Pierpaolo Giannoccolo[#]

MIGRATION, FISCAL COMPETITION AND BRAIN DRAIN ([*])

Quaderni- Working Papers DSE n°

Keywords: Fiscal Competition inside Europe, migration of high educated workers, symmetric Nash Equilibria, potential Brain Drain, one generation model and OLG extension.

JEL Classifications: C71, F150, F22, H23, H30, I29.

Abstact: In this paper I present a simple model in which I analyse the impact of labor mobility on redistributive policies and the provision of education as a publicly provided good. I've analysed different extensions of the model (Maximin objective function of the government and utilitarian function, symmetric and asymmetric population, one generation and two generation model) for check the results in different specifications of the world. The results obtained are in agreement to the literature: less redistribution and less provision of public good with respect to the efficient value (which could be obtained in the absence of mobility or in the presence of coordination among jurisdictions) The aim of this work is to underline this particular aspect of the fiscal competition: when we add up the two negative effects due to the absence of coordination among jurisdictions, the loss of efficiency is more accentuate. This result is important in the European contest because we have an increase of labor mobility. For this reason the possibility of brain drain added to fiscal competition and to an increase of the mobility can be more dangerous for the European jurisdictions and a coordination is necessary in the education policies and in the redistribution policies within the European Union.

[#] Department of Economics, University of Bologna, Italy
e-mail giannoccolop@libero.it
web page www.dse.unibo.it/giannoccolo

[*] I thank Maurice Marchand for useful comments and suggestions. I gratefully acknowledge financial support from Ente Luigi Einaudi grant. Furthermore, I thank Francesco Zanetti and Flavia Cortelezzi for useful comments
.

1. INTRODUCTION

Much of the literature on fiscal competition among independent jurisdictions has focused on the question of whether or not, in absence of coordination, tax rates will be set at efficient levels. While much attention has thus been devoted to the impact of fiscal competition on the overall level of public spending, little consideration has been given to its impact on the composition of public spending. Particularly, an important aspect of fiscal competition is to study what happens when the public good, subjected to the competition, is education and its effect on the redistributive policies. In the following chapter I'll first expose some important results of the literature on fiscal competition[1]. Then I present a simple model in which I analyse the impact of labor mobility on redistributive policies and the provision of education as a publicly provided good. I've analysed different extensions of the model (Maximin objective function of the government and utilitarian function, symmetric and asymmetric population, one generation and two generation model) for check the results in different specifications of the world. The model is exposed in the third chapter and the results obtained are in agreement to the literature: less redistribution and less provision of public good with respect to the efficient value (which could be obtained in the absence of mobility or in the presence of coordination among jurisdictions). The aim of this work is to underline this particular aspect of the fiscal competition: when we add up the two negative effects due to the absence of coordination among jurisdictions, the loss of efficiency is more accentuate. This result is important in the European contest because we have an increase of labor mobility and because, thanks to the treaty of Rome, the citizens of the other jurisdictions within the system must have fiscal treatments that are identical for all the citizens of the region in which they work. For this reason the possibility of brain drain added to fiscal competition and to an increase of the mobility can be more dangerous for the European jurisdictions and a coordination is necessary in the education policies and in the redistribution policies within the European Union[2].

2. SURVEY OF THE LITERATURE

Most of the economic analysis of fiscal competition[3] has focused on two fundamental aspects of the economic theory: mobility and redistribution and mobility and the optimal provision of public goods. In this paper I consider the two literatures jointly, so in the following paragraphs I analyse briefly how these two aspects are treated by the literature.

---

[1] The studies on the impact of labor mobility on redistributive policies are numerous and the results are those expected: "*redistribution is generally lower than in autarky or than within a cooperative setting*".
Moreover, there are numerous studies on mobility and optimal provision of public goods and the standard conclusion is that there is an externality associated with an individual's migration that generally leads to an inefficient distribution of population across regions. Finally, some studies are focused on the impact of labor mobility on the provision of education.

[2] The evidence suggests that the countries of the Europe are experiencing inter-regional movements of labour and capital of significance magnitude. Different authors have studied the necessity of fiscal harmonization in Europe:
Buchanan, J. M. (1950); Burda, M. and C. Wyplosz (1992); Cnossen, S. (1990); Edwards, J. and M. Keen, (1996); Inman R. P. and D. L. Rubinfeld (1992); Janeba, E. and W. Peters (1993); Martin, R. and M. S. Steinen (1997); Obstfeld, W. and G. Peri (1998); Patsouratis, A. V. (1990); Sinn, H.-W. (1990); Smith, S. (1990); Braunerhjelm, P., R. Faini, V. Norman, F. Ruane and P. Seabright (2000).

2.1 Mobility and Redistribution

The results of the literature on labor mobility and redistribution are those expected: redistribution is generally lower than in autarky or than within a cooperative setting. The redistributive policies result in a kind of adverse selection: redistribution creates locational incentives that attract those who benefit from these policies (the poor) and repel contributors (taxpayers)[4]. Most studies conclude that the mobility of taxpayers reduces the ability of local governments to use taxes and transfers to redistribute income locally and most authors argue that international fiscal policy coordination can be desirable because national fiscal policy choices have consequences for efficiency in the international allocation of resources or for the international distribution of welfare. The literature on factor mobility and redistribution was early exclusively devoted to the setting of a federal state. The recent studies have concentrated on economic unions, also called confederations. In a federation there is central authority, which does not exist in a confederation. The issue of subsidiarity is also less pervasive in a federation than in a confederation.

2.1.1 Conventional wisdom

The conventional wisdom is that labor mobility across regions constraints each of them in its ability to pursue redistributive polices. If we consider a region in which there are just two groups of workers, skilled and unskilled, and we suppose that neither group is internationally mobile, then, redistribution such as taxation of the skilled with of transfer to the unskilled workers are in principle impeded only by potential work disincentives. When we suppose that the skilled become mobile, they will migrate to regions levying lower taxes. If the region of reference is small relative to the world market for skilled workers, we have two results: first, the tax on the skilled has no effect on their after-tax incomes and, second, it generates an inefficiency cost which is borne by the immobile unskilled workers. Then the redistribution becomes unsustainable, and if the unskilled workers are mobile instead of the skilled, the same unsustainability result obtains: regions offering relatively generous transfers to unskilled will experience an influx of unskilled and this undermines the financial feasibility of initial redistributive program. Whether the skilled or the unskilled can move are two cases often discussed separately. The so- called brain drain is often considered separately from the immigration of low skilled individuals. If a region can attract skilled or more generally high-income individuals, it will enlarge its tax base and achieve a better redistribution to the benefit of its low skilled residents.

2.2 Mobility and optimal provision of public goods

Greater mobility of capital and labor, consumers and taxpayers (individual and corporate) implies that international differences in taxes and the supply of public goods can induce migration of each of these. The increased mobility of persons, as well as of goods, services and financial capital raises the potential for fiscal spillovers across borders creating incentives for fiscal policy cooperation. The issue of fiscal policy

[3] Different authors have studied the role of tax competition, recent studies can be found in: Bucovetsky, S. and J. D. Wilson (1991-a); Bucovetsky, S. and J. D. Wilson (1991-b); Wildasin, David E. (1994); Wildasin, D. E. (1987); Wilson, J .D. (1988); Wilson, J .D (1999).

coordination in an international setting is a problem of determining the appropriate level of government making different sets of fiscal policy choices. The benefits and the costs of different public goods and externalities are realized on a variety of spatial scales. While some public goods (or bads) provide benefits (costs) on a global scale, others generate benefits only to users in a particular place, region or nation. In an economy with costless consumer mobility, it is known that a system in which individual jurisdictions compete in the supply of local public goods financed by using the local tax base will, in general, not yield a Pareto optimal equilibrium allocation. There is a large literature on the efficiency properties of a system of competing regional jurisdictions. One strand is the fiscal externality literature. It examines the problem associated with the attainment of an efficient regional population distribution. The standard conclusion in the fiscal externality literature is that there is an externality associated with an individual's migration that generally leads to an inefficient distribution of population across regions[5]. Cooperative policy-making is also often proposed for income stabilization in the presence of international and trade and capital mobility.

2.3 A particular kind of public good: "education"

The greater mobility of labor in many parts of the word, and especially the increased integration of the European Union, have motivated extensive re-examination of economic and social policies. Increased labor mobility undermines the ability of those who pay for these policies to capture their full benefits, thus eroding essential political support. In the absence of some form of interjurisdictional compensation for these trans-local benefits, large flows of skilled labor can be expected to erode the political support for local public funding of higher education (M. Justman and J-F. Thisse, 2000). Education in general accounts for as much as of 5% of GNP, and 10% or more of public spending in advanced industrialized countries, with public funding covering, on average, almost 90% of education costs in these countries. Higher education typically accounts for 15-20% of overall education expenditures. Migration of skilled labor implies that those who pay the bill for public higher education may find it difficult to fully capture its benefits. In the past this "brain-drain" was a unidirectional flow of highly skilled labor from third –word countries. More recently, increased integration of labor markets, especially within the European Union, has drawn attention to problems that arise from bi-directional movement of skilled labor between similarly developed countries. Important is also analyse the students mobility[6]and how the education can influence the productivity growth of national economies[7].

[4] "Mobility and redistribution" are analyzed by different authors: Cremer, H., V. Fourgeaud, M. L. Moriero, M. Marchand and P. Pestieau (1996); Epple, D. and T. Romer (1991).

[5] Non-optimality may occur because in moving from one region to another a migrant does not account for the effect of his moving on the tax price of the public good in the region he leaves (the tax prices rises) or enters (the tax price falls).

[6] J. Gordon and P. Jallade (1996) report that in 1993/94 "mobile" foreign students in EU countries (i.e., students not previously resident in the country where they were studying) numbered over 95,000. Similar inter-state mobility occurs in United States. In both cases, the impact of tertiary education transcends the boundaries of the local (sub-federal) jurisdictions where it is founded and where its budget are politically determined.

[7] National per capita endowment of human capital can diverge as a result of different lump sum fiscal policies and subsidies to education (as well as taste differences). Uncoordinated supplies of national public goods can lead to significant departures from allocative efficiency. The brain drain is one example of the possibility in an economy with heterogeneous individuals, since countries would compete to attract individuals with high endowments.

3. MODEL

Initial assumptions

In these initial assumptions I refer, for simplify, directly to the overlapping generation version of the model. In realty the O.L.G. analysis is done only in the last part of this paper and for the others parts we refer to one-generation model. I assume two regions, indexed $i = 1,2$, with the same initial population.

I assume the population, born in each region, constant and equal to one half.

$$N_{1,t} = N_{2,t} = \tfrac{1}{2} \ \forall \mathrm{t}$$

Each worker lives two periods: (young and old).

$t-1$	t	$t+1$
y_{t-1}	o_{t-1}	
	y_t	o_t

In each time we have then the co-existence of two generation of workers, then the total population of each region in time t is normalized to unity. For each generation, there are two types of workers. h_i denotes the amount of educated workers and l_i the amount of non-educated workers who are both at work in region i.

Within each type, each worker is endowed with one unit of homogeneous labor that he supplies inelastically in his region of residence. The two types of labour are perfect substitutes: w^e and w^n are numbers of efficiency units of labour per worker. There are fixed coefficients of transformation: one efficient unit of labour produces one unit of consumption good. In other word, competitive firms produce the private good with a fixed-coefficient production specification

$$f(h_i, l_i) = w^e h_i + w^n l_i \ \text{ for } i = 1,2$$

Where h_i and l_i are the only inputs necessary for production. All the firms in both regions have access to the same technology. With competitive labour and commodity markets, the firms pay each type of workers a wage equal to its marginal product, identical in the two regions: w^e to educated workers and w^n to non-educated workers. By assumption: $w^n < w^e$, otherwise it would certainly not be appropriate to provide education. Only the government of each region provides education[8]. The government decides the amount of education s_i(i.e. the number of school places). Only the workers with young and middle age can be educated and we assume that the government decides in each time the optimal provision of education for

[8] It's possible consider the non-educated workers as the individuals with at least the secondary level of education and the educated workers the individuals which are more specialized. In this case the model can be more realistic and we can analyzed the brain drain phenomena when, as we see below, we assume that only the educated people are mobile.

each generation of workers. The cost of education is the same in each region and in each time and I suppose a quadratic cost function. Then, for each government the total education costs are: $\left(a+\frac{b}{2}s_{it}\right)s_{it}$ with $a>0, b>0 \quad \forall t$.

The regional authority maximizes, for each generation, the utility of his citizens[9] by the use of lump sum taxes (t_i^e) to educated workers and lump sum subsidies (t_i^n) to the non-educated workers. Then the non educated's available income is $x_i^n = w^n + t_i^n$ and the educated's available income is $x_i^e = w^e - t_i^e$.

Each Government must satisfy, for each generation, the budget constraint

$$t_i^e h_i - (a+\frac{b}{2}s_i)s_i - t_i^n l_i = 0.$$

Mobility specifications

Only the educated people (young and old age) are mobile.

Individuals are heterogeneous only with respect to their attachment to home. This parameter is denoted by n, with $n \in [-1,1]$.

We assume that the distribution of n_t is conditional on the education status of the worker.

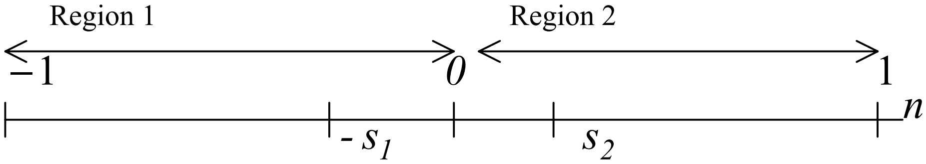
We assume that for each generation and in each time:

For Region 1: $\begin{cases} \text{non educated workers have} & n_t \in (-1;-s_{1t}) \\ \text{educated workers have} & n_t \in (-s_{1t};0) \end{cases} \quad \forall \text{t}$

i.e. the former has the lowest values of n_t, and the latter the highest ones.

For Region 2: $\begin{cases} \text{non educated worker have} & n_t \in (s_{2t};1) \\ \text{educated worker have} & n_t \in (0;s_{2t}) \end{cases}$

i.e. the former has the highest values of n_t, and the latter the lowest ones.



Educated individuals of type n_t derive utility from consumption of their net income and from their region of residence according to:

---

[9] We analyse two different cases: Maximin, subjective maximization.

$$\begin{cases} U(w^e - t_{1t}^e) - \dfrac{1}{2k_t}\mathrm{n}_t & \text{if he lives in region 1} \\ U(w^e - t_{2t}^e) + \dfrac{1}{2k_t}\mathrm{n}_t & \text{if he lives in region 2} \end{cases}$$

Where $\frac{1}{2k_t}\mathrm{n}_t$ measures the non-pecuniary benefit from living in region 2 and $-\frac{1}{2k_t}\mathrm{n}_t$ measures the benefit from living in region 1. Educated individuals, maximizing their utility, are free to choose their region of residence when they are young and when they are middle age, and take the behaviours of all other agents as given.

If $t_{1t}^e = t_{2t}^e$ all educated individuals with $-1 \le n < 0$ would prefer to live in region 1; educated individuals with $\left(\frac{1}{2} < n \le 1\right)$ would prefer to live in region 2 and educated individual with $n = 0$ would be indifferent between the two regions. The migration equilibrium will be characterized by the marginal educated individual, denoted by $\hat{n}$, being indifferent between locating in either region, individuals with n less then $\hat{n}$ locate in region 1 and individuals with n greater than $\hat{n}$ locate in the region 2.

$$U(w^e - t_{1t}^e) - \frac{\hat{n}_t}{2k_t} = U(w^e - t_{2t}^e) + \frac{\hat{n}_t}{2k_t}$$

$$U(w^e - t_{1t}^e) - \frac{n_t}{2k_t} < U(w^e - t_{2t}^e) + \frac{n_t}{2k_t} \qquad \forall n_t < \hat{n}_t$$

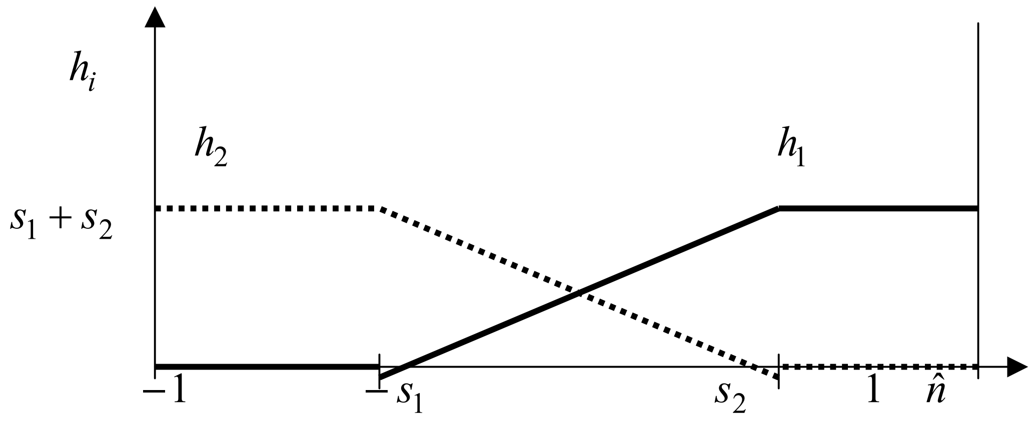
$$U(w^e - t_{1t}^e) - \frac{n_t}{2k_t} > U(w^e - t_{2t}^e) + \frac{n_t}{2k_t} \qquad \forall n_t > \hat{n}_t$$

The number of educated workers being resident in region i $(h_{it} + h_{it-1})$ can be different from the number of people having been educated in region i $(s_{it} + s_{it-1})$.

The distribution of the h_{it}'s $(i = 1,2)$ is the following:

$$h_{1t} = \begin{cases} s_{1t} + \hat{n}_t & \text{for } \hat{n}_t \in (-s_{1t}; s_{2t}) \\ 0 & \text{for } \hat{n}_t \le -s_{1t} \\ s_{1t} + s_{2t} & \text{for } \hat{n}_t \ge s_{2t} \end{cases}$$

$$h_{2t} = \begin{cases} s_{2t} - \hat{n}_t & \text{for } \hat{n}_t \in (-s_{1t}; s_{2t}) \\ s_{1t} + s_{2t} & \text{for } \hat{n}_t \le -s_{1t} \\ 0 & \text{for } \hat{n}_t \ge s_{2t} \end{cases}$$



<u>One generation model – Maximin case</u>

In the next paragraphs I assume that there is only one generation[10] so I can concentrate my analysis on the role of the fiscal competition in the redistribution of income and in the provision of the education.

Autarky solutions

We consider then the autarky case in which we have no mobility of educated workers. In this case the policies of the other governments don't have any effect on the decision of a government. This is equivalent to having a Central Authority that coordinates the politics of the two regions, so there aren't negative externalities from fiscal competition. Then this case can be used as a benchmark to which we can compare the results obtained in the other cases. In this case there isn't migration so $h_i = s_i$ is the total amount of educated workers and $l_i = 1 - s_i$ is the total amount of non-educated workers.

Assume that each government maximizes the utility of the worst-off workers, i.e. use a Maximin criterion. In autarky, with lump sum taxes and transfers, nothing prevents the government from equalizing the available incomes of the two types of workers.

Therefore each government chooses s and t^e so as

$$\underset{s,t^e,t^n}{Max}\, U(w^n + t^n) \tag{1}$$

$$U(w^e + t^e) = U(w^n + t^n)$$

$$t^e h \geq (1-s)t^n + (a + \frac{b}{2}s)s$$

or equivalently:

[10] The one generation model is a particolar case of the O.L.G. model in which there isn't differences between the two generations that can be considered as a unique generation normalized to one.

$$\underset{s,t^e,t^n}{Max}\ t^n \tag{2}$$

$$w^e + t^e = w^n + t^n$$

$$t^e h \ \geq (1-s\ )t^n + (a + \frac{b}{2}s\ )s$$

or equivalently:

$$\underset{s,t^n}{Max}\ t^n \tag{3}$$

$$t^n \leq s(w^e - w^n) - as - \frac{b}{2}s^2$$

or equivalently:

$$\underset{s}{Max}\ \ s(w^e - w^n) - as - \frac{b}{2}s^2 \tag{4}$$

Solving problem (4), we obtain

$$Foc(s): \quad w^e - w^n = a + bs \tag{5}$$

Then the autarkic solutions is

$$w^e - w^n = a + bs \tag{6}$$

or $s = \dfrac{w^e - w^n}{b} - \dfrac{a}{b}$ (7)

According to economic intuition, an increase in the education costs through either a or b implies a decrease in the optimal provision of education places, while an increase in the difference of productivities makes s rise. The lump-sum tax and transfer are chosen so as to equate the available incomes of educated and non – educated people and satisfy the government's budget constraint.

Mobility solutions

We consider the case in which only educated people can migrate. In this case the policies of the other region have effects on the decisions of a region, so each government must take in account what the other does. In this case, in its maximization problem each government must take in account the possibility that there isn't a correspondence between the education provided and the educated people resident in their region.

Variable $\hat{n}$, which characterizes the effect of these policies on the decision to migrate, defines the educated worker who is indifferent between remaining in his region or migrating in the other region.

$$\hat{n} = k\left[U(w^e - t_1^e) - U(w^e - t_2^e)\right] \tag{8}$$

where $\hat{n}$ changes in response to difference in the taxations in the two regions.

According to economic intuition, we have: $\hat{n}\left(\underset{-}{t_1^e}, \underset{+}{t_2^e}\right)$

since $\dfrac{\partial \hat{n}}{\partial t_1^e} = -kU'(w^e - t_1^e) < 0$ (9)

and $\dfrac{\partial \hat{n}}{\partial t_2^e} = kU'(w^e - t_2^e) > 0$ (10)

Contrary to autarky it is now possible that with Maximin objectives in the two regions fiscal competition makes the available income of non-educated workers lower that of educated at the Nash equilibrium. We therefore proceed by solving in a first step the following problem and then check in a second step whether the Nash equilibrium satisfies $w^e - t_1^e > w^n + t_1^n$.

Therefore in the first step we solve the following problem for the region 1:

$$\underset{s_1, t_1^n}{Max} \quad t_1^n \tag{11}$$

$$t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_1)s_1 - (1 - s_1)t_1^n = 0$$

or equivalently

$$\underset{s_1, t_1^n}{Max} \; t_1^n = \frac{t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_1)s_1}{1 - s_1} \tag{12}$$

In (11) and (12) the choices of s_1 and t_1^e are simultaneously made.

In the following paragraphs I analyse, together with this case, another one in which the s_i 's are chosen in a first stage (anticipating their impact on the equilibrium of the second stage) and the t_i^e 's in a second stage.

Mobility solution: Nash equilibrium with s_i *and* t_i^e *simultaneously determined*

In this case each government chooses its values s_i and t_i^e simultaneously for a given policy of the other region and taking in account the possibility of migration of some educated workers as a response to its choices.

For region 1 we have:

$$\underset{s_1, t_1^e}{Max} \; t_1^n = \frac{t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_1)s_1}{1 - s_1} \tag{13}$$

Solving (13) we have

$$Foc(t_1^e):\ \frac{\left(s_1+\hat{n}(t_1^e,t_2^e)\right)-t_1^e\,\frac{\partial \hat{n}(t_1^e,t_2^e)}{\partial t_1^e}}{1-s_1}=0 \tag{14}$$

$$Foc(s_1):\quad \frac{t_1^e-a-bs_1}{1-s_1}+\frac{t_1^e\left(s_1+\hat{n}(t_1^e,t_2^e)\right)-as_1-\frac{b}{2}s_1^2}{(1-s_1)^2}=0 \tag{15}$$

Therefore, in the symmetric Nash equilibrium in which $\hat{n}(t_1^e,t_2^e)=\frac{1}{2}$, we obtain

$$t_1^e=-\frac{s_1}{\frac{\partial \hat{n}(t_1^e,t_2^e)}{\partial t_1^e}} \tag{16}$$

From (9), (16) becomes:

$$t_1^e=\frac{s_1}{kU'(w^e-t_1^e)} \tag{17}$$

At the symmetric Nash equilibrium, $Foc(s_1)$ yields:

$$\frac{t_1^e-a-bs_1}{1-s_1}+\frac{t_1^n}{1-s_1}=0 \tag{18}$$

or $t_1^e+t_1^n=a+bs_1$ (19)

or $s_1=\frac{t_1^e+t_1^n}{b}-\frac{a}{b}$ (20)

Two cases can be distinguished for different values of k. The higher the value of k the stronger the fiscal competition between the two regions.

Case 1: high value of k: strong fiscal competition

Solving the system of three equations in three unknown:

$$t_1^e=\frac{s_1}{kU'(w^e-t_1^e)} \tag{17}$$

$$s_1=\frac{t_1^e+t_1^n}{b}-\frac{a}{b} \tag{20}$$

The budget constraint yields $w^e-t_1^e>w^n+t_1^n$

Then the symmetric Nash Equilibrium is then given by (17) and (20).

Since $t_1^e + t_1^n < w^e - w^n$ (21)

s_1 is lower than in autarkic case. Therefore in this case, fiscal competition lowers both the provision of education and the magnitude of income redistribution.

Case 2: low value of k : weak fiscal competition

Solving (17) and (20) and the budget constraint yields $w^e - t_1^e < w^n + t_1^n$

In this case, the solution is such that the available income of non-educated workers turns out to be higher than that of the educated ones. As a consequence, with a Maximin objective the additional constraint: $w^e - t_1^e \geq w^n + t_1^n$, must be introduced into each government's problem. Therefore, the same solution as in the autarky case is obtained. Summing up, there is a critical value k_c such that for $k < k_c$ the autarky solution prevails at the Nash equilibrium while for $k \geq k_c$, redistribution and education provision are lower.

Mobility solution: Nash equilibrium with s_i and t_i^e determined in different steps

In this more realistic case, the s_i 's are chosen in a first stage (anticipating their impact on the equilibrium of the second stage) and the t_i^e 's in a second stage. As in the previous case, the government must take into account the possibility of migration of some educated workers when it increases their income tax.

We can determine the Nash Equilibrium by backward induction (starting from the second stage and then solving the first stage) Assuming that k is high enough for $U(w^n + t^n) < U(w^e - t^e)$ at the Nash equilibrium.

Second stage

$$\underset{t_1^e}{Max}\ t_1^n = \frac{t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_1)s_1}{1 - s_1} \tag{22}$$

Solving (22) we have

$$Foc(t_1):\ \frac{\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) + t_1^e \frac{\partial \hat{n}(t_1^e, t_2^e)}{\partial t_1^e}}{1 - s_1} = 0 \tag{23}$$

that yields

$$t_1^e = \frac{s_1}{kU'(w^e - t_1^e)} + \frac{\hat{n}(t_1^e, t_2^e)}{kU'(w^e - t_1^e)} \tag{24}$$

It's equivalent for country 2

$$t_2^e = \frac{s_2}{kU'(w^e - t_2^e)} - \frac{\hat{n}(t_1^e, t_2^e)}{kU'(w^e - t_2^e)} \tag{25}$$

Before to analyse the first stage we need to study the properties of (24) and (25). In Appendix (A) I obtain:

$$\frac{dt_1^e}{ds_1} > 0 \quad \frac{dt_1^e}{ds_2} > 0 \tag{26}$$

$$\frac{dt_2^e}{ds_1} > 0 \quad \frac{dt_2^e}{ds_2} > 0 \tag{27}$$

First stage

$$\underset{s_1}{Max}\ t_1^n = \frac{t_1^e(s_1,s_2)\left(s_1 + \hat{n}\left(t_1^e(s_1,s_2), t_2^e(s_1,s_2)\right)\right) - (a + \frac{b}{2}s_1)s_1}{1 - s_1}, \tag{28}$$

$$Foc(s_1): \begin{array}{l} \dfrac{t_1^e(s_1,s_2)\left(1 + \frac{\partial \hat{n}(*)}{\partial s_1}\right) + \left(s_1 + \hat{n}(*)\right)\frac{\partial t_1^e}{\partial s_1} - (a + bs_1)s_1}{1 - s_1} + \\ + \dfrac{t_1^e(s_1,s_2)\left(s_1 + \hat{n}\left(t_1^e, t_2^e\right)\right) - (a + \frac{b}{2}s_1)s_1}{(1 - s_1)^2} = 0 \end{array} \tag{29}$$

the (29) from (24) and (A7) in the appendix A, becomes

$$\frac{t_1^e + t_1^e \dfrac{B^1\left[B^2 + A^2\right]}{A^1B^2 - A^2B^1}}{\frac{1}{2} - s_1} + \frac{\left[s_1 + \hat{n}(*)\right]\dfrac{-B^2}{A^1B^2 - A^2B^1} - a - bs_1 + t_1^n}{\frac{1}{2} - s_1} = 0 \tag{30}$$

or equivalently

$$t_1^e\left[1 + \tfrac{A^2B^1}{A^1B^2 - A^2B^1}\right] - a - bs_1 + t_1^n = 0 \tag{31}$$

$$t_1^e\left[1 + X\right] - a - bs_1 + t_1^n = 0 \tag{32}$$

Where, from (A2)

$$X = \tfrac{A^2B^1}{A^1B^2 - A^2B^1} > 0 \tag{33}$$

For the region (2) we obtain:

$$t_2^e\left[1+Y\right]-a-bs_2+t_2^n=0 \qquad (34)$$

$$Y=\frac{A^2B^1}{A^1B^2-A^2B^1}>0 \qquad (35)$$

Where $X=Y>0$

The (24), by the symmetry becomes:

$$t_1^e=\frac{s_1}{kU'(w^e-t_1^e)} \qquad (24')$$

$$s_1=\frac{t_1^e(1+X)+t_1^n}{b}-\frac{a}{b} \qquad (36)$$

In the conclusions we compare the Nash equilibria obtained in the one-step-mobility case [k low: equation (17) and (20)], and the results obtained in the mobility case with two steps [equation (24') and (36)].

From (26) we know that $\frac{dt_1^e}{ds_1}>0$, from (36) and (24') we can observe that, from the positive value of X, we have in the case with two steps a greater redistribution of income and a greater provision of education respect the one step case. These results are justified from the fact that in the two steps case we reduce the uncertainty and the opportunity of each jurisdiction to attract the educated workers by using the fiscal competition. Then we obtain in the two steps case a solution that it's more closed to the optimal redistribution and optimal provision of education (autarkic case) respect to the one step case.

<u>One generation model - Subjective Utility function</u>

In these paragraphs I analyse the case in which the Government has a subjective utility function. Then the regional authority maximizes the utility of the workers, using a different objective function by the use of lump sum taxes (t_i^e) to educated workers and lump sum subsidies (t_i^n) to the non-educated workers.

Autarky solutions

In this case there isn't migration so $h_i=s_i$ is the total amount of educated workers and $l_i=1-s_i$ is the total amount of non-educated workers. Assume that each government maximizes the utility of the workers using a new objective function. Therefore each government chooses s_i and t_i^e so as

$$\underset{s_1,t_1^e}{Max}\, U\left(w^n+\frac{t_1^e s_1-(a+\frac{b}{2}s_1)s_1}{1-s_1}\right)+\alpha U\left(w^e-t_1^e\right) \qquad (37)$$

with $0\leq\alpha\leq1$

Solving problem (37), we obtain

$$Foc(t_1): U'(y_1^n)\left(\frac{s_1}{1-s_1}\right) - \alpha U'(y_1^e) = 0 \tag{38}$$

$$Foc(s_1): \frac{\partial U(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1-s_1}\right) = 0 \tag{39}$$

where:

$$y_1^e = w^e - t_1^e$$

$$y_1^n = w^n + t_1^n$$

$$U'(y_1^e) = \frac{\partial U(y_1^e)}{\partial t_1^e}$$

$$U'(y_1^n) = \frac{\partial U(y_1^n)}{\partial t_1^e}$$

From (38) and (39) we have

$$U'(y_1^n) = \left(\frac{1-s_1}{s_1}\right)\alpha U'(y_1^e) \tag{40}$$

$$t_1^e - a - bs_1 + t_1^n = 0 \tag{41}$$

According with the economic intuition, we have [Appendix B: (B2) and (B3)]

$$\frac{dt_1^e}{d\alpha} < 0 \qquad \frac{ds_1}{d\alpha} < 0 \tag{42}$$

An increase in the parameter α, which implies a greater weight of the utility of educated in the maximization of the government, has a negative effect in the redistribution and in the provision of education. When $\alpha = 0$ we are in the Maximin case.

$$U'(y_1^n) = U'(y_1^e) \tag{43}$$

which implies that the utilities of the non-educated workers must be equal to the utilities of educated workers.

$$U(y_1^n) = U(y_1^e) \tag{44}$$

$$t_1^e + t_1^n = w^e - w^n \tag{45}$$

Then (41) becomes

$$s_1 = \frac{w^e - w^n}{b} - \frac{a}{b} \tag{46}$$

Then we obtain, with a subjective utility function, the same solutions seen in the Autarkic case with a Maximin criterion.

Mobility solutions

We consider the same mobility specifications seen in the previous paragraphs.

Assume that each government maximizes the utility of the workers using the objective function seen in the autarkic case. In this case, in its maximization problem each government must take in account the possibility that there isn't a correspondence between the education provided and the educated people resident in their region. Then we have, as in Maximin case:

$h_1 = s_1 + \hat{n}$ and $h_2 = s_2 - \hat{n}$.

$\hat{n} = k\left[U(w^e - t_1^e) - U(w^e - t_2^e)\right]$

Mobility solution: Nash equilibrium with s_i and t_i^e simultaneously determined

Each government chooses its values s_i and t_i^e simultaneously for a given policy of the other region and taking in account the possibility of migration of some educated workers as a response to its choices.

For region 1 we have:

$$\underset{s_1, t_1^e}{Max} \quad U\left(w^n + \frac{t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_1)s_1}{1 - s_1} \right) + \alpha U\left(w^e - t_1^e\right) \tag{47}$$

Solving (47) we have

$$Foc(t_1^e):\ U'(y_1^n)\frac{s_1 + \hat{n}(t_1^e, t_2^e) + t_1^e \frac{\partial \hat{n}(t_1^e, t_2^e)}{\partial t_1^e}}{1 - s_1} - \alpha U'(y_1^e) = 0 \tag{48}$$

$$Foc(s_1):\quad \frac{\partial U(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1 - s_1}\right) = 0 \tag{49}$$

Therefore, in the symmetric Nash equilibrium in which $\hat{n}(t_1^e, t_2^e) = 0$, we obtain

$$t_1^e\left[U'(y_1^n)\left(-kU'(y_1^e)\right)\right] = (1 - s_1)\alpha U'(y_1^e) - s_1 U'(y_1^n) \tag{50}$$

or, equivalently

$$t_1^e = \frac{s_1}{kU'(y_1^e)} - \frac{(1 - s_1)\alpha}{kU'(y_1^n)} \tag{51}$$

from $Foc(s_1)$ we obtain

$$t_1^e + t_1^n = a + bs_1 \tag{52}$$

$$s_1 = \frac{t_1^e + t_1^n}{b} - \frac{a}{b} \tag{53}$$

According with the economic intuition, we have [Appendix C: (C2) and (C3)]

$$\frac{dt_1^e}{d\alpha} < 0 \qquad \frac{ds_1}{d\alpha} < 0 \tag{54}$$

An increase in the parameter α, which implies a greater weight of the utility of educated in the maximization of the government, has a negative effect in the redistribution and in the provision of education.

When $\alpha = 0$, the (51) is the same of (17)

$$t_1^e = \frac{s_1}{kU'(w^e - t_1^e)} \tag{17}$$

When $\alpha > 0$, we have that $\frac{s_1}{kU'_{1e}} - \frac{(1-s_1)\alpha}{kU'_{1n}} < \frac{s_1}{kU'_{1e}}$

then we have in this case less redistribution and less provision of education with respect to the Maximin case.

Mobility solution: different value of α between Regions

For region 1 we have:

$$\underset{s_1, t_1^e}{Max} \quad U\left(w^n + \frac{t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_1)s_1}{1 - s_1} \right) + \alpha U\left(w^e - t_1^e\right) \tag{55}$$

For region 2 we have:

$$\underset{s_2, t_2^e}{Max} \quad U\left(w^n + \frac{t_2^e\left(s_2 - \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_2)s_2}{1 - s_2} \right) + \beta U(w^e - t_2^e) \tag{56}$$

Solving (55) we have

$$Foc(t_1^e): \ U'(y_1^n)\frac{s_1 + \hat{n}(t_1^e, t_2^e) + t_1^e \frac{\partial \hat{n}(t_1^e, t_2^e)}{\partial t_1^e}}{1 - s_1} - \alpha U'(y_1^e) = 0 \tag{57}$$

$$Foc(s_1): \quad \frac{\partial U(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1 - s_1}\right) = 0 \tag{58}$$

$$t_1^e = \frac{s_1}{kU'(y_1^e)} - \frac{(1-s_1)\alpha}{kU'(y_1^n)} + \frac{\hat{n}(*)}{kU'(y_1^e)} \tag{59}$$

from $Foc(s_1)$ we obtain

$$t_1^e + t_1^n = a + bs_1 \tag{60}$$

$$s_1 = \frac{t_1^e + t_1^n}{b} - \frac{a}{b} \tag{61}$$

Solving (56) we have

$$Foc(t_2^e): U'(y_2^n)\frac{s_2 - \hat{n}(t_1^e, t_2^e) - t_1^e \frac{\partial \hat{n}(t_1^e, t_2^e)}{\partial t_2^e}}{1-s_2} - \beta U'(y_2^e) = 0 \tag{62}$$

$$Foc(s_2): \quad \frac{\partial U(y_2^n)}{\partial y_2^n}\left(\frac{t_2^e - a - bs_2 + t_2^n}{1-s_2}\right) = 0 \tag{63}$$

$$t_2^e = \frac{s_2}{kU'(y_2^e)} - \frac{(1-s_2)\beta}{kU'(y_2^n)} - \frac{\hat{n}(*)}{kU'(y_2^e)} \tag{64}$$

from $Foc(s_2)$ we obtain

$$t_2^e + t_2^n = a + bs_2 \tag{65}$$

$$s_2 = \frac{t_2^e + t_2^n}{b} - \frac{a}{b} \tag{66}$$

When $\alpha = \beta$ we have $t_1^e = t_2^e$, then we have a symmetric Nash Equilibrium seen in previous case. When $\alpha > \beta$ we have an ambiguous effect in the redistribution as we can see in the Appendix (D).From one side the Region 1 has a less redistribution of the Region 2 (and consequently has less provision of education from the 60) from the fact that $\alpha > \beta$. From other side the fact that $t_1^e < t_2^e$ gives to Region 1 the possibility to attract educated people from the other region, $\hat{n}(t_1^e, t_2^e) > 0$, then the Region 1 can increase t_1^e as we can see from the (59).

One generation model - Asymmetric population

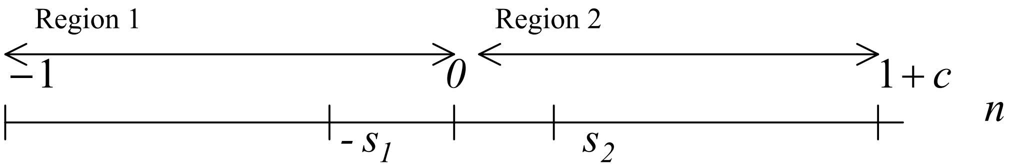
Let's assume that the two regions, indexed $i = 1,2$ have different initial population; the total population of the first region is normalized to unity.

$$N_1^0 = 1 \quad N_2^0 = 1 + c$$

The mobility specifications are the same seen before. $n \in [-1, 1+c]$

For Region 1: $\begin{cases} \text{non educated workers have} & n \in (-1; -s_1) \\ \text{educated workers have} & n \in (-s_1; 0) \end{cases}$

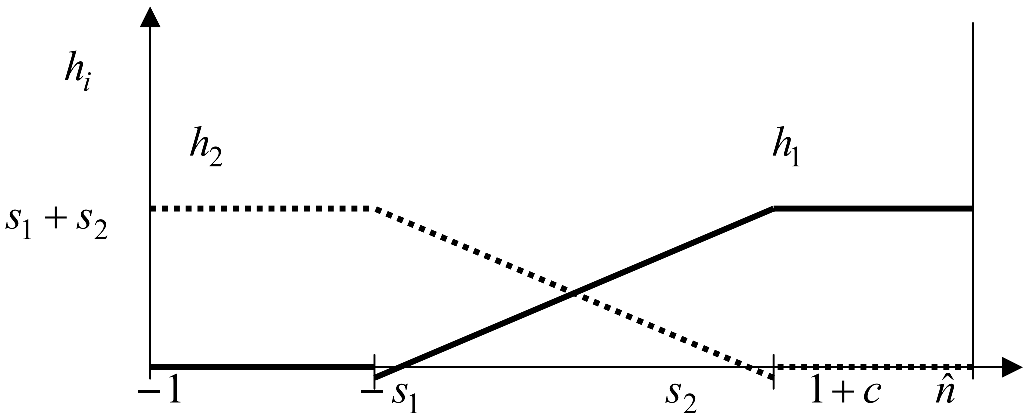
For Region 2: $\begin{cases} \text{non educated worker have} & n \in (s_2; 1+c) \\ \text{educated worker have} & n \in (0; s_2) \end{cases}$



The distribution of the h_i's $(i = 1,2)$ is the following:

$$h_1 = \begin{cases} s_1 + \hat{n} & \text{for} \quad \hat{n} \in (-s_1; s_2) \\ 0 & \text{for} \quad \hat{n} \le -s_1 \\ s_1 + s_2 & \text{for} \quad \hat{n} \ge s_2 \end{cases}$$

$$h_2 = \begin{cases} s_2 - \hat{n} & \text{for} \quad \hat{n} \in (-s_1; s_2) \\ s_1 + s_2 & \text{for} \quad \hat{n} \le -s_1 \\ 0 & \text{for} \quad \hat{n} \ge s_2 \end{cases}$$



Mobility solution

In this case each government chooses its values s_i and t_i^e simultaneously for a given policy of the other region and taking in account the possibility of migration of some educated workers as a response to its choices. For region 1 we have:

$$\underset{s_1, t_1^e}{Max}\ t_1^n = \frac{t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_1)s_1}{1 - s_1} \tag{67}$$

Solving (67) we have

$$Foc(t_1^e): \frac{\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) + t_1^e \frac{\partial \hat{n}(t_1^e, t_2^e)}{\partial t_1^e}}{1 - s_1} = 0 \tag{68}$$

$$Foc(s_1): \quad \frac{t_1^e - a - bs_1}{1 - s_1} + \frac{t_1^e\left(s_1 + \hat{n}(t_1^e, t_2^e)\right) - as_1 - \frac{b}{2}s_1^2}{\left(1 - s_1\right)^2} = 0 \tag{69}$$

From (9), (67) becomes:

$$t_1^e = \frac{s_1 + \hat{n}}{kU'(w^e - t_1^e)} \tag{70}$$

The $Foc(s_1)$ yields:

$$\frac{t_1^e - a - bs_1}{1 - s_1} + \frac{t_1^n}{1 - s_1} = 0 \tag{71}$$

or $t_1^e + t_1^n = a + bs_1$ (72)

or $s_1 = \frac{t_1^e + t_1^n}{b} - \frac{a}{b}$ (73)

For region 2 we have:

$$\underset{s_2, t_2^e}{Max}\ t_2^n = \frac{t_2^e\left(s_2 - \hat{n}(t_1^e, t_2^e)\right) - (a + \frac{b}{2}s_2)s_2}{1 + c - s_2} \tag{74}$$

Solving (74) we have

$$Foc(t_2^e): \frac{\left(s_1 - \hat{n}(t_1^e, t_2^e)\right) - t_2^e \frac{\partial \hat{n}(t_1^e, t_2^e)}{\partial t_1^e}}{1 + c - s_2} = 0 \tag{75}$$

$$Foc(s_2): \quad \frac{t_2^e - a - bs_2}{1 + c - s_2} + \frac{t_2^e\left(s_2 - \hat{n}(t_1^e, t_2^e)\right) - as_2 - \frac{b}{2}s_2^2}{\left(1 + c - s_2\right)^2} = 0 \tag{76}$$

From (9), (75) becomes:

$$t_2^e = \frac{s_2 - \hat{n}}{kU'(w^e - t_2^e)} \tag{77}$$

The $Foc(s_2)$ yields:

$$\frac{t_2^e - a - bs_2}{1 + c - s_2} + \frac{t_2^n}{1 + c - s_2} = 0 \tag{78}$$

or $\quad t_2^e + t_2^n = a + bs_2 \quad (79)$

or $\quad s_2 = \frac{t_2^e + t_2^n}{b} - \frac{a}{b} \quad (80)$

Two cases can be distinguished for different values of k. The higher the value of k the stronger the fiscal competition between the two regions. We have for both regions the same results obtained in the symmetric case.

Parametric Solutions

To solve the asymmetric case we must have a parametric solution. Assume a quadratic utility function

$$U(w^e - t_1^e) = (w^e - t_1^e)^2$$

Then (70), (72), (77) and (79) become

$$t_1^e = \frac{s_1 + k(w^e - t_1^e)^2 - k(w^e - t_2^e)^2}{2k(w^e - t_1^e)} \tag{81}$$

$$t_1^e + \frac{t_1^e\left[s_1 + k(w^e - t_1^e)^2 - k(w^e - t_2^e)^2\right] - (a + \frac{b}{2}s_1)s_1}{1 - s_1} = a + bs_1 \tag{82}$$

$$t_2^e = \frac{s_2 - k(w^e - t_1^e)^2 + k(w^e - t_2^e)^2}{2k(w^e - t_2^e)} \tag{83}$$

$$t_2^e + \frac{t_2^e\left[s_2 - k(w^e - t_1^e)^2 + k(w^e - t_2^e)^2\right] - (a + \frac{b}{2}s_2)s_2}{1 + c - s_2} = a + bs_2 \tag{84}$$

Solving these four equations we obtain the following best responses[11]:

$$-39x^3 + 884x^2 + (59 + 59y^2 + 420y)x + y^4 + 398y^2 + 40y - 1 = 0$$

[11] For simplicity we assume these value for the parameters:
$b = 2; a = 1; w^e = 10; k = 1$

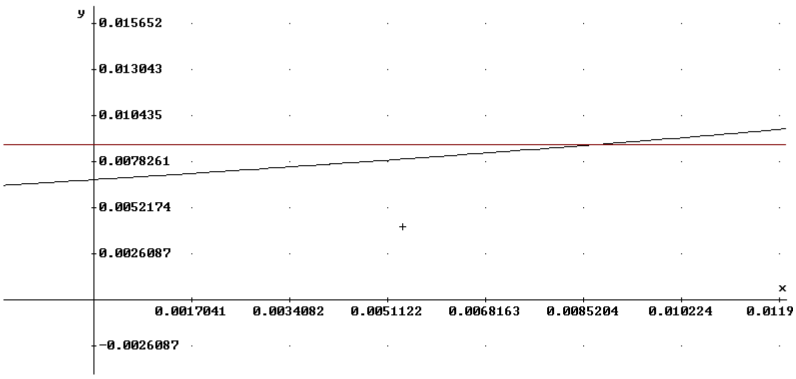
$$-39y^3+(884+2c)y^2+(59-29c+59x^2+420x)y+x^4+(398-c)x^2+$$
$$+(40+20c)x-1=0$$

Where $x=t_1^e$
$y=t_2^e$

In the following pictures we simulate how the value of x and y change when:

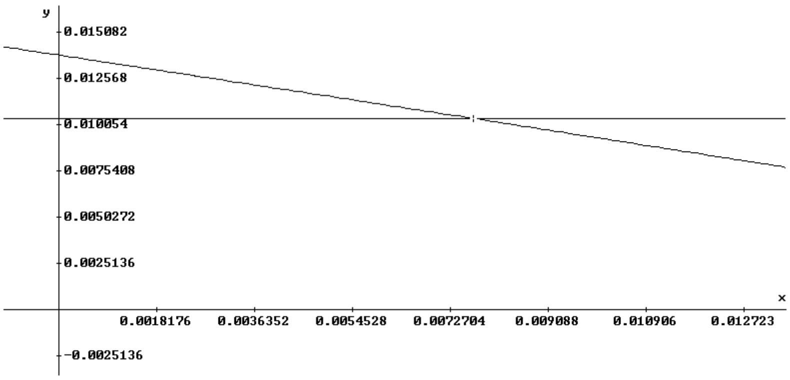
1) The two regions have the same population (c=0)
2) The region two has population equal to 1.5 (c=0.5)
3) The region two has twice population of region one (c=1)

1) The two regions have the same population (c=0)



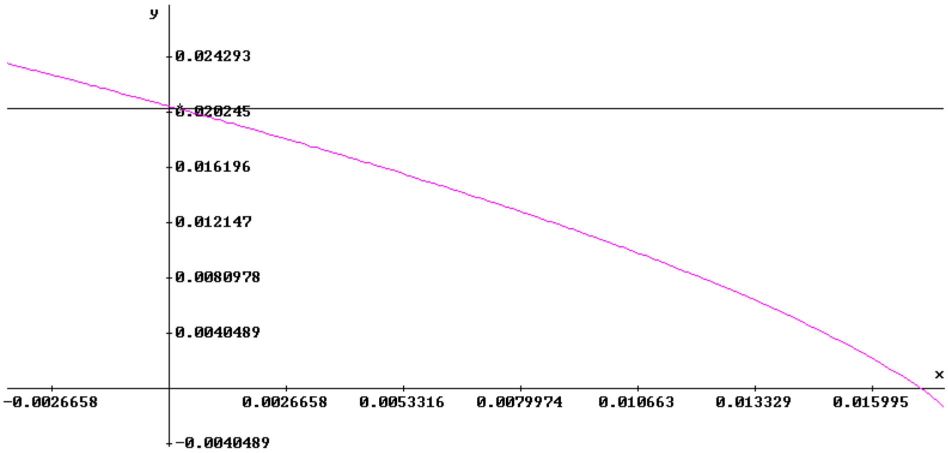
$$x=t_1^e=0.0087\ \ y=t_2^e=0.0087\ \ \hat{n}=0\quad s_1=s_2=0.17384\quad h_1=h_2=0.17384$$

2) The region two has population equal to 1.5 (c=0.5)



$$x=t_1^e=0.00769\ y=t_2^e=0.01038\ \ \hat{n}=0.05375$$
$$s_1=0.09993\ s_2=0.26113\ \ h_1=0.15368\ \ h_2=0.20738$$

3) The region two has twice population of region one (c=1)



$x = t_1^e = 0.00024$ $y = t_2^e = 0.02059$ $\hat{n} = 0.4059$ $s_1 = -0.4010$ $s_2 = 0.81685$

$h_1 = 0.4059$ $h_2 = 0.41095$

These results, according with the economic intuition, show that when we have asymmetric population the region with less population has less taxation of the other region. These results are justified because for the bigger region is convenient lose same educated worker and have greater income to redistribute (the impact of the fiscal competition of the smaller region it's non stronger enough to justify low taxes). The region 1 has a provision of education negative (it's impossible so we can assume equal to zero) because it's convenient attract educated worker from region two and have no educated worker formed at home.

Overlapping generation model

In this paragraph assume that exist two different generations and that is verified the entire hypothesis seen in the initial assumptions. The main difference is that the two generations have different propensity to migrate, then I study what is the impact of this more realistic assumption on the results obtained in the previous specifications of the model. In the previous paragraphs (with only one generation) I've assumed that the regional authority maximizes the utility of his citizens (Maximin case and subjective maximization) by the use of lump sum taxes (t_i^e) to educated workers and lump sum subsidies (t_i^n) to the non-educated workers. In this OLG version I assume that the regional authority tries to capture in each time, by the use of lump sum taxes (t_i^e) to educated workers, the gain dues to the education provided in each time. Furthermore we assume that, for each generation we have $w^e - t^e \geq w^n$, otherwise it's not convenient be educated. Then exist a value maximum of t_i^e and we assume that it's known. $t_i^e \in (0, t^{\max})$

Autarky solutions

We consider then the autarky case in which we have no mobility of educated workers. In this case the policies of the other governments don't have any effect on the decision of a government. This is equivalent to having a Central Authority that coordinates the politics of the two regions, so there aren't negative externalities from fiscal competition. Then this case can be used as a benchmark to which we can compare the results obtained in the other cases. In this case there isn't migration so $h_{it} + h_{it-1} = s_{it} + s_{it-1}$ is the total amount of educated workers. The regional authority tries to capture in each time, by the use of lump sum taxes (t_i^e) to educated workers, the gain dues to the education provided in each time. Therefore each government chooses s and t^e. We know that exist a value maximum of t_i^e and we assume that it's known $t_i^e \in (0, t^{\max})$. Then, from the fact that in autarkic case the educated workers can't emigrate, the optimal value of the taxation is the maximum.

Where $t^{\max} = w^e - w^n$ (85)

$$\underset{s_{t-1};s_t}{Max} : R_t \tag{86}$$

with

$$R_t = [s_{t-1} + s_{t-1}]t^{\max} - s_{t-1}(a + \frac{b}{2}s_{t-1}) - s_t(a + \frac{b}{2}s_t)$$

$$FOC(s_t): \quad t^{\max} - a - bs_t = 0 \tag{87}$$

$$FOC(s_{t-1}): \quad t^{\max} - a - bs_{t-1} = 0 \tag{88}$$

Then the solution is:

$$s_t = s_{t-1} = s \tag{89}$$

$$s = \frac{t^{\max} - a}{b} \tag{90}$$

From the (85) the solution is:

$$s = \frac{(w^e - w^n) - a}{b} \tag{91}$$

with $s \in (0,1)$

According to economic intuition, an increase in the education costs through either a or b implies a decrease in the optimal provision of education places, while an increase in the difference of productivities makes s rise.[12]

Mobility solutions

We consider the case in which only educated people (young and old age) can migrate. In this case the policies of the other region have effects on the decisions of a region, so each government must take in account what the other does.

[12]It's important note that the equations (90) corresponding to the same solution obtained in the one generation model.

In this case, in its maximization problem each government must take in account the possibility that there isn't a correspondence between the education provided and the educated people resident in their region.

Variable $\hat{n}_t$, which characterizes the effect of these policies on the decision to migrate, defines the educated worker who is indifferent between remaining in his region or migrating in the other region.

$$\hat{n}_t = k_t \left[U(w^e - t^e_{1,t}) - U(w^e - t^e_{2,t}) \right] \tag{92}$$

where $\hat{n}_t$ changes in response to difference in the taxations in the two regions.

According to economic intuition, we have: $\hat{n}_t\left(\underset{-}{t^e_{1t}}, \underset{+}{t^e_{2t}}\right)$

$$\text{since} \quad \frac{\partial \hat{n}_t}{\partial t^e_{1t}} = -k_t U'(w^e - t^e_{1t}) < 0 \tag{93}$$

$$\text{and} \quad \frac{\partial \hat{n}_t}{\partial t^e_{2t}} = k_t U'(w^e - t^e_{2t}) > 0 \tag{94}$$

Contrary to autarky it is now possible that fiscal competition between the two regions makes the lump-sum tax for educated worker lower then its maximum value.

The mobility case can be analyzed in two different cases:

1) ($k_t = k_{t-1} = k \quad \forall t$) **One generation model**.

2) ($k_t \neq k_{t-1} \quad \forall t$) **Two generation model**.

each generation of workers has different propensity to migrate, then we can have two situation:

2.1) ($t^e_{i1,t-1} = t^e_{i,t} = t^e_i \quad \forall i = 1,2$) **Equal taxation between generations**.

2.2) ($t^e_{i1,t-1} \neq t^e_{i,t} \quad \forall i = 1,2$) **Different taxation between generations**.

1) One generation model.

It's possible demonstrate that the analysis of this case it's a particular case of the model studied in the first part of this paper were each generation of workers has the same propensity to migrate and there isn't no other differences between the generation of workers. $\hat{n} = k\left[U(w^e - t^e_1) - U(w^e - t^e_2) \right]$

In this version of the model we have $t^n_1 = 0$ (we don't care of the redistribution problem) but all the analysis are the same seen in the one-generation model.

2.1) Equal taxation between generations.

$\hat{n}_{t-1} = k_{t-1}\left[U(w^e - t^e_1) - U(w^e - t^e_2) \right]$ for the old generation.

$$\hat{n}_t = k_t \left[U(w^e - t_1^e) - U(w^e - t_2^e) \right]$$ for the young generation.

In this case each government chooses its values s_i and t_i^e simultaneously for a given policy of the other region and taking in account the possibility of migration of some educated workers as a response to its choices. For region 1 we have:

$$\underset{s_{1,t-1};s_{1,t};t_1^e}{Max} : R_t \tag{95}$$

$$R_t = \left[\left(s_{1,t-1} + \hat{n}_{t-1} \right) + \left(s_{1,t} + \hat{n}_t \right) \right] t_1^e - s_{1,t-1}(a + \frac{b}{2} s_{1,t-1}) - s_{1,t}(a + \frac{b}{2} s_{1,t})$$

Solving (95) we have

$$FOC(t_1^e) : \left(s_{1,t-1} + \hat{n}_{t-1} \right) + \left(s_{1,t} + \hat{n}_t \right) + \left(\frac{\partial \hat{n}_{t-1}}{\partial t_1^e} + \frac{\partial \hat{n}_t}{\partial t_1^e} \right) t_1^e = 0 \tag{96}$$

$$FOC(s_{1,t}) : \quad t_1^e - a - b s_{1,t} = 0 \tag{97}$$

$$FOC(s_{1,t-1}) : \ t_1^e - a - b s_{1,t-1} = 0 \tag{97'}$$

From (96) and (97) we obtain:

$$s_{1,t} = s_{1,t-1} = s_1 \tag{98}$$

where $s_1 = \dfrac{t_1 - a}{b}$ (99)

From (92), (93) and (96) we have:

$$2s_1 + \hat{n}_t + \hat{n}_{t-1} - (k_{t-1} + k_t) \left[U'(w^e - t_1^e) \right] t_1^e = 0 \tag{100}$$

Therefore, in the symmetric Nash equilibrium in which $\hat{n}_t = \hat{n}_{t-1} = 0$, we obtain

$$t_1^e = \frac{s_1}{\frac{1}{2}(k_{t-1} + k_t) U'(w^e - t_1^e)} \tag{101}$$

If we consider that

$$\bar{k} = \tfrac{1}{2}(k_{t-1} + k_t)$$

We can compare these results with the solution obtained in the one-generation model. When we introduce different generations and we assume equal taxation the government must apply a level of taxation that take in account an average of the propensity to migrate of each generation.[13]

[13] As in the one generation model I have demonstrate that by introducing the mobility of educated workers and solving the symmetric Nash Equilibrium, two cases can be distinguished for different values of k. The higher the value of k the stronger the fiscal competition between the two regions.
Summing up, there is a critical value k_c such that for $k < k_c$ the autarky solution prevails at the Nash equilibrium while for $k \geq k_c$, redistribution and education provision are lower.

2.2) Different taxation between generations.

$$\hat{n}_{t-1} = k_{t-1}\left[U(w^e - t^e_{1,t-1}) - U(w^e - t^e_{2,t-1})\right]$$ for the old generation.

$$\hat{n}_t = k_t\left[U(w^e - t^e_{1,t}) - U(w^e - t^e_{2,t})\right]$$ for the young generation.

In this case each government chooses its values s_i and t_i^e simultaneously for a given policy of the other region and taking in account the possibility of migration of some educated workers as a response to its choices. For region 1 we have:

$$\underset{\substack{s_{1,t-1};s_{1,t} \\ t^e_{1,t-1};t^e_{1,t}}}{Max} : R_t \tag{102}$$

$$R_t = \left(s_{1,t-1} + \hat{n}_{t-1}\right)t^e_{1,t-1} + \left(s_{1,t} + \hat{n}_t\right)t^e_{1,t} - s_{1,t-1}(a + \frac{b}{2}s_{1,t-1}) - s_{1,t}(a + \frac{b}{2}s_{1,t})$$

Solving (102) we have

$$FOC(t^e_{1,t-1}): \qquad \left(s_{1,t-1} + \hat{n}_{t-1}\right) + \frac{\partial \hat{n}_{t-1}}{\partial t^e_{1,t-1}} t^e_{1.t-1} = 0 \tag{103}$$

$$FOC(t^e_{1,t}): \qquad \left(s_{1,t} + \hat{n}_t\right) + \frac{\partial \hat{n}_t}{\partial t^e_{1,t}} t^e_{1.t} = 0 \tag{104}$$

$$FOC(s_{1,t}): \qquad t^e_{1,t} - a - bs_{1,t} = 0 \tag{105}$$

$$FOC(s_{1,t-1}): \quad t^e_{1,t-1} - a - bs_{1,t-1} = 0 \tag{106}$$

From (105) and (106) we obtain:

$$s_{1,t} = \frac{t^e_{1,t} - a}{b} \tag{107}$$

$$s_{1,t-1} = \frac{t^e_{1,t-1} - a}{b} \tag{108}$$

From (92), (93), (103) and (104) we obtain

$$s_{1,t} + \hat{n}_t - k_t U'(w^e - t^e_{1,t})t^e_{1,t} = 0 \tag{109}$$

$$s_{1,t-1} + \hat{n}_{t-1} - k_{t-1}U'(w^e - t^e_{1,t-1})t^e_{1,t-1} = 0 \tag{110}$$

Therefore, in the symmetric Nash equilibrium in which $\hat{n}_t = \hat{n}_{t-1} = 0$, we obtain

$$t^e_{1,t} = \frac{s_{1,t}}{k_t U'(w^e - t^e_{1,t})} \tag{111}$$

$$t_{1,t-1}^{e} = \frac{s_{1,t-1}}{k_{t-1}U'(w^{e} - t_{1,t-1}^{e})} \tag{112}$$

We can compare these results with the solution obtained in the one-generation model.

When we introduce different generations and we assume equal taxation the government must apply a level of taxation that take in account the propensity to migrate of each generation. Then if the government known the exact value of this propensity and can combine for each generation his specific tax we obtain the result seen in the note (14).

Dynamic of the model

In this simple model we assume the population constant and equal in the two regions, then the dynamic are due to the change in the propensity to migrate of the different generations.

We can analyse two different cases:

Case A

If $k_t = k_{t-1} = k \quad \forall \mathrm{t}$ (so we are in the first case analyzed before) then we are in one-generation model, which are static.

Case B

If $k_t \neq k_{t-1} \quad \forall \mathrm{t}$, and we can be sure that for each generation there is a constant trend in the change of the value of the propensity to migrate, then we can study the dynamic of the Nash Equilibria that we have found. The symmetric Nash Equilibria in the "different taxation" case are:

$$t_{1,t}^{e} = \frac{s_{1,t}}{k_{t}U'(w^{e} - t_{1,t}^{e})} \tag{111}$$

$$t_{1,t-1}^{e} = \frac{s_{1,t-1}}{k_{t-1}U'(w^{e} - t_{1,t-1}^{e})} \tag{112}$$

and in the previous analysis I've found that

$$\frac{\partial t_{1,t}^{e}}{\partial s_{1,t}} > 0 \tag{113}$$

From (111) $\quad \dfrac{\partial t_{1,t}^{e}}{\partial k_{1,t}} < 0$ (114)

From the (111) and the (113) it' possible to write

$$s_{1,t} = \Phi\left(t_{1,t}^{e}\right) \text{ with } \Phi'\left(t_{1,t}^{e}\right) > 0 \tag{115}$$

Then $\dfrac{dt_{1,t}^{e}}{dk_{1,t}} = \dfrac{\partial t_{1,t}^{e}}{\partial k_{1,t}} + \dfrac{\partial t_{1,t}^{e}}{\partial s_{1,t}} * \dfrac{\partial t_{1,t}^{e}}{\partial k_{1,t}}$ (116)

Two different cases can be distinguish

Case B,1: $k_t > k_{t-1} \quad \forall \mathrm{t}$ *and* $t_{i1,t-1}^{e} \neq t_{i,t}^{e}$

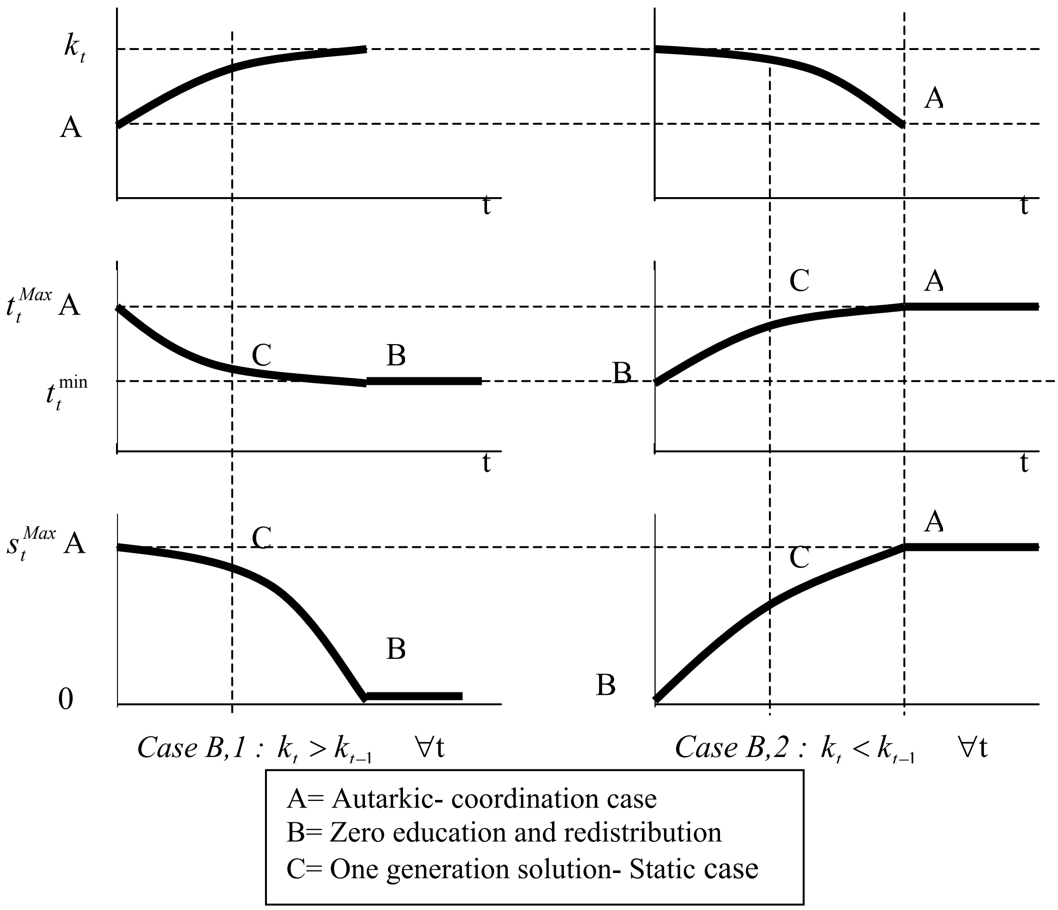
The new generations are more mobile of the old ones

In this case the (116) becomes $\dfrac{dt_{1,t}^{e}}{dk_{1,t}} < 0$

Case B,2: $k_t < k_{t-1} \quad \forall \mathrm{t}$ *and* $t_{i1,t-1}^{e} \neq t_{i,t}^{e}$

The new generations are more mobile of the old ones

In this case the (116) becomes $\dfrac{dt_{1,t}^{e}}{dk_{1,t}} > 0$



Dynamic of s and steady state

I study the dynamic of s in the case B,1 and B,2 seen before

Case B,1: $k_t > k_{t-1} \quad \forall t$

$$\frac{dt_{1,t}^e}{dk_{1,t}} < 0$$

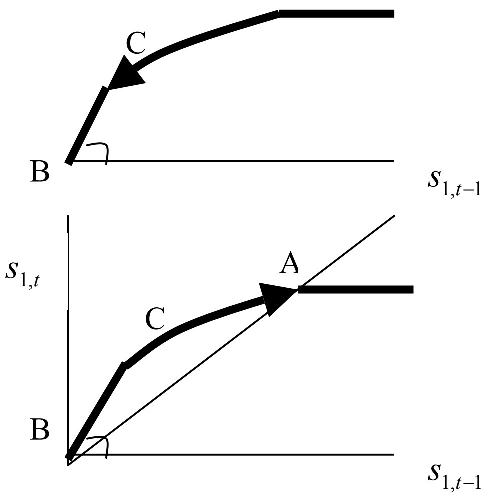
$$\frac{ds_{1,t}}{dk_{1,t}} = \frac{\partial t_{1,t}^e}{\partial s_{1,t}} * \frac{\partial t_{1,t}^e}{\partial k_{1,t}} < 0$$

Then the unique Steady State is the Zero education case (point B in the picture)

Case B,2: $k_t < k_{t-1} \quad \forall t$

$$\frac{dt_{1,t}^e}{dk_{1,t}} > 0$$

$$\frac{ds_{1,t}}{dk_{1,t}} = \frac{\partial t_{1,t}^e}{\partial s_{1,t}} * \frac{\partial t_{1,t}^e}{\partial k_{1,t}} > 0$$



Case B,1 :
$k_t > k_{t-1} \quad \forall t$

Case B,2 :
$k_t < k_{t-1} \quad \forall t$

A= Autarkic- coordination case
B= Zero education and redistribution
C= One generation solution- Static

4. Concluding remarks

$\alpha = \beta = 0$		
Autarkic Solutions	Mobility Solutions (one step)	Mobility Solutions (two steps)
$t_1^e = w^e - w^n - t_1^n$ $t_1^e = t_2^e$	$t_1^e = \dfrac{s_1}{kU'(w^e - t_1^e)}$ (18) $t_1^e = t_2^e$	$t_1^e = \dfrac{s_1}{kU'(w^e - t_1^e)}$ (24') $t_1^e = t_2^e$
$s_1 = \dfrac{t_1^e + t_1^n}{b} - \dfrac{a}{b}$ (7) $s_1 = s_2$	$s_1 = \dfrac{t_1^e + t_1^n}{b} - \dfrac{a}{b}$ (20) $s_1 = s_2$	$s_1 = \dfrac{t_1^e(1+X) + t_1^n}{b} - \dfrac{a}{b}$ (36) $s_1 = s_2$ $X = Y > 0$

$\alpha = \beta > 0$	
Autarkic Solutions	Mobility Solutions (one step)
$t_1^e > w^e - w^n - t_1^n$ $t_1^e = t_2^e$	$t_1^e = \dfrac{s_1}{kU'(y_1^e)} - \dfrac{(1-s_1)\alpha}{kU'(y_1^n)}$ (51) $t_1^e = t_2^e$
$s_1 = \dfrac{t_1^e + t_1^n}{b} - \dfrac{a}{b}$ (46) $s_1 = s_2$	$s_1 = \dfrac{t_1^e + t_1^n}{b} - \dfrac{a}{b}$ (53) $s_1 = s_2$

$\alpha \neq \beta$	
Autarkic Solutions	Mobility Solutions (one step)
$t_1^e > w^e - w^n - t_1^n$ $t_1^e \neq t_2^e$	$t_1^e = \dfrac{s_1}{kU'(y_1^e)} - \dfrac{(1-s_1)\alpha}{kU'(y_1^n)} + \dfrac{n(*)}{kU'(y_1^e)}$ (59) $t_2^e = \dfrac{s_2}{kU'(y_2^e)} - \dfrac{(1-s_2)\beta}{kU'(y_2^n)} - \dfrac{\hat{n}(*)}{U'(y_2^e)}$ (64)
$s_1 = \dfrac{t_1^e + t_1^n}{b} - \dfrac{a}{b}$ (46) $s_1 \neq s_2$	$s_1 = \dfrac{t_1^e + t_1^n}{b} - \dfrac{a}{b}$ (61) $s_2 = \dfrac{t_2^e + t_2^n}{b} - \dfrac{a}{b}$ (66)

In this schedules are summarized the results obtained in the previous chapters.
The results obtained are in agreement to the literature: less redistribution and less provision of public good with respect to the efficient value (which could be obtained in the absence of mobility or in the presence of coordination among jurisdictions). In the first schedule I reassume the solutions obtained when $\alpha = \beta = 0$, the Maximin case, and what happened if we consider the two steps ore the one step case[14]. In the second schedule, I reassume the solutions obtained when $\alpha = \beta > 0$ and According to the literature when we increase the weight of the utility of educated in the maximization of the government, we have a negative effect in the redistribution and in the provision of education. Furthermore, more relevant is the fact that when $\alpha \neq \beta$, third schedule, we introduce a new tool of competition that the Jurisdiction can use and we obtain a greater inefficiency in redistribution and in provision of education. In the asymmetric case I have analysed the possibility to have different initial population. In this case it's not possible found a symmetric Nash Equilibrium than I've simulated the different equilibria obtained by different parametric specifications (assuming the population of region 1 the 50% more then the region 2, with the same population and with twice population) and I've compared the three cases. According to the economic intuitions I've found that when increase the difference among population the region with less population has less taxation respect the other because the role of the Fiscal Competition of the smaller region it's no stronger enough to justify low taxes for the bigger. In the O.L.G. specification I've analysed the role of the parameter k (propensity to migrate). Using this specification I can introduce the analysis of the model in a dynamic context. The results are that if the new generation are more mobile, then the fiscal competition increase and increase the gain that we can obtain with coordination among jurisdictions; otherwise, if the new generations are less mobile the necessity of coordination are no stronger and we come back (for lower values of k) in the autarkic case independently the level of coordination among regions.
Resuming, in the previous paragraphs are analysed different specifications of the model that emphasise a particular aspect of the fiscal competition: when we add up the two negative effects due to the absence of coordination among jurisdictions, the loss of efficiency is more accentuate. Then we can conclude that the Jurisdiction can compete to the others by use different levels of taxation, different level of provision of public goods (especially education), different subjective utility functions or with combination of these three tools. In the model we have found different results, in some case these results can be compared and in others it's impossible to do, but in all case the conclusion is the same: a more realistic analysis of the fiscal competition can be done if and only if we try to analyse all the different tools that each jurisdiction use to compete with the others. Otherwise, if we focus our analysis in only one tool, we could sub-estimate the inefficiency due to the mobility and propose solutions that are inadequate to resolve these problems.

[14] First justification at this hypothesis is that it is more realistic assumption. Second possibility is that a central authority can decide to coordinate the jurisdictions only respect the taxation to educated workers (more simple to check) and giving the possibility to each jurisdiction to choose independently the level of provision of education.

Appendix A

Using the theorem of implicit functions we differentiate the system given by (24) and (25) and we obtain:

$$\begin{aligned} A^1 dt_1^e + B^1 dt_2^e + ds_1 &= 0 \\ A^2 dt_1^e + B^2 dt_2^e + ds_2 &= 0 \end{aligned} \tag{A1}$$

Solutions of the (A1), by using the Cramer Rule are:

$$\begin{aligned} \frac{dt_1^e}{ds_1} &= \frac{-B^2}{A^1B^2 - A^2B^1} \quad \frac{dt_1^e}{ds_2} = \frac{A^2}{A^1B^2 - A^2B^1} \\ \frac{dt_2^e}{ds_1} &= \frac{B^1}{A^1B^2 - A^2B^1} \quad \frac{dt_2^e}{ds_2} = \frac{-A^1}{A^1B^2 - A^2B^1} \end{aligned} \tag{A2}$$

Where:

$$\begin{aligned} A^1 &= -kU'(y_1^e) + kt_1^e U''(y_1^e) - kU'(y_1^e) = SOC(t_1^e) < 0 \\ A^2 &= kU'(y_2^e) > 0 \\ B^1 &= kU'(y_1^e) > 0 \\ B^2 &= -kU'(y_2^e) + kt_2^e U''(y_2^e) - kU'(y_2^e) = SOC(t_{21}^e) < 0 \end{aligned}$$

From $A^1B^2 > A^2B^1$ the A(2) becomes

$$\begin{aligned} \frac{dt_1^e}{ds_1} > 0 \quad &, \quad \frac{dt_1^e}{ds_2} > 0 \\ \frac{dt_2^e}{ds_1} > 0 \quad &, \quad \frac{dt_2^e}{ds_2} > 0 \end{aligned} \tag{A4}$$

Furthermore, we have

$$\hat{n}\left(t_1^e(s_1, s_2), t_2^e(s_1, s_2)\right) \tag{A5}$$

$$\frac{\partial \hat{n}}{\partial s_1} = \frac{\partial \hat{n}}{\partial t_1^e}\frac{\partial t_1^e}{\partial s_1} + \frac{\partial \hat{n}}{\partial t_2^e}\frac{\partial t_2^e}{\partial s_1} = \frac{kU'(y_2^e)\left[B^1\right] + kU'(y_1^e)\left[B^2\right]}{A^1B^2 - A^2B^1} \tag{A6}$$

$$\frac{\partial \hat{n}}{\partial s_2} = \frac{\partial \hat{n}}{\partial t_1^e}\frac{\partial t_1^e}{\partial s_2} + \frac{\partial \hat{n}}{\partial t_2^e}\frac{\partial t_2^e}{\partial s_2} = \frac{-kU'(y_2^e)\left[A^1\right] - kU'(y_1^e)\left[A^2\right]}{A^1B^2 - A^2B^1} \tag{A7}$$

or equivalently

$$\frac{\partial \hat{n}}{\partial s_1} = \frac{B^1\left[B^2 + A^2\right]}{A^1B^2 - A^2B^1} < 0 \tag{A8}$$

$$\frac{\partial \hat{n}}{\partial s_2} = \frac{-A^2\left[A^1 + B^1\right]}{A^1B^2 - A^2B^1} > 0 \tag{A9}$$

Appendix B

Using the theorem of implicit functions we differentiate the system given by (38) and (41) and we obtain:

$$Adt_1^e + Bds_1 + Cd\alpha = 0$$
$$Ddt_1^e + Eds_1 = 0 \qquad \text{(B1)}$$

Where

$$A = \frac{\partial U(y_1^n)}{\partial y_1^n}\left(\frac{s_1}{1-s_1}\right)^2 - \alpha U''(y_1^e) = S.O.C.(t_1^e) > 0$$

$$B = U'(y_1^n)\left[\frac{1}{1-s_1} + \frac{s_1}{(1-s_1)^2}\right] + \left(\frac{s_1}{1-s_1}\right)\frac{\partial U'(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1-s_1}\right)$$

$$C = -U'(y_1^e) < 0$$

$$D = 1 + \frac{1}{1-s_1} > 0$$

$$E = -b + \frac{t_1^e - a - bs_1 + t_1^n}{1-s_1}$$

From $Foc(s_1)$ the second side of B and E are equal to zero then:

$$B = U'(y_1^n)\left[\frac{1}{(1-s_1)^2}\right] > 0$$
$$E = -b < 0$$

The (B1) becomes

$$\frac{dt_1^e}{d\alpha} = \frac{CE}{AE - BD}$$
$$\frac{ds_1}{d\alpha} = \frac{-CD}{AE - BD} \qquad \text{(B2)}$$

Then

$$\frac{dt_1^e}{d\alpha} < 0$$
$$\frac{ds_1}{d\alpha} < 0 \qquad \text{(B3)}$$

Appendix C

Using the theorem of implicit functions we differentiate the system given by (51) and (52) and we obtain:

$$\begin{aligned} & A^1 dt_1^e + B^1 ds_1 + C^1 d\alpha = 0 \\ & D dt_1^e + E ds_1 = 0 \end{aligned} \tag{C1}$$

Where

$$A^1 = -U''(y_1^n)\left(\frac{s_1}{1-s_1}\right)kU'(y_1^e) + U'(y_1^n)kU''(y_1^e) - \alpha U''(y_1^e) +$$

$$- s_1 U''(y_1^n)\left(\frac{s_1}{1-s_1}\right) = S.O.C.(t_1^e) > 0$$

$$B^1 = -t_1^e U'(y_1^e)\frac{\partial U'(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1-s_1}\right) +$$

$$+ \alpha U'(y_1^e) - U'(y_1^n) - s_1 \frac{\partial U'(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1-s_1}\right)$$

$$C^1 = -(1-s_1)U'(y_1^e) < 0$$

$$D = 1 + \frac{1}{1-s_1} > 0$$

$$E = -b + \frac{t_1^e - a - bs_1 + t_1^n}{1-s_1}$$

Using the $Foc(s_1)$ B^1 and E can be rewrite:

$$B^1 = \alpha U'(y_1^e) - U'(y_1^n)$$

$$E = -b$$

The (C1) becomes

$$\frac{dt_1^e}{d\alpha} = \frac{C^1 E}{A^1 E - B^1 D} \qquad \frac{ds_1}{d\alpha} = \frac{-C^1 D}{A^1 E - B^1 D} \tag{C2}$$

$$\frac{dt_1^e}{d\alpha} < 0 \qquad \frac{ds_1}{d\alpha} < 0 \tag{C3}$$

Appendix D

Using the theorem of implicit functions we differentiate the system given by (59), (60), (64) and (65) and we obtain:

$$
\begin{aligned}
&A^1 dt_1^e + B^1 ds_1 + C^1 d\alpha + D^1 dt_2^e = 0 \\
&A^2 dt_1^e + B^2 ds_2 + C^2 d\beta + D^2 dt_2^e = 0 \\
&E^1 dt_1^e + F ds_1 + G^1 dt_2^e = 0 \\
&E^2 dt_1^e + F ds_2 + G^2 dt_2^e = 0
\end{aligned}
\tag{D1}
$$

Where:

$$
\begin{aligned}
A^1 &= -U'(y_1^n)kU'(y_1^e) - t_1^e U''(y_1^n)\left(\frac{t_1^e - a - bs_1 + t_1^n}{\frac{1}{2} - s_1}\right)kU'(y_1^e) + t_1^e U''(y_1^n)kU'(y_1^e) + \\
&+ \hat{n}(*)U''(y_1^n)\left(\frac{t_1^e - a - bs_1 + t_1^n}{1 - s_1}\right) - U'(y_1^n)kU'(y_1^e) + \\
&+ (1 - s_1)\alpha U''(y_1^e) + s_1 U''(y_1^n)\left(\frac{t_1^e - a - bs_1 + t_1^n}{1 - s_1}\right) < 0
\end{aligned}
$$

From the $Foc(s_1)$, becomes

$$A^1 = -U'(y_1^n)kU'(y_1^e) + t_1^e U''(y_1^n)kU'(y_1^e) + (1 - s_1)\alpha U''(y_1^e) > 0$$

$$A^2 = kU'(y_1^e)\left[\frac{(1 - s_2)\beta U'(y_2^e) + \left(U'(y_2^n)\right)^2}{U'(y_2^n)}\right] > 0$$

$$
\begin{aligned}
B^1 &= -t_1^e kU'(y_1^e)\frac{\partial U'(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1 - s_1}\right) + \alpha U'(y_1^e) + \\
&+ \left(\hat{n}(*) + s_1\right)\frac{\partial U'(y_1^n)}{\partial y_1^n}\left(\frac{t_1^e - a - bs_1 + t_1^n}{1 - s_1}\right) + U'(y_1^n)
\end{aligned}
$$

From the $Foc(s_1)$, becomes

$$B^1 = \alpha U'(y_1^e) + U'(y_1^n) > 0 \qquad B^2 = \beta U'(y_2^e) + U'(y_2^n) > 0$$

$$C^1 = -(1 - s_1)U'(y_1^e) < 0 \qquad C^2 = -(1 - s_2)U'(y_2^e) < 0$$

$$D^1 = t_2^e\left(-kU'(y_2^e)\right)kU'(y_1^e)\frac{\partial U'(y_1^n)}{\partial y_1^n} + kU'(y_2^e)U'(y_2^n) + \hat{n}(*)\frac{\partial U'(y_1^n)}{\partial y_1^n}kU'(y_2^e)$$

from $Foc(t_1^e)$, becomes

$$D^1 = kU'(y_2^e)\left[\frac{(1 - s_1)\alpha U'(y_1^e) + \left(U'(y_1^n)\right)^2}{U'(y_1^n)}\right] > 0$$

$$D^2 = -U'(y_2^n)kU'(y_2^e) + t_2^e U''(y_2^n)kU'(y_2^e) + (1-s_2)\beta U''(y_2^e) > 0$$

$$E^1 = 1 + \left(\frac{\hat{n}(*) + s_1 + t_1^e \frac{\partial \hat{n}(*)}{\partial t_1^e}}{1-s_1} \right)$$

$$E^1 = 1 + \frac{1}{1-s_1} \left(\hat{n}(*) + s_1 - t_1^e \frac{\partial U'(y_1^e)}{2k} \right)$$

from $Foc(t_1^e)$, becomes

$$E^1 = 1 + \alpha \frac{U'(y_1^e)}{U'(y_1^n)} > 0 \quad E^2 = 1 + \beta \frac{U'(y_2^e)}{U'(y_2^n)} > 0 \quad F = \left(\frac{t_1^e + t_1^n}{1-s_1} \right) - b$$

From the $Foc(s_1)$, becomes

$$F = a \qquad G^1 = t_1^e kU'(y_2^e) > 0 \qquad G^2 = t_2^e kU'(y_1^e) > 0$$

Solving the (D1) we obtain, using the Cramer rule

$$\frac{dt_1^e}{d\alpha} = \frac{-C^1 F\left[(B^2 G^2) + (D^2 G^1)\right]}{\nabla}$$

$$\frac{dt_1^e}{d\beta} = \frac{-C^2\left[G^1(B^1 G^2) - FD^1)\right]}{\nabla}$$

$$\frac{dt_2^e}{d\alpha} = \frac{C^1 F\left[(A^2 G^2 - E^2 D^2) + (D^2 E^1)\right]}{\nabla}$$

$$\frac{dt_2^e}{d\beta} = \frac{-C^2\left[G^2(A^1 F - B^1 E^1) + FG^1(E^1 - E^2)\right]}{\nabla}$$

$$\frac{ds_1}{d\alpha} = \frac{-C^1\left[-E^2(D^2 G^1) + G^2(A^2 G^1 - B^2 E^2)\right]}{\nabla}$$

$$\frac{ds_1}{d\beta} = \frac{C^2\left[G^1(A^1 G^2 - D^1 E^2)\right]}{\nabla}$$

$$\frac{ds_2}{d\alpha} = \frac{C^1 F\left[(A^2 G^1 - B^2 E^1) + (B^2 E^2)\right]}{\nabla}$$

$$\frac{ds_2}{d\beta} = \frac{-C^2\left[G^1(A^1 F - B^1 E^2)\right]}{\nabla}$$

$$\begin{aligned}\nabla = &-B^2\left[D^1 F(E^1 - E^2) + G^2(A^1 F - E^1 B^1)\right] + \\ &+ G^1\left[B^1(A^2 G^2 - E^2 B^2) + F(A^1 D^2 - A^2 D^1)\right]\end{aligned}$$

References

1. *Braunerhjelm, P., R. Faini, V. Norman, F. Ruane and P. Seabright* "Integration and The Regions of Europe: How the right policies can prevent Polarization". Monitoring European Integration 10, Centre for Economic Policy Research, 2000.
2. *Buchanan, J. M.,* "Federalism and Fiscal Equity." European Econ. Review 40 (September 1950): 589-99.
3. *Bucovetsky, S. and J. D. Wilson* "Asymmetric tax competition" Journal of Urban Economics (1991).
4. *Bucovetsky, S. and J. D. Wilson* "Tax Competition with two tax instruments" Regional Science and Urban Economics 21 (1991) 333-350.
5. *Burda, M. and C. Wyplosz* "Human Capital investment and migration in an integrated Europe", European Economic Review, 36, 1992: pp. 677-684.
6. *Cnossen, S.* "The case for tax diversity in the European Community". European Economic Review, 34, 1990, 471-9.
7. *Cremer, H., V. Fourgeaud, M. L. Moriero, M. Marchand and P. Pestieau* "Mobility and Redistribution: a survey", Public Finance. 1996.
8. *Edwards, J. and M. Keen.* "Tax Competition and the Leviathan", European Economic Review, 1996.
9. *Epple, D. and T. Romer* "Mobility and Redistribution" JPE, 1991, 99, 828-858.
10. *Gordon J. and P. Jallade* "Spontaneous Student Mobility in the European Union: A Statistical Survey" European Journal of Education 31, 1996, 133-152.
11. *Inman R. P. and D. L Rubinfeld.* "Fiscal Federalism in Europe: Lessons from the United States Experience." European Econ. Rev 36 (April 1992): 654-60.
12. *Janeba, E. and W. Peters* "Tax Evasion, Tax Competition and the gains from non-discrimination: the case of interest taxation in Europe" The Economic Journal, 109, January 93-101.
13. *Joustman, M. and J. –F. Thisse* "Local Public Funding of Higher Education when Skilled Labor is imperfectly mobile". International Tax and Public Finance, 7, 247-258. 2000.
14. *Martin, R. and M. S. Steinen* "State Aid, Regional Policy and Locational Competition in the European Union", European Urban and Regional Studies 4, 1997, pp 19-31.
15. *Obstfeld, W. and G. Peri* "Regional Non-Adjustment and Fiscal Policy" Economic Policy: A European Forum 26, 1998, pp 205-47.
16. *Patsouratis, A. V.* "Fiscal Decentralization in the EEC Countries" Public Finance, 3, 1990: 423-439.
17. *Sinn, H.W.,* "Tax Harmonization and Tax Co-operation in Europe" European Economic Review, May 1990, 34, 489-504.
18. *Smith, S.,* "The European Community's Priorities in Tax Policy" Institute for Fiscal Studies (London) Working Paper W90/2, 1990.
19. *Wildasin, D. E.* "Income redistribution and migration", Canadian Journal of Economics, 1994.
20. *Wildasin, D. E.* "Nash Equilibria in Models of Fiscal Competition." JPE 95, August 1987: 835-56.
21. *Wilson, J .D.* "A theory of inter regional tax Competition", Journal of urban Economics, 1988.
22. *Wilson, J .D.* "Theories of Tax Competition", National tax journal, 1999.