility with High Frequency Data

The RV measure is an estimator for the total quadratic variation, namely, it converges in probability, as the sampling frequency increases, to the continuous volatility component if there are no jumps. Instead, it converges to the sum of continuous and discontinuous volatility components if at least one jump occurs. As explained in Andersen et al. (2012), it is possible to use the daily RV measures, the *ex-post* volatility observations, to construct the *ex-ante* volatility forecasts. This is possible simply by using standard ARMA time series tools but it is important to take into account the difference with GARCH-type forecasting. The fundamental difference is that in the former case the risk manager treats volatility as observed while in the latter framework volatility is inferred from past returns conditional on a specific model. The idea behind the RV is the following: even if prices are not available on continuous basis, prices are recorded at higher frequency than daily. Using these squared returns a daily RV could easily be computed. In this way the ex-post volatility is considered as observable at each point in time.

More precisely, the RV on day t based on returns at the Δ intraday frequency is

$$RV_t(\Delta) \equiv \sum_{j=1}^{N(\Delta)} r_{t,j}^2$$

where $r_{t,j} = p_{t-1+j\Delta} - p_{t-1+(j-1)\Delta}$ and $p_{t-1+j\Delta}$ is the log-price at the end of the jth interval on day t and $N(\Delta)$ is the number of the observations available at day t recorded at Δ frequency. In the absence of microstructure noise, as $\Delta \to 0$ the RV estimator approaches the integrated variance of the underlying continuous-time stochastic volatility process on day t:

$$RV_t \longrightarrow_p IV_t \qquad \text{where} \qquad IV_t = \int_{t-1}^{t} \sigma^2(\tau)\, d\tau$$

Furthermore, in this paper I assume that the the underlying price process is characterized by discontinuities. Indeed, the previous convergence is not valid but the RV estimators approaches in probability to the sum of the integrated volatility and the variation due to jumps that occurred on day t:

$$RV_t \longrightarrow_p \int_{t-1}^{t} \sigma^2(\tau)\, d\tau \quad + \quad \sum_{j=1}^{\zeta_t} J_{t,j}^2$$

If jumps ($J_{t,j}$) are absent, the second term vanishes and the realized volatility consistently estimates the integrated volatility. A nonparametric estimate of the continuous volatility component is obtained by using the bipower variation (BV) measures:

$$BV_t \equiv \frac{\pi}{2} \frac{N(\Delta)}{N(\Delta)-1} \sum_{j=1}^{N(\Delta)-1} |r_{t,j}||r_{t,j+1}| \tag{1}$$

Furthermore, the contribution to the total return variation stemming from the jump component (JV_t) is consistently estimated by

$$RV_t - BV_t \longrightarrow_p \sum_{j=1}^{\zeta_t} J_{t,j}^2$$

Considering the suggestion of Barndorff-Nielsen and Shephard (2004) the empirical measurements are truncated at zero in order to ensure that all of the daily estimates are nonnegative:

$$JV_t = max\{RV_t - BV_t, 0\} \tag{2}$$

According to Andersen et al. (2007), this truncation reduces the problem of measurement error with fixed sampling frequency but it captures a large number of nonzero small positive values in the jump component series. These small positive values can be treated both as part of the continuous sample path variation process or as measurement errors
In order to identify statistically significant jumps, i.e. large values of $RV_t - BV_t$, the authors suggest the use of the following statistic:

$$Z_t = \frac{log(RV_t) - log(BV_t)}{\sqrt{N(\Delta)^{-1}(\mu_1^{-4} + 2\mu_1^{-2} - 5)TQ_t BV_t^{-2}}} \longrightarrow_d N(0,1) \tag{3}$$

where $\mu_1 = \sqrt{2/\pi}$. In the denominator appears the realized tripower variation (TQ) that is the estimator of the integrated quarticity as required for a standard deviation notion of scale:

$$TQ_t = N(\Delta)\mu_{4/3}^{-3} \sum_{j=3}^{N(\Delta)} |r_{t,j}|^{4/3}|r_{t,j+1}|^{4/3}|r_{t,j+2}|^{4/3}$$

where $\mu_{4/3} = 2^{2/3}\Gamma(7/6)\Gamma(1/2)$. The significant jumps and the continuos component are identified and estimated respectively as:

$$\begin{aligned} JV_t &= \mathbb{1}_{\{Z_t > \Phi_\alpha\}}(RV_t - BV_t) \\ CV_t &= RV_t - JV_t = \mathbb{1}_{\{Z_t \leq \Phi_\alpha\}} RV_t - \mathbb{1}_{\{Z_t > \Phi_\alpha\}} BV_t \end{aligned} \tag{4}$$

where $\mathbb{1}$ is the indicator function and Φ_α is the α quantile of a Standard Normal cumulative distribution function.

Corsi et al. (2010) show that the nonparametric estimator BV can be strongly biased in finite sample because of the presence of consecutive jumps and they define a new nonparametric estimator, called Threshold Bipower Variation (TBV). In particular, TBV corrects for the positive bias of BV in the case of consecutive jumps:

$$TBV_t = \mu_1^{-2} \sum_{j=2}^{N(\Delta)} |r_{t,j}||r_{t,j+1)}|\mathbb{1}_{\{|r_{t,j}|^2 < \theta_j\}}\mathbb{1}_{\{|r_{t,j+1|}|^2 < \theta_{j+1}\}}$$

where θ is strictly positive random threshold function equal to $\hat{V}_t c_\theta^2$, $\hat{V}_t$ is an auxiliary estimator and c_θ^2 is a scale-free constant that allows to change the threshold. The jump detection test presented by Corsi et al. (2010) is the following:

$$\texttt{C-Tz} = N(\Delta)^{-1/2} \frac{(RV_t - TBV_t)RV_t^{-1}}{\sqrt{(\frac{\pi^2}{4} + \pi - 5)max\{1, \frac{TTriPV_t}{TBV_t^2}\}}} \longrightarrow_d N(0,1) \tag{5}$$

where $TTriPV$ is a quarticity estimator which is obtained by multiplying the TBV by $\mu_{4/3}^{-3}$. Also in this case the jumps and the continuos component are identified and estimated respectively as:

$$\begin{aligned} JV_t &= \mathbb{1}_{\{\texttt{C-Tz}_t > \Phi_\alpha\}}(RV_t - TBV_t) \\ CV_t &= RV_t - JV_t = \mathbb{1}_{\{\texttt{C-Tz}_t \leq \Phi_\alpha\}} RV_t - \mathbb{1}_{\{\texttt{C-Tz}_t > \Phi_\alpha\}} TBV_t \end{aligned} \tag{6}$$

The other measure chosen in this work is the Range volatility (RA) presented by Parkinson (1980):

$$RA_t = \frac{1}{4 \log 2} (\log(H_t) - \log(L_t))^2 \tag{7}$$

This estimator is constructed by taking the highest price (H) and the lowest price (L) for each day as summary of the intraday activity, i.e. the full path process. Its major empirical advantage is that for many assets these informations are ready available. Alizadeh et al. (2002), it is affected by a much lower measurement error than the RV estimator, it is more robust to microstructure noise in a stochastic volatility framework and it allows to extract efficiently latent volatility. On the one hand, the RA estimator contains informations comparable to those embedded in RV. On the other hand, RA is easy to compute also for those assets that are not frequently traded. Indeed, this estimator has advantages typical of both HF data and daily observations.

2.2 Forecasting volatility using High Frequency Data

In the literature, there is no consensus if jumps help to forecast volatility. In this sense, this work can be useful in order to understand if allowing for an explicit jump component is important to forecast volatility, independently of the sampling frequency of the price process. Moreover, if different sampling frequencies (daily and 5-minutes) are considered then a discrimination between the two kinds of data used, can be done.

For all forecasting models that I am going to describe in this section, I define a log specification both for inducing normality and for ensuring positivity of volatility forecasts [11]. The natural starting point in forecasting volatility is to use an Autoregressive (AR) specification[12]. The first model for both RV and RA is the AR model. In particular, an AR(8) model is identified for both RV measure and for Range estimator[13]. The AR specification is easy to implement but it does not capture the volatility long-range dependence due to the slowly

[11] Volatility forecasts at each time is obtained by applying the exponential transformation.

[12] It is also possible to use an ARMA model to forecast volatility in order to consider some measurement errors since the empirical sampling is not done in continuous time.

[13] The identification procedure for the order of both AR models is done by exploiting the sample autocorrelation and the sample partial autocorrelation function, by running both AIC and BIC information criteria and significance of single parameters. Then I check the properties of the residuals: they are normal and the Ljung Box test does not reject the null of no autocorrelation at any significance level.

decaying autocorrelation of returns. As an alternative, it is possible to use the Heterogenous Autoregressive model proposed by Corsi (2009). This model can be seen as an approximation of long memory model with an important advantage: it is easier to implement than the pure long-memory model (see Andersen et al., 2007, Corsi and Renó, 2009). Indeed, the second forecasting model for both volatility measures is the Heterogeneous Autoregressive model (HAR). The aggregate measures for the daily, weekly and monthly realized volatility are computed as sum of past realized volatilities over different horizons:

$$RV_t^{(N)} = \frac{1}{N} RV_t + \cdots + RV_{t-N+1} \tag{8}$$

where N is typically equal to 1, 5 or 22 according to if the time scale is daily, weekly or monthly.

Then, HAR-RV becomes:

$$\log RV_{t+h} = \beta_0 + \beta_1 \log RV_{t+h-1} + \beta_2 \log RV_{t+h-1}^{(5)} + \beta_3 \log RV_{t+h-1}^{(22)} + \epsilon_t \tag{9}$$

where ϵ_t is IID zero mean and finite variance noise [14]

Moreover, as suggested in Corsi and Renó (2009), the heterogeneous structure applies also to leverage effect. As a consequence, volatility forecasts are obtained by considering asymmetric responses of realized volatility to previous daily, weekly and monthly negative returns. The past aggregated negative returns are constructed as:

$$l_t^{(N)} = \frac{1}{N} (r_t + \cdots + r_{t-N+1}) \mathbb{1}_{\{(r_t + \cdots + r_{t-N+1}) < 0\}} \tag{10}$$

Then the L-HAR model is defined as:

$$\begin{aligned} \log RV_{t+h} =& \beta_0 + \beta_1 \log RV_{t+h-1} + \beta_2 \log RV_{t+h-1}^{(5)} + \beta_3 \log RV_{t+h-1}^{(22)} + \\ & \beta_4 l_{t+h-1} + \beta_5 l_{t+h-1}^{(5)} + \beta_6 l_{t+h-1}^{(22)} + \epsilon_t \end{aligned} \tag{11}$$

The explanatory variables of the HAR-RV model can be decomposed into continuous and

[14] Corsi and Renó (2009) model the dynamic of the latent quadratic variation, call it $\tilde{\sigma}_t$. Suppose that $\hat{V}_t$ is a generic unbiased estimator of $\tilde{\sigma}_t$ and $log(\tilde{\sigma}_t) = log(\hat{V}_t) + \omega_t$ where ω_t is a zero mean and finite variance measurement error. Then ϵ_t is independent from ω_t.

jump components, in this way the forecasting model obtained is:

$$
\begin{aligned}
\log RV_{t+h} =& \beta_0 + \beta_1 \log CV_{t+h-1} + \beta_2 \log CV^{(5)}_{t+h-1} + \beta_3 \log CV^{(22)}_{t+h-1} + \\
& \beta_4 \log\left(1 + JV_{t+h-1}\right) + \beta_5 \log\left(1 + JV^{(5)}_{t+h-1}\right) + \beta_6 \log\left(1 + JV^{(22)}_{t+h-1}\right) + \epsilon_t
\end{aligned} \tag{12}
$$

Depending on how the jump component is detected three different forecasted realized volatility are obtained. First, the HAR-Jumps is obtained according to (2) and for the continuous component to (1). Second, the HAR-CV-JV model is obtained following Andersen et al. (2007), namely according to (4). The last model, HAR-C-J is defined according to (6) following the estimation strategy presented in Corsi and Renó (2009).

If a cascade leverage structure is considered as in (10) then the forecasting volatility model becomes:

$$
\begin{aligned}
\log RV_{t+h} =& \beta_0 + \beta_1 \log CV_{t+h-1} + \beta_2 \log CV^{(5)}_{t+h-1} + \beta_3 \log CV^{(22)}_{t+h-1} + \\
& \beta_4 \log\left(1 + JV_{t+h-1}\right) + \beta_5 \log\left(1 + JV^{(5)}_{t+h-1}\right) + \beta_6 \log\left(1 + JV^{(22)}_{t+h-1}\right) + \\
& \beta_7 l_{t+h-1} + \beta_8 l^{(5)}_{t+h-1} + \beta_9 l^{(22)}_{t+h-1} + \epsilon_t
\end{aligned} \tag{13}
$$

As before, according to the estimators used for the volatility components, I obtain the LHAR-Jumps, LHAR-CV-JV and LHAR-C-J models.

In order to asses the forecast ability of the RA, I extend the idea of the heterogeneity in the time horizons of investors in the financial markets and I define two different forecasting models, in addition to the AR(8) model:

$$
\log RA_{t+h} = \beta_0 + \beta_1 \log RA_{t+h-1} + \beta_2 \log RA^{(5)}_{t+h-1} + \beta_3 \log RA^{(22)}_{t+h-1} + \epsilon_t \tag{14}
$$

called Range-HAR and

$$
\begin{aligned}
\log RA_{t+h} =& \beta_0 + \beta_1 \log RA_{t+h-1} + \beta_2 \log RA^{(5)}_{t+h-1} + \beta_3 \log RA^{(22)}_{t+h-1} + \\
& \beta_4 l_{t+h-1} + \beta_5 l^{(5)}_{t+h-1} + \beta_6 l^{(22)}_{t+h-1} + \epsilon_t
\end{aligned} \tag{15}
$$

called Range-L-HAR.

2.3 Forecasting volatility using daily data

The first specification for the continuous volatility component is the GARJI model:

$$
\begin{aligned}
R_t &= \mu + \sigma_t z_t + \sum_{i=1}^{N_t} X_t^{(i)} && (16)\\
\lambda_t &= \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_{t-1} && (17)\\
\sigma_t^2 &= \gamma + g(\Lambda, \mathcal{F}_{t-1})\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 && (18)\\
g(\Lambda, \mathcal{F}_{t-1}) &= exp(\alpha + \alpha_j E(N_t|\mathcal{F}_{t-1}) && (19)\\
&+ \mathbb{1}_{\{\epsilon_{t-1}<0\}}[\alpha_a + \alpha_{a,j}E(N_t|\mathcal{F}_{t-1})])
\end{aligned}
$$

where $\epsilon_t = \epsilon_{1,t} + \epsilon_{2,t} = \sigma_t z_t + \sum_{i=1}^{N_t} X_t^{(i)}$, $z_t \sim \mathcal{N}(0,1)$, $N_t \sim Poisson(\lambda_t)$, $X_t^{(j)} \sim \mathcal{N}(\mu, \omega^2)$ and $\xi_{t-1} = E[N_{t-1}|\mathcal{F}_{t-1}) - \lambda_{t-1}$.

As explained in Maheu and McCurdy (2004), the last equation allows for the introduction of a differential impact if past news are deemed good or bad. If past news are business as usual, in the sense that no jumps occurred, and are positive, then the impact on current volatility will be $exp(\alpha)\epsilon_{t-1}^2$. If no jump takes place but news are bad, the volatility impact becomes $exp(\alpha + \alpha_a)\epsilon_{t-1}^2$. If a jump takes place, with good news, the impact is $exp(\alpha + \alpha_j)\epsilon_{t-1}^2$. If a jump takes place, with bad news, then the impact becomes $exp(\alpha + \alpha_j + \alpha_a + \alpha_{a,j})\epsilon_{t-1}^2$.

The arrival rate of jumps is assumed to follow a non homogeneous Poisson process while jump size is described by a Normal distribution. In this way, the single impact of extraordinary news on volatility is identified through the combination of parameters in $g(\Lambda, \mathcal{F}_{t-1})$. The idea of the authors is the following: the conditional variance of returns is a combination of a smoothly evolving continuous-state GARCH component and a discrete jump component. In addition previous realization of both innovations, $\epsilon_{1,t}$ and $\epsilon_{2,t}$ affect expected volatility through the GARCH component of the conditional variance. This feedback is important because once return innovations are realized, there may be strategic or liquidity tradings related to the propagation of the news which are further sources of volatility clustering[15]. With this model it is possible to allow for several asymmetric responses to past returns in-

[15] A source of jumps to price can be important and unusual news, such as earnings surprise (result as an extreme movement in price) while less extreme movements in price can be due to typical news events, such as liquidity trading and strategic trading.

novations and then obtain a richer characterization of volatility dynamics, especially with respect to events in the tail of the distribution (jumps).

In particular $E[N_{t-1}|\mathcal{F}_{t-1})$ is the ex-post assessment of the expected number of jumps that occurred from $t-2$ to $t-1$ and it is equal to $\sum_{j=0}^{\infty} jP(N_{t-1} = j|\mathcal{F}_{t-1})$. Therefore ξ_{t-1} is the change in the econometrician's conditional forecast on N_{t-1} as the information set is updated, it is the difference between the expected value and the actual one. As shown by Maheu and McCurdy (2004) this expression may be inferred using Bayes' formula:

$$P(N_t = j|\mathcal{F}_{t-1}) = \frac{f(R_t|N_t = j, \mathcal{F}_{t-1})P(N_t = j|\mathcal{F}_{t-1})}{f(R_t|\mathcal{F}_{t-1})} \quad \text{for} \quad j = 0, 1, 2, \ldots \tag{20}$$

Indeed, conditional on knowing λ_t, σ_t, and the number of jumps that took place over a time interval, $N_t = j$, the density of R_t in terms of observable is Normal:

$$f(R_t|\mathcal{F}_{t-1}) = \sum_{j=0}^{\infty} f(R_t|N_t = j, \mathcal{F}_{t-1}) \times P(N_t = j|\mathcal{F}_{t-1}) \tag{21}$$

where

$$f(R_t|N_t = j, \mathcal{F}_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma_t^2 + j\delta^2)}} \exp\left(-\frac{(R_t - \mu + \theta\lambda_t - \theta j)^2}{2(\sigma_t^2 + j\delta^2)}\right) \tag{22}$$

Naturally the likelihood function is defined starting from (22), where $\tilde{\theta}$ is the vector of the parameters of interest, i.e. $\tilde{\theta} = (\gamma, \rho, \theta, \delta^2, \alpha, \alpha_j, \alpha_a, \alpha_{aj}, \omega, \beta, \lambda_0, \mu)$:

$$\mathcal{L}(R_t|N_t = j, \mathcal{F}_{t-1}; \tilde{\theta}) = \prod_{t=1}^{T} f(R_t|N_t = j, \mathcal{F}_{t-1}) \tag{23}$$

and the log-likelihood is:

$$l(R_t|N_t = j, \mathcal{F}_{t-1}; \tilde{\theta}) = \sum_{t=1}^{T} \log f(R_t|N_t = j, \mathcal{F}_{t-1}) \tag{24}$$

The maximum number of jumps in each day in the filter (20) is set equal to 10. This is becasue, as suggested in Maheu and McCurdy (2004), the conditional Poisson distribution has almost zero probability in the tails for values of $N_t \geq 10$.

In order to isolate the role of jumps, I estimate a nested version of the GARJI model, i.e. ARJI, which is obtained by imposing $\alpha_j = \alpha_a = \alpha_{a,j} = 0$.

In addition, I consider the GARCH-*t* model and Beta-*t*-GARCH model for conditional volatility. The aim is to understand if the ARJI model can provide a better fit to the empirical distribution of the data and a better quantile forecast with respect to volatility specifications based on fat tails, such as *t*-Student. In particular, Beta-*t*-GARCH presents a more sophisticated volatility specification with respect to GARCH-*t* model. The former consists of an observation driven model based on the idea that the specification of the conditional volatility as a linear combination of squared observations is taken for granted but, as a consequence, it responds too much to extreme observations and the effect is slow to dissipate. Harvey and Luati (2014) define a class of models (DCS) in which the observations are generated by a conditional heavy-tailed distribution with time-varying scale parameters and where the dynamics are driven by the score of the conditional distribution. In this way, Beta-*t*-GARCH counts the innovation outliers but also the additive outliers.

3 Computing and comparing VaR forecasts

The VaR is defined as the $100\alpha\%$ quantile of the distribution of returns. The probability that the return of a portfolio over a t holding period will fall below the VaR is equal to $100\alpha\%$. The predicted VaRs are based on the predicted volatility and they depend on the assumption on the conditional density of daily returns. The one day-ahead VaR prediction at time $t+1$ conditional on the information set at time t is:

$$\widehat{VaR}_{t+1|t} = \sqrt{\widehat{\sigma}^2_{t+1|t}} F_t^{-1}(\alpha) \tag{25}$$

In (25) $\widehat{\sigma}^2_{t+1|t}$ is the returns variance, estimated in both parametric and non-parametric models, $F_t^{-1}(\alpha)$ is the inverse of the cumulative distribution of daily returns while α indicates the degree of significance level. In the case of HF data $\widehat{\sigma}^2_{t+1|t}$ is equal to $\widehat{RV}_t$ or $\widehat{RA}_t$ estimated as explained in the section 2.2 while for GARJI model the returns variance is not simply the modified GARCH dynamic but it also consist of the variance due to jumps (Hung et al., 2008):

$$\widehat{VaR}_{t+1|t} = \sqrt{\widehat{\sigma}^2_{t+1|t} + (\widehat{\theta}^2_t + \widehat{\delta}^2_t)\widehat{\lambda}_t} \widetilde{F}_t^{-1}(\alpha) \tag{26}$$

where $\widetilde{F}_t^{-1}(\alpha) = F_t^{-1}(\alpha) + \frac{1}{6}((F_t^{-1}(\alpha))^2 - 1)Sk(R_t|t\mathcal{F}_{t-1})$ and $Sk(R_t|t\mathcal{F}_{t-1})$ is the conditional return skewness computed after estimating the model. Once obtained VaR forecasts, I assess the relative performance of the models through the violation[16] rate and the quality of the estimates by applying backtesting methods[17].

A violation occurs when a realized return is greater than the estimated ones (VaR). The violation rate is defined as the total number of violations divided by the total number of one period-forecasts[18]The tests used in this paper are the Unconditional Coverage (L_{UC}) and Conditional Coverage (L_{CC}) tests suggested respectively by Kupiec (1995) and Christoffersen (1998). The $L_{UC}$and L_{CC} are the most popular tests among practitioners and academics. This is because they are very simple to implement and because they are incorporated in the Basel accords requirements [19]. These two motivations represent also the reason why both tests are used also in the academic literature. The L_{UC} and the L_{CC}tests assess the *adequacy* of the model by considering the number of VaR exceptions, i.e. days when returns exceed VaR estimates. If the number of exceptions is less than the selected significance level would indicate, the system overestimates risk; on the contrary too many exceptions signal underestimation of risk. In particular, the first test examines whether the frequency of exceptions over some specified time interval is in line with the selected significance level. A good VaR model produces not only the "correct" amount of exceptions but also exceptions that are independent each other and, in turn, not clustered over time. The test of conditional coverage takes into account for the number of exceptions and when the exceptions occur.

The tick loss function considered is defined as Binary loss function (BLF) which counts the number of exceptions, that are verified when the loss is larger than the forecasted VaR:

[16]In the testing literature exception is used instead of violation because the former is referred, as I explain later, to a loss function. The loss function changes according to the test applied and the motivation behind the testing strategies.

[17]The backtesting tests give the possibility to interpret the results and then the quality of the forecasting model choose in inferential terms.

[18]As well explained in Gençay et al. (2003) at$q$ th quantile, the model predictions are expected to underpredict the realized return $\alpha = (1 - q)$ percent of the time. A high number of exceptions implies that the model excessively underestimates the realized return. If the exception ratio at the q th quantile is greater than α percent, this implies excessive underprediction of the realized return. If the number of exceptions is less than α percent at the q th quantile, there is excessive overprediction of the realized return by the underlying model.

[19]See Nieto and Ruiz, 2016 for a review on VaR forecasting and evaluation through backtesting.

$$BLF_{t+1} = \begin{cases} 1 & \text{if} \quad R_{t+1} < \widehat{VaR}_{t+1|t} \\ 0 & \text{if} \quad R_{t+1} \geq \widehat{VaR}_{t+1|t} \end{cases} \tag{27}$$

where $\widehat{VaR}_{t+1|t}$ is the estimated VaR at time t that refers to the period $t+1$.
The Likelihood Ratio test of unconditional coverage tests the null hypothesis that the true probability of occurrence of an exception over a given period is equal to α:

$$H_0: \quad p = \alpha$$
$$H_1: \quad p \neq \alpha$$

where $\widehat{p} = \frac{n_0}{n_1+n_0}$ is the unconditional coverage (the empirical coverage rate) or the failure rate and n_0 and n_1 denote, respectively, the number of exceptions observed in the sample size and the number of non-exceptions.
The unconditional test statistic is given by:

$$LR_{UC} = -2\log\left(\frac{(1-\alpha)^{n_1}\alpha^{n_0}}{(1-\widehat{p})^{n_1}\widehat{p}^{n_0}}\right) \sim \chi^2(1) \tag{28}$$

So, under the null hypothesis the significance level used to forecast VaRs and the empirical coverage rate are equal. The test of conditional coverage proposed by Christoffersen (1998) is an extended version of the previous one taking into consideration whether the probability of an exception on any day depends on the exception occurrence in the previous day. The loss function in constructed as in (27) and the log-likelihood testing framework is as in (28) including a separate statistic for the independence of exceptions. Define the number of days when outcome j occurs given that outcome i occurred on the previous day as n_{ij} and the probability of observing an exception conditional on outcome i of the previous day as π_i. Summarizing:

$$\pi_0 = \frac{n_{01}}{n_{00}+n_{01}} \qquad \pi_1 = \frac{n_{11}}{n_{10}+n_{11}} \qquad \pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}} \tag{29}$$

The independence test statistic is given by:

$$LR_{IND} = -2\log\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right) \tag{30}$$

Under the null hypothesis the first two probabilities in (29) are equal, i.e. the exceptions do not occur in cluster. Summing the statistics (28) and (30) the conditional coverage statistic is obtained, i.e. $LR_{CC} = LR_{UC} + LR_{IND}$ and it is distributed as a χ^2 with two degrees of freedom since two is the number of possible outcomes in the sequence in (27). In order to avoid the possibility that the models considered passing the joint test but fail either the coverage or the independence test I choose to run LR_{CC} and also its decomposition in LR_{UC} and LR_{IND}.

4 Data and Empirical results

4.1 Data

In order to assess the informational content of HF and daily data, I use S&P 500 index from 5 Jan.1996 to 30 Dec.2005 for both samples.

The total number of trading days is equal to 2516 which coincides with the number of daily returns. In the top panel of Figure 1 the level of the S&P 500 index is presented. The corresponding daily returns are displayed in the bottom panel of Figure 1. Given the literature on the effects of microstructure noise of estimates of RV and the forecast performance of RV models based on different sampling frequency, I use 5-minutes data for a total of $197,689$ observations. I compute 5-minutes intraday returns as the log-difference of the closing prices in two subsequent periods of time. The daily returns are computed taking the last closing prices in each trading day. The range volatility at each date is calculated as scaled log difference between the highest and the lowest price in a trading day. Table 1 reports the descriptive statistics of S&P 500 index for RA_t, RV_t and its decomposition in BV_t and JV_t. In particular JV_t is computed as $max\{RV_t - BV_t, 0\}$[20]. A number of interesting features are founded. Firstly, returns exhibit negative asymmetry and leptokurtosis. As shown in Ander-

[20]The summary statistics of the continuous and disontinuous components computed according to Andersen et al. (2007) and Corsi et al. (2010) are not reported because are very similar to those presented in Table 1.

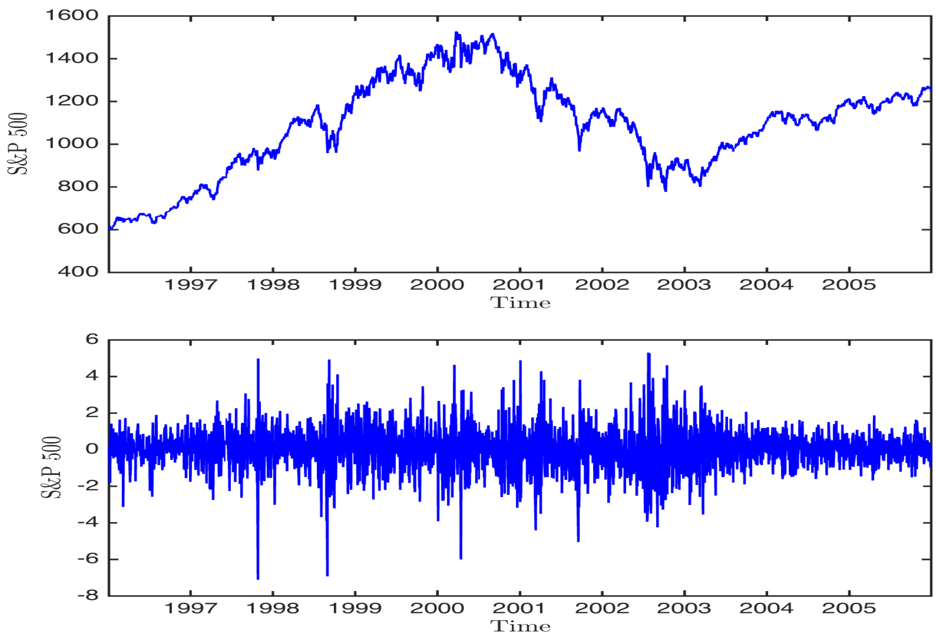

Figure 1: Top: daily S&P 500 index from 5 Jan.1996 to 30 Dec.2005. The horizontal axis corresponds to time while the vertical axis displays the value of the index. Bottom: daily S&P 500 percentage returns calculated by $r_t = log(p_t/p_{t-1})$, where p_t is the value of the index at time t.

Table 1: Summary Statistics. The rows report the sample mean, standard deviation, skewnwss, kurtosis, sample minimum and maximum for the daily returns (R_t), the standardized daily returns ($R_t/\sqrt{RV_t}$) the daily realized volatility (RV_t), the daily bipower variation (BV_t), the daily jump component (JV_t) and the daily range estimator (RA_t). Returns are expressed in percentage.

	R_t	$R_t/\sqrt{RV_t}$	RV_t	BV_t	JV_t	RA_t
Mean	0.0279	0.1378	0.8250	7.93E-05	3.15E-06	9.70E-05
St. Dev.	1.1520	1.3138	1.0097	9.85E-05	1.00E-05	1.52E-04
Skewness	-0.0951	0.253	4.8721	4.8786	19.9283	7.1671
Kurtosis	5.9165	2.8505	39.1013	39.3401	659.3967	84.1687
Min	-7.1127	-3.6092	0.0281	0.0281	0	0.0206
Max	5.3080	4.7161	11.890	11.890	3.6200	25.931

sen et al. (2007) the daily returns standardized with respect to the square root of the ex-post realized volatility are closed to Gaussian. In fact its mean and asymmetry are close to zero, its variance is close to one while its kurtosis is near to 3. This result is clear from Figure 2 in which the empirical density distribution is plotted with the normal density distribution for $R_t/\sqrt{RV_t}$. Moreover if I compare RV_t and BV_t the latter is less noisy than the former, considering the role of jumps. Finally, jump process does not show any Gaussian feature [21].

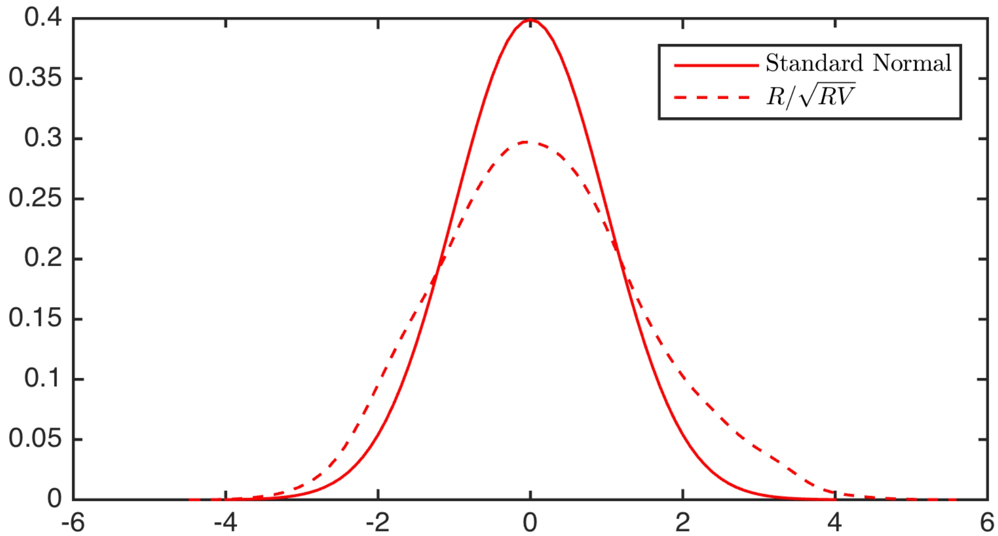

Figure 2: Standardized log-returns distribution of the S&P 500 index.The standard normal distribution (solid line) is compared with the standardized log-returns distribution (dashed line).

Figure 3 shows the plot of RV_t, BV_t, JV_t and RA_t estimators. It is evident that RV_t, BV_t and JV_t follow a similar pattern and the latter tends to be higher when RV_t is higher. JV_t exhibits a relatively small degree of persistence as consequence of the clustering effect. Not surprisingly, RA_t follows the same pattern of RV_t since both of them are ex-post volatility measures.

4.1.1 Estimation results based on daily data

Table 2, provides parameter estimates for both the GARJI and ARJI model applied to the S&P500. The parameter estimates are presented separating the diffusion component from

[21] In particular, jumps computed according to (6) exhibit a higher mean with respect to those computed according to (4), given that the former exploits the possibility of consecutive jumps.

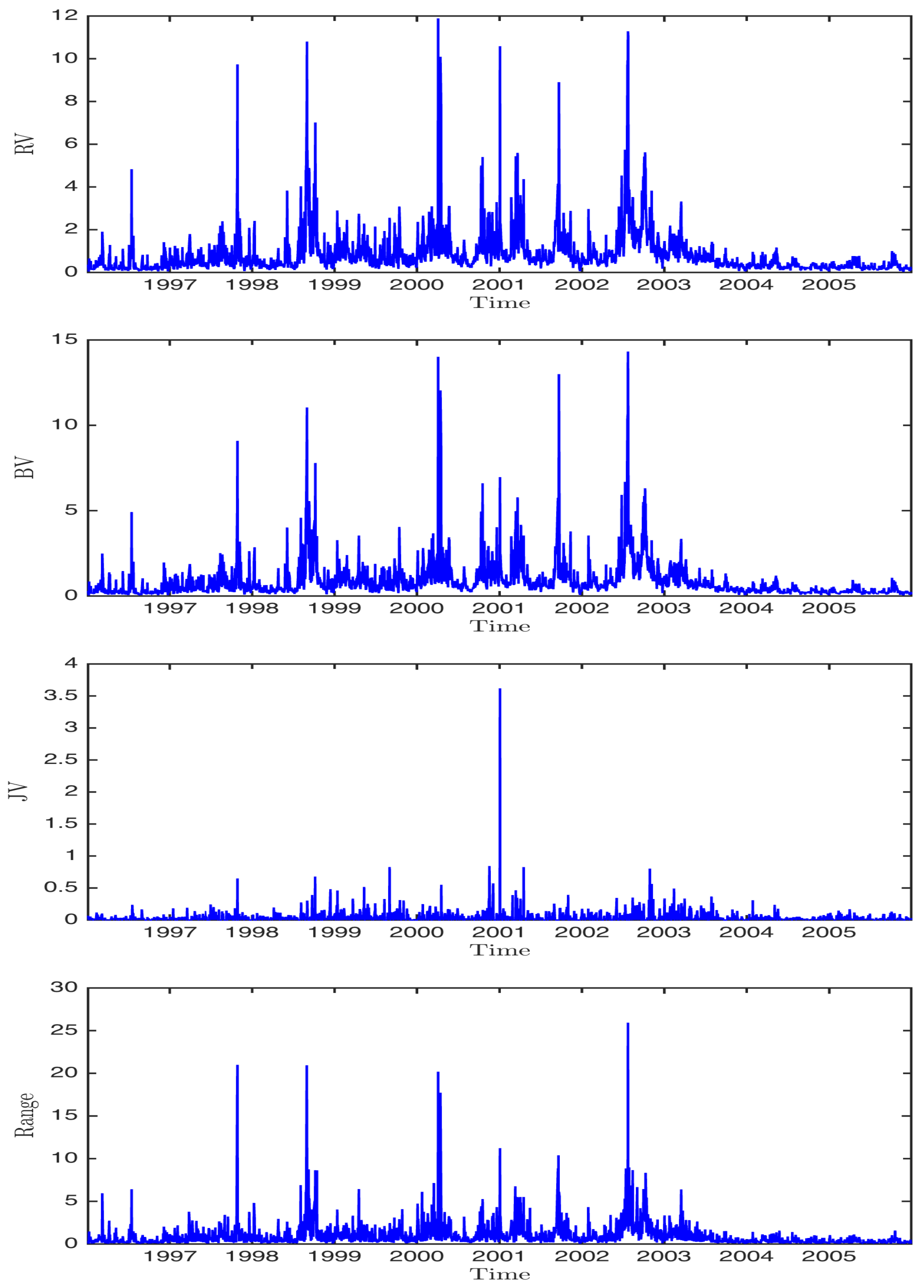

Figure 3: Top: RV_t computed using 5- minutes data from 5 Jan.1996 to 30 Dec.2005. Second: BV_t computed using 5- minutes data from 5 Jan.1996 to 30 Dec.2005. Third: $JV_t = max\{RV_t - BV_t, 0\}$ is computed using 5- minutes data from 5 Jan.1996 to 30 Dec.2005. Bottom: Range estimator computed using daily data from 5 Jan.1996 to 30 Dec.2005. Time is on the horizontal axis.

the jump component. First, both parameters ρ and γ are significantly different from zero. The former represents the persistence of the arrival process of jumps that is quite high for both models implying the presence of jump clustering. The latter, γ, measures the change in the conditional forecast of the number of jumps due to the last day information. The significance of these two parameters suggests that the arrival process of jumps can deviate from its unconditional mean. The implied unconditional jump intensity is 0.8727 while the average variance due to jumps is equal to 0.5516: the index is volatile. This result is confirmed by the average proportion of conditional variance explained by jumps which is equal to 0.3068, jumps explained almost the 23% of the total returns variance. Moreover the jump size mean θ is negative for both model and the most interesting feature is that it affects conditional skewness and conditional kurtosis. The sign of θ indicates that large negative return realizations due to jumps are associated with an immediate increase in the variance explaining the contemporaneous leverage effect: when jumps are realized they tend to have a negative effect on returns. In particular the average conditional skewness is equal to -0.2766 while the average conditional kurtosis is equal to 3.2814. Furthermore the feedback coefficient $g(\Lambda, \mathcal{F}_{t-1})$ tends to be smaller when at least one jump occurs because the total innovation is larger after jumps . Considering the first column of Table 2, the feedback coefficient associated with good news and no jump is equal to 0.0005 and it increases if one jump occurs, i.e. 0.0010. If no jumps occur and if news are bad the coefficient is equal to 0.0411; it is equal to 0.0348 in case of bad news if one jump occurs. These results provide evidence for the asymmetric effect of good and bad news and they show that the asymmetry associated to bad news is more important in the absence of jumps, namely for normal innovations. In fact the difference between the coefficient estimates for both good and bad news in the case of no jumps and one jump are quite similar. This means that news associated with jump innovations is incorporated more quickly into current prices. The second column of Table 2 presents the estimated parameters for the model with $\alpha_j = \alpha_a = \alpha_{a,j} = 0$. With this specification and through the LR test it is possible to understand if the asymmetric effect of good versus bad news is statistically significant: the asymmetric news effect is statistically significant.

Table 2: GARJI and ARJI models estimates. ARJI model is obtained assuming $\alpha_j = \alpha_a = \alpha_{a,j} = 0$. Standard errors are in parenthesis.

Process	Parameters	S&P 500	
		GARJI	ARJI
Diffusion			
	μ	0.0106 (1.9839)	0.0153 (2.1142)
	ω	0.0036 (0.0005)	0.0034 (0.0005)
	α	-7.7048 (0.4332)	-4.7623 (0.3063)
	α_j	0.8096 (0.7538)	-
	α_a	4.5131 (0.4213)	-
	$\alpha_{a,j}$	-0.9776 (0.7204)	-
	β	0.9696 (0.0002)	0.9787 (0.0000)
Jump			
	λ_0	0.0211 (0.0039)	0.0229 (0.0052)
	ρ	0.9758 (0.0025)	0.9757 (0.0030)
	γ	0.5262 (0.0501)	0.4792 (0.0641)
	θ	-0.9895 (0.3985)	-0.9793 (0.4501)
	δ^2	0.0005 (0.0000)	0.0000 (0.0000)
Log-likelihood		-3570.8	-3574.4

4.1.2 Estimation results based on high frequency data

All the estimates presented in Table 3, Table 4 and Table 5 are computed employing the OLS method over the entire sample period, i.e. from 5 Jan. 1996 to 30 Dec. 2005, for the S&P500 index. Table 3 and Table 4 show the results for the models presented in Section 2.2 for models based on RV, its decomposition in BV and JV and the cascade structure for the leverage effect.

The coefficients of the continuous component expressed as daily, weekly and monthly measures, respectively β_1,β_2 and β_3 are significants in all models. Moreover, jump components appear to be fundamental to forecast one step ahead volatility; the predictive power is larger for those specifications that allow for RV decomposed in its continuous and discontinuous components, regardless the identified method used for jump magnitude. Furthermore, the estimates for the aggregate leverage variables are negatives (as expected) and significant. Moreover, the predictive power increases adding the cascade structure for the leverage

Table 3: Estimation of models based on high frequency data: AR(8), HAR, L-HAR, HARC-Jumps, LHARC-Jumps. The coefficients refer to models presented in the Section 2.2. Standard errors are in parenthesis.

Parameter	AR(8)		HAR		L-HAR		HARC-Jumps		LHARC-Jumps	
β_0	-0,0615	(0,0130)	-0,1086	(0,0125)	-0,3916	(0,0474)	-0,0670	(0,0260)	-0,3658	(0,0552)
β_1	0,3877	(0,0195)	0,4020	(0,0189)	0,2763	(0,0199)	0,4047	(0,0189)	0,2799	(0,0199)
β_2	0,1670	(0,0211)	0,3537	(0,0317)	0,3257	(0,0336)	0,3452	(0,0318)	0,3218	(0,0335)
β_3	0,0693	(0,0212)	0,1735	(0,0314)	0,2029	(0,0372)	0,1691	(0,0331)	0,1891	(0,0387)
β_4	0,0902	(0,0213)	-	-	-0,2374	(0,0152)	-0,0886	(0,1554)	0,0009	(0,1477)
β_5	0,0831	(0,0213)	-	-	-0,1962	(0,0422)	-0,0828	(0,3146)	-0,1692	(0,2987)
β_6	0,0332	(0,0213)	-	-	-0,0840	(0,0907)	0,1147	(0,6055)	0,4391	(0,5776)
β_7	0,0567	(0,0210)	-	-	-	-	-	-	-0,2349	(0,0152)
β_8	0,0316	(0,0195)	-	-	-	-	-	-	-0,1914	(0,0422)
β_9	-	-	-	-	-	-	-	-	-0,0862	(0,0911)
Obs.	2494		2494		2494		2494		2494	
R^2	0,6463		0,6444		0,6744		0,6459		0,6752	
Adj. R^2	0,6452		0,6439		0,6736		0,6450		0,6740	

regressors.

This finding confirms the different reaction of daily volatility to negative returns. The estimates of the forecasting models based on the Range estimator are reported in Table 5. The coefficients of the HAR specification are statistically significant; these results imply a heterogeneous structure also for RA volatility measure. The highest predictive power is recorded for the L-HAR model. Indeed also, in this case, the heterogenous structure in the leverage effect has an important role in predicting future volatility.

4.2 VaR accuracy results

To assess the model's capability of predicting future volatility, I report the results of the Kupiec (1995) and the Christoffersen (1998) tests described in the Section 3. Both tests address the accuracy of VaR models and their results interpretation give insights into volatility models usefulness to risk managers and supervisory authorities. The tests are computed for both models based on HF data and on daily data. In evaluating models performance, the available sample is divided into two subsamples. The in-sample period is equal to 1677 ob-

Table 4: Estimation of models based on high frequency data: HAR-CV-JV,LHAR-CV-JV, HAR-C-J, LHAR-C-J. The coefficients refer to models presented in the Section 2.2. Standard errors are in parenthesis.

Parameter	HAR-CV-JV		LHAR-CV-JV		HAR-C-J		LHAR-C-J	
β_0	-0,0666	(0,0244)	-0,3671	(0,0549)	-0,0482	(0,0182)	-0,3225	(0,0512)
β_1	0,4040	(0,0188)	0,2792	(0,0198)	0,4085	(0,0185)	0,2906	(0,0195)
β_2	0,3450	(0,0317)	0,3219	(0,0334)	0,3115	(0,0310)	0,2942	(0,0327)
β_3	0,1711	(0,0327)	0,1902	(0,0385)	0,1942	(0,0313)	0,2153	(0,0371)
β_4	-0,0938	(0,1545)	0,0059	(0,1468)	-0,1345	(0,1004)	-0,1242	(0,0954)
β_5	-0,0676	(0,3115)	-0,1511	(0,2958)	0,4000	(0,1733)	0,2607	(0,1649)
β_6	0,0435	(0,5948)	0,4172	(0,5686)	-0,4836	(0,2960)	-0,2292	(0,2827)
β_7	-	-	-0,2351	(0,0152)	-	-	-0,2310	(0,0151)
β_8	-	-	-0,1909	(0,0422)	-	-	-0,1888	(0,0419)
β_9	-	-	-0,0869	(0,0913)	-	-	-0,0604	(0,0908)
Obs.	2494		2494		2494		2494	
R^2	0,6458		0,6752		0,6497		0,6781	
Adj. R^2	0,6450		0,6740		0,6489		0,6769	

Table 5: Estimation of models based on Range estimator: AR(8), HAR, L-HAR. The coefficients refer to models presented in Section 2.2. Standard errors are in parenthesis.

Parameter	AR(8)		HAR		L-HAR	
β_0	-0,0937	(0,0212)	-0,2694	(0,0194)	-0,8699	(0,0724)
β_1	0,1094	(0,0204)	0,0993	(0,0205)	-0,0455	(0,0224)
β_2	0,2054	(0,0205)	0,4580	(0,0422)	0,2970	(0,0484)
β_3	0,1212	(0,0209)	0,3255	(0,0463)	0,3486	(0,0566)
β_4	0,0918	(0,0209)	-	-	-0,3069	(0,0260)
β_5	0,1002	(0,0209)	-	-	-0,4060	(0,0707)
β_6	0,0791	(0,0209)	-	-	-0,4148	(0,1479)
β_7	0,0573	(0,0205)	-	-	-	-
β_8	0,0938	(0,0204)	-	-	-	-
β_9	-	-	-	-	-	-
Obs.	2494		2494		2494	
$\hat{R2}$	0,3964		0,3914		0,4337	
Adj. $\hat{R2}$	0,3945		0,3907		0,4323	

servations, around 2/3 of the total sample, while the out-of-sample period is around 1/3 of the total sample, equal to 839 observations. A rolling window procedure is used to implement the backtesting procedure and, in turn, to choose among different specifications. After estimating the alternative VaR models, the one-day-ahead VaR estimate is computing using the in-sample period. Then the in-sample period is moved forward by one period and the estimation is run again. This procedure is repeated step by step for the remaining 839 days, until the end of the sample. For both tests the expected number of exceedances is chosen equal to 5% and 1% level[22]. Table 6 and Table 7 shows the VaR accuracy results at both 5% and 1% level, respectively, for all models presented in Section 2.2 and Section 2.3. The economic value per se of the HF-data forecasting models is assessed looking at the first part of Table 6 and Table 7: All models that allow for explicit jumps and leverage components do not reject the null at 1% while LHAR-C-J (jumps specified according to Corsi et al., 2010) is the only model that does not reject the null conditional coverage at both αs level. In fact, for this model, the average number of violations for the VaR at 5% level is the closest to the true probability of occurrence of an exception over one day.

Instead, looking at the accuracy of daily data models, GARCH-*t* and Beta-*t*-GARCH do not reject the null of conditional coverage at 5%, while all models pass the L_{CC} test at 1% level. Comparing this last result with the accuracy of the LHAR-C-J model, both GARCH-*t* and Beta-*t*-GARCH provide an average number of violations closer to the theoretical one. AR(8) provides accurate VaR measures if the Range estimator is used to proxy the latent volatility. Even if the statistical significance of all βs parameters in both Range HAR and L-HAR models give insight on the possibility to extend the heterogeneous structure to such forecasting models (see Table 5), these models do not pass accuracy tests at both considered level.

Indeed, the VaR forecasts according to both L_{UC} and L_{CC} are more accurate for daily data than HF-data models.

Furthermore, allowing for an explicit jump component improves over HF-based VaR performance at 1% level. No matter what the jump identification strategy is chosen, all models (HARC-Jumps, HAR-CV-JV, and HAR-C-J) do not reject the null of unconditional and con-

[22]Both tests are also implemented to 10% level and the results are shown in the Appendix A. The quantile required by Basel accords is 1%. Financial institutions, recently, has implemented stress tests which require VaR forecasts for level smaller than 1%.

Table 6: VaR accurancy at 5%. The first column shows the model chosen in order to compute the VaR forecasts. H is the average number of violations computed for each model. VaR is the average VaR forecasts. LR_{UC}, LR_{CC} and LR_{IND} represent the pvalue associated to the Kupiec (1995) and Christoffersen (1998) tests. All tests are evaluated at 1% significance level.

Model	H	VaR	LR_{UC}	LR_{IND}	LR_{CC}
AR(8)	0.074	-1.276	0.003	0.493	0.009
HAR	0.075	-1.267	0.002	0.134	0.002
L-HAR	0.073	-1.268	0.004	0.096	0.004
HARC-Jumps	0.076	-1.261	0.001	0.597	0.004
LHARC-Jumps	0.074	-1.266	0.003	0.114	0.003
HAR-CV-JV	0.076	-1.260	0.001	0.597	0.004
LHAR-CV-JV	0.074	-1.265	0.003	0.114	0.003
HAR-C-J	0.075	-1.257	0.002	0.544	0.006
LHAR-C-J	0.072	-1.261	0.007	0.399	0.018
GARJI	0.029	-1.646	0.002	0.030	0.001
ARGJI	0.024	-1.673	0.000	0.087	0.000
GARCH-t	0.032	-2.221	0.012	0.279	0.023
Beta-t-GARCH	0.032	-2.195	0.012	0.279	0.023
Range AR(8)	0.070	-1.255	0.010	0.664	0.034
Range HAR	0.075	-1.233	0.002	0.055	0.001
Range L-HAR	0.098	-1.135	0.000	0.453	0.000

ditional coverage at 1% significance level. At odds, the null is rejected for VaR computed at 5% level. For what concerns daily-data models, accounting for an explicit jump component (GARJI, ARJI) or supposing a fat-tails distribution for log-returns gives the same VaR accuracy at 1% in terms of L_{UC} and L_{CC}. Allowing for an explicit jump factor in the conditional log-returns distribution provides more accurate VaR measure, in addition to important information about the market response to outside news.

Another focus of this paper is represented by the leverage effect. Looking at Table 6 and Table 7 , leverage effect has an important role in improving volatility forecasts and, in turn, VaR accuracy. In fact, at least for daily data and HF-data, models that allow for an asymmetric volatility response to price movements, do not reject the null of conditional coverage, passing both Kupiec (1995) and Christoffersen (1998) tests at 1% level. Surprisingly, L-HAR model at 1% level generates the same proportion of hits (13%) of LHAR-CV-JV and LHARC-Jumps, involving an equal value for the L_{CC} statistic. This means that adding jumps as an explanatory variable in the forecasting volatility model does not improve over VaR accuracy if a leverage component is considered.

Table 7: VaR accurancy at 1%. The first column shows the model chosen in order to compute the VaR forecasts. H is the average number of violations computed for each model. VaR is the average VaR forecasts. LR_{UC}, LR_{CC} and LR_{IND} represent the pvalue associated to the Kupiec (1995) and Christoffersen (1998) tests. All tests are evaluated at 1% significance level.

Model	H	VaR	LR_{UC}	LR_{IND}	LR_{CC}
AR(8)	0.017	-1.805	0.075	0.490	0.162
HAR	0.019	-1.791	0.019	0.034	0.007
L-HAR	0.013	-1.794	0.385	0.129	0.217
HARC-Jumps	0.018	-1.783	0.039	0.025	0.010
LHARC-Jumps	0.013	-1.790	0.385	0.129	0.217
HAR-CV-JV	0.018	-1.782	0.039	0.025	0.010
LHAR-CV-JV	0.013	-1.790	0.385	0.129	0.217
HAR-C-J	0.017	-1.778	0.075	0.019	0.013
LHAR-C-J	0.015	-1.783	0.138	0.191	0.142
GARJI	0.004	-2.423	0.031	0.883	0.098
ARGJI	0.002	-2.486	0.008	0.922	0.029
GARCH-t	0.008	-3.449	0.622	0.731	0.835
Beta-t-GARCH	0.010	-3.409	0.894	0.695	0.918
Range AR(8)	0.020	-1.775	0.009	0.350	0.021
Range HAR	0.023	-1.744	0.002	0.070	0.001
Range L-HAR	0.029	-1.606	0.000	0.180	0.000

A slightly different result is registered for the HAR-C-J and the L-HAR-C-J models, underlying a superior ability of jump identification strategy proposed by Corsi et al. (2010). Summing up, daily-data models are preferred to HF-data models when the VaR is required at 5% level[23]. At 1% VaR level, all daily data models pass the Kupiec (1995) and the Christoffersen (1998) tests, at odd of HF-data models. For this data category, only the more sophisticated volatility forecasting models give accurate VaR forecasts. Finally, both jumps and leverage effect are important factors in order to obtain reliable VaR measures.

5 Conclusion

This paper assesses the economic value of different forecasting volatility models, in terms of informational content embedded in the HF observations and daily data. In order to do so, I compare the performance of HF-data and daily data models in a VaR framework. Two key assumptions are introduced: jumps in price and leverage effect in volatility dynamics.
Specifically, I consider various specifications of HF-data models for volatility forecast, which differs along three main dimensions: different time-horizons for investors, separation of continuous and discontinuous volatility components and, finally, a cascade dynamic for the leverage effect. I also consider different variants of the daily data models, in form of GARJI models either with or without an asymmetric effect of news on volatility, as well as in form of two fat-tails models, namely the GARCH-*t* and the Beta-*t* GARCH models. All these models are compared with a correspondent and equivalent model, based on the Range volatility measure; the latter is expected to estimate a level of volatility which is intermediate with respect to those measured by HF-data and daily data models. This analysis highlights important issues. First, it stresses the importance of the sampling frequency for data needed in economic applications such as the VaR measurement. Second, it emphasizes the strict relationship between VaR measures and the type of model used to forecast volatility. In sum, daily-data models are preferred to HF-data models at 5% and 1% VaR level.
The accuracy of the VaR measure significantly improves when introducing both an explicit jump component and a fat-tails distribution in forecasting volatility models. Specifically, independently from the data frequency, allowing for jumps in price (or providing fat-tails) and leverage effects translates in more accurate VaR measure. However, introducing jumps allows risk managers to have relevant information on the market reaction to outside news.

[23] From Table 8 in the Appendix A only the AR(8) model passes all accuracy tests. This result can be interpreted in favor of more sophisticated forecasting models when the α level required is less conservative.

Appendix A VaR accurancy at 10% level

Table 8: VaR accurancy at 10% level. The first column shows the model chosen in order to compute the VaR forecasts. H is the average number of violations computed for each model. VaR is the average VaR forecasts. LR_{UC}, LR_{CC} and LR_{IND} represent the pvalue associated to the Kupiec (1995) and Christoffersen (1998) tests. All tests are evaluated at 1% significance level.

Model	H	VaR	LR_{UC}	LR_{IND}	LR_{CC}
AR(8)	0.132	-1.276	0.003	0.670	0.010
HAR	0.139	-1.267	0.000	0.817	0.001
L-HAR	0.136	-1.268	0.001	0.881	0.004
HARC-Jumps	0.142	-1.261	0.000	0.945	0.001
LHARC-Jumps	0.138	-1.266	0.000	0.987	0.002
HAR-CV-JV	0.142	-1.260	0.000	0.726	0.001
LHAR-CV-JV	0.137	-1.265	0.001	0.949	0.003
HAR-C-J	0.147	-1.257	0.000	0.979	0.000
LHAR-C-J	0.141	-1.261	0.000	0.913	0.001
GARJI	0.070	-1.646	0.003	0.165	0.004
ARGJI	0.068	-1.673	0.001	0.279	0.003
GARCH-t	0.052	-2.221	0.000	0.029	0.000
Beta-t-GARCH	0.052	-2.195	0.000	0.029	0.000
Range AR(8)	0.143	-1.255	0.000	0.559	0.000
Range HAR	0.145	-1.233	0.000	0.621	0.000
Range L-HAR	0.166	-1.135	0.000	0.439	0.000

References

Aït-Sahalia, Y. and J. Jacod (2010). Analyzing the spectrum of asset returns: Jump and volatility components in high frequency data. Technical report, National Bureau of Economic Research.

Aït-Sahalia, Y. and L. Mancini (2008). Out of sample forecasts of quadratic variation. *Journal of Econometrics 147*(1), 17–33.

Aït-Sahalia, Y., P. A. Mykland, and L. Zhang (2005). How often to sample a continuous-time process in the presence of market microstructure noise. *Review of Financial studies 18*(2), 351–416.

Aït-Sahalia, Y., P. A. Mykland, and L. Zhang (2011). Ultra high frequency volatility estimation with dependent microstructure noise. *Journal of Econometrics 160*(1), 160–175.

Alizadeh, S., M. W. Brandt, and F. X. Diebold (2002). Range-based estimation of stochastic volatility models. *The Journal of Finance 57*(3), 1047–1091.

Andersen, T. G. and T. Bollerslev (1998). Answering the skeptics: Yes, standard volatility models do provide accurate forecasts. *International economic review*, 885–905.

Andersen, T. G., T. Bollerslev, P. F. Christoffersen, and F. X. Diebold (2012). Financial risk measurement for financial risk management. Technical report, National Bureau of Economic Research.

Andersen, T. G., T. Bollerslev, and F. X. Diebold (2007). Roughing it up: Including jump components in the measurement, modeling, and forecasting of return volatility. *The Review of Economics and Statistics 89*(4), 701–720.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and H. Ebens (2001a). The distribution of realized stock return volatility. *Journal of financial economics 61*(1), 43–76.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys (2003). Modeling and forecasting realized volatility. *Econometrica 71*(2), 579–625.

Andersen, T. G., T. Bollerslev, F. X. Diebold, and C. Vega (2002). Micro effects of macro announcements: Real-time price discovery in foreign exchange. Technical report, National bureau of economic research.

Andersen, T. G., T. Bollerslev, P. Frederiksen, and M. Ørregaard Nielsen (2010). Continuous-time models, realized volatilities, and testable distributional implications for daily stock returns. *Journal of Applied Econometrics 25*(2), 233–261.

Barndorff-Nielsen, O. E. (2002). Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology) 64*(2), 253–280.

Barndorff-Nielsen, O. E. and N. Shephard (2004). Power and bipower variation with stochastic volatility and jumps. *Journal of financial econometrics 2*(1), 1–37.

Bates, D. S. (1996). Jumps and stochastic volatility: Exchange rate processes implicit in deutsche mark options. *Review of financial studies 9*(1), 69–107.

Bates, D. S. (2000). Post-'87 crash fears in the s&p 500 futures option market. *Journal of Econometrics 94*(1), 181–238.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics 31*(3), 307–327.

Bollerslev, T. and V. Todorov (2011). Tails, fears, and risk premia. *The Journal of Finance 66*(6), 2165–2211.

Brownlees, C. T. and G. M. Gallo (2010). Comparison of volatility measures: a risk management perspective. *Journal of Financial Econometrics 8*(1), 29–56.

Chan, W. H. and J. M. Maheu (2002). Conditional jump dynamics in stock market returns. *Journal of Business & Economic Statistics 20*(3), 377–389.

Chiu, C.-L., M.-C. Lee, and J.-C. Hung (2005). Estimation of value-at-risk under jump dynamics and asymmetric information. *Applied Financial Economics 15*(15), 1095–1106.

Christensen, K., R. C. Oomen, and M. Podolskij (2014). Fact or friction: Jumps at ultra high frequency. *Journal of Financial Economics 114*(3), 576–599.

Christoffersen, P., R. Elkamhi, B. Feunou, and K. Jacobs (2009). Option valuation with conditional heteroskedasticity and nonnormality. *Review of Financial studies*, hhp078.

Christoffersen, P. F. (1998). Evaluating interval forecasts. *International economic review*, 841–862.

Christoffersen, P. F. and F. X. Diebold (2000). How relevant is volatility forecasting for financial risk management? *Review of Economics and Statistics 82*(1), 12–22.

Clements, M. P., A. B. Galvão, and J. H. Kim (2008). Quantile forecasts of daily exchange rate returns from forecasts of realized volatility. *Journal of Empirical Finance 15*(4), 729–750.

Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, nbp001.

Corsi, F., D. Pirino, and R. Reno (2010). Threshold bipower variation and the impact of jumps on volatility forecasting. *Journal of Econometrics 159*(2), 276–288.

Corsi, F. and R. Renó (2009). Har volatility modelling with heterogeneous leverage and jumps. *Available at SSRN 1316953*.

Creal, D., S. J. Koopman, and A. Lucas (2013). Generalized autoregressive score models with applications. *Journal of Applied Econometrics 28*(5), 777–795.

Diebold, F. X. and R. S. Mariano (2012). Comparing predictive accuracy. *Journal of Business & economic statistics*.

Duan, J.-C. et al. (1995). The garch option pricing model. *Mathematical finance 5*(1), 13–32.

Duan, J.-C., P. Ritchken, and Z. Sun (2006). Approximating garch-jump models, jump-diffusion processes, and option pricing. *Mathematical Finance 16*(1), 21–52.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, 987–1007.

Engle, R. F. and S. Manganelli (2004). Caviar: Conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics 22*(4), 367–381.

Gençay, R., F. Selçuk, and A. Ulugülyağci (2003). High volatility, thick tails and extreme value theory in value-at-risk estimation. *Insurance: Mathematics and Economics 33*(2), 337–356.

Ghysels, E., P. Santa-Clara, and R. Valkanov (2004). The midas touch: Mixed data sampling regression models. *Finance*.

Giot, P. and S. Laurent (2004). Modelling daily value-at-risk using realized volatility and arch type models. *journal of Empirical Finance 11*(3), 379–398.

Hansen, P. R. and A. Lunde (2006). Realized variance and market microstructure noise. *Journal of Business & Economic Statistics 24*(2), 127–161.

Harvey, A. and A. Luati (2014). Filtering with heavy tails. *Journal of the American Statistical Association 109*(507), 1112–1122.

Heston, S. L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of financial studies 6*(2), 327–343.

Heyde, C., S. Kou, and X. Peng (2006). What is a good risk measure: Bridging the gaps between data, coherent risk measures and insurance risk measures. *Preprint, Columbia University*.

Huang, X. and G. Tauchen (2005). The relative contribution of jumps to total price variance. *Journal of financial econometrics 3*(4), 456–499.

Hung, J.-C., M.-C. Lee, and H.-C. Liu (2008). Estimation of value-at-risk for energy commodities via fat-tailed garch models. *Energy Economics 30*(3), 1173–1191.

Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *THE J. OF DERIVATIVES 3*(2).

Maheu, J. M. and T. H. McCurdy (2004). News arrival, jump dynamics, and volatility components for individual stock returns. *The Journal of Finance 59*(2), 755–793.

Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of financial economics 3*(1), 125–144.

Müller, U. A., M. M. Dacorogna, R. D. Davé, R. B. Olsen, O. V. Pictet, and J. E. von Weizsäcker (1997). Volatilities of different time resolutions—analyzing the dynamics of market components. *Journal of Empirical Finance 4*(2), 213–239.

Nieto, M. R. and E. Ruiz (2016). Frontiers in var forecasting and backtesting. *International Journal of Forecasting 32*(2), 475–501.

Parkinson, M. (1980). The extreme value method for estimating the variance of the rate of return. *Journal of Business*, 61–65.

Todorov, V. and G. Tauchen (2011). Volatility jumps. *Journal of Business & Economic Statistics 29*(3), 356–371.

Wu, L. (2003). Jumps and dynamic asset allocation. *Review of Quantitative Finance and Accounting 20*(3), 207–243.

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