Evaluating Monetary Policy Regimes: The Role of Nominal Rigidities

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Abstract

I compare four basic monetary policy rules belonging to the interest-rate pegging rules class in two different analystical frameworks representing the way through which nominal rigidities are designed. A first model consider the Calvo-Woodford mechanism of price adjustment, as has become customary in the literature on monetary policy rules. A second model, instead, considers the cost of price adjustment function, as proposed by Rotemberg (1982). The two models are simulated to find the optimal monetary policy rule, maximizing the welfare of the representative agent. The results show that the optimal monetary policy rule for the model based on the Calvo price setting method is a simple contemporaneous interest-pegging rule with a lagged nominal interest rate. However, with a model based on the cost of price adjustment, the optimal rule is given by an interest rate rule with a expected inflation, contemporaneous output gap and a lagged interes rate term as arguments. Impulse respnse functions for different vallues of the parameters of the respective optimal monetary poliy function are plotted for both models, for an expansionary public expenditure shock and a contractionary monetary policy shock. The calculation of the model are empirically robust. For the first time fiscal policy is explicitly inserted in the model through a 'Passive' tax policy rule. An explicit analysi f about the determinacy of the equilibrium is contained in the model.

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1 Introduction

In the present paper I will explore how different hypothesis on the way through which nominal rigidities enter into the model will affect the search of an optimal monetary policy rule. In the literature the workhorse model considered so far has been the work by Rotemberg and Woodford (1997, 1998, 1999), where nominal rigidities enter in the model through a simple price adjustment mechanism designed according to Calvo (1983), where nominal price rigidity derives from the imperfect price adjustment done by each firm after a shock. This approach has several advantages. Probably, the more appealing feature of this approach is given by his analytical simplicity which translates into the possibility of constructing an aggregate supply function in terms of the expected inflation and output gap in a very intuitive way. However, the main problem with this approach is represented by the fact that the instance of time revision after a shock derives from an exogenous factor: the fraction of firm adjusting prices in each period after a shock is exogenously given and is not explicitly derived from an optimization problem.

Probably, a more rigorous approach would include an explicit optimization function of the price set by each firm under the hypothesis considered by Rotemberg (1982) and recently implemented in simple Monetary Real Business Cycle model by Kim (2000). In the present paper I compare the performance of monetary policy rules for two models: the first adopts the nominal rigidities along the Calvo approach, while the other includes the price adjustment cost approach outlined by Rotemberg (1982) and Kim (2000). The results show that the choice of nominal rigidities is not completely neutral with respect to the optimality of monetary policy function. In fact, I have found that for a model with nominal rigidities designed along the Calvo-Woodford approach, the optimal monetary policy rule is given by a Taylor-type rule where nominal interest rate reacts to contemporaneous inflation output gap with a lagged interest rate.

On the other hand, in a model with cost of price adjustment, the optimal monetary policy rule is given by an interest-rate pegging rule with expected inflation, the contemporaneous output gap and a lagged nominal interest rate, along the same lines proposed by Clarida, Galì and Gertler (1999). This discrepancy is due to the explicit forward-looking character of the price setting mechanism characterizing the price adjustment cost model. The forward looking character of the price setting mechanism introduces a channel for the definition of the expected inflation term, which is included in the monetary policy rule.

A dissatisfactory aspect of the existing literature is given by the way in which money enters into the model. In order to obtain sound welfare results, in the current litereature it is generally assumed a strongly separable utility function in money, consumption and labor effort. This approach has the shortcoming of considering shocks to money demand as pure preference shocks, things which are difficult to be empirically tested. A different approach instead, would insert money through transaction cost approach. This is precisely the approach taken here: shocks to money demand can now be seen as shocks to the transaction cost function, with a stronger empirical flavor than the preference approach. Therefore, in this framework, the utility function will be additively separable in consumption an labor effort.

Another element of novelty given by the approach taken in this paper is given by the explicit consideration of fiscal policy: in the existing literature, fiscal policy is *assumed* to be 'Ricardian', i.e. it considers implicitly the existence of a solvency constraint on the government, as carefully discussed in Rotemberg and Woodford (1999) and especially Woodford (2000). On the other hand, the present paper takes explicitly into consideration the role of the government budget constraint into the model simulation and solution and in the comparative evaluation of the various monetary policy rules.

As discussed by Cochrane (1998, 1999), Leeper (1991), Sims (1994) and Woodford (1996, 2000), fiscal policy requirements on the future path of the primary surpluses are a necessary complement to the monetary policy rules in order to achieve full price stability. In the present model I assume a fiscal policy rule making taxes as reacting to the outstanding stock of real public debt. Such policy rule is defined 'Passive' in Leeper's terminology, or 'Ricardian' in Woodford's terminology, without any explicit link with the notion of the Ricardian equivalence. This gives to the model an higher degree of generality.

The discussion on determinacy/indeterminacy of the Rational Expectation Equilibrium in dynamic intertemporal models with monetary policy rules has been carefully considered by Woodford (2000) and especially Bullard and Mitra (1999, 2000). The benchmark model adopted by the current literature considers an Aggregate Supply function without real money balances - or nominal interest rate - via the money demand expression derived from portfolio optimization of the representative agent. This is mainly due to the particular approach adopted by those authors in setting up the role of money in these models. In fact, with money inserted in an utility function strongly separable in all its arguments, it is natural to derive an aggregate supply function completely independent upon monetary variables (real money balances or nominal interest rate). Instead, the Aggregate Supply function derived in the present model (where money enters via transaction costs in the Representative Agents's budget constraint) turns out to be dependent upon real money balances and the interest rate, adding a further dimension to the various shocks of the model.

The paper is organized as follows. In the next section the reader is introduced to the common elements of the two models, through the description of the demand side, together with the choice problems (intertemporal and intratemporal) of the Representative Agent. In this section there is also a description of the government's role and of the equilibrium of the overall economy. The following section starts with the description of the supply side for both models: I start first with the model based on Calvo (1983) and Rotemberg and Woodford (1999). The model based on the Cost of Price Adjustment à la Rotemberg (1982) - Kim (2000) will follow. Monetary policy rules employed in the simulations are described in a new section, followed by a discussion on the calibration of the parameters of the model. The discussion on the optimal monetary policy function is preceded by a section on the conditions to be respected by the monetary policy reaction function parameters in order to achieve determinacy of the Rational Expectation Equilibrium. Then, the welfare analysis with the search for the optimal monetary policy function will follow together with a discussion on the goodness of fit of the models simulated with the parameter configurations for their respective optimal monetary policy function. Finally, the analysis of the impulse-response function for the two models closes the presentation of results. A section with final comments ends the paper.

2 The demand side

2.1 The intertemporal and intra-temporal consumer's problem.

In what follows I am going to describe the common features of the models used for the evaluation of monetary policy functions. The economy is populated by a large number of representative agents indexed by j on the real line between 0 and 1, each of them will maximize the following stream of utility discounted with a rate β, $0 < \beta < 1$, depending only upon consumption C_t^j and the work effort L_t^j:

$$U_t^j = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^j, L_t^j\right) \tag{1}$$

The instantaneous utility function $u\left(C_t^j, L_t^j\right)$ is given by:

$$u\left(C_t^j, L_t^j\right) = \frac{\left(C_t^j\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{\left(L_t^j\right)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \tag{2}$$

with $\sigma > 1$, and $\eta > 1$. In (2) I assumed a strongly separable utility function in consumption and labor effort. This can be a strong limitation for dynamic models like that considered here. However, the utility function above considered avoids nonsensical welfare calculations because it does not show the cross substitution effect between C_t^j and L_t^j. This utility function is very close to the special case considered in Rotemberg and Woodford (1999), and Woodford (1999), apart from the absence of real money balances.

The representative agent maximizes the utility function $(1)-(2)$ subjected to the following budget constraint:

$$\int_0^1 p_t(i) C_t^j(i) \left[1+\xi_t^j f\left(V_t^j\right)\right] di + B_t^j + M_t^j = \left(1+i_{t-1}\right) B_{t-1}^j + M_{t-1}^j + w_t^j L_t^j + P_t \Omega_t^j - P_t T_t^j \qquad (3)$$

As in the traditional models of monopolistic competition, I consider the existence of a continuous set of goods indexed by i on the real line between 0 and 1. Under this view, the variable C_t^j is an index of all the goods produced in this economy. In the formulation under (3) for the representative agent budget constraint, I assume that the wealth of the agent is given by nominal bonds and money. In the resources available for consumption are given by interest earned on government bonds[1], $\left(1+i_{t-1}\right) B_{t-1}^j$, money M_t^j, labor income $w_t^j L_t^j$ and nominal profits $P_t \Omega_t^j$. The decision to consume is costly for the agent since a fraction of its resources is spent in the transaction technology $f\left(V_t^j(i)\right)$, where V_t^j is the velocity of money for agent j. V_t^j is defined as follows:

$$V_t^j = \frac{P_t M_t^j}{C_t^j} \qquad (4)$$

where C_t^j is the aggregate demand expressed on all goods by agent j, and P_t is the general price index. I consider here a very simple formulation for the velocity function, as in Sims (1994). In fact, $f\left(V_t^j\right)$ is assumed to be linear and it is given by: $f\left(V_t^j\right) = V_t^j$. This is done in order to keep calculations simple and to provide a set of results for a model which can be considered as a possible benchmark, open to further extensions.

In the above formulation, ξ_t^j is a transaction cost parameter and represents the size of the total transaction cost that has to be paid by the single agent in each consumption transaction.

To allow for money demand shocks I assume a stochastic behavior for ξ_t^j, which can be represented with an AR(1) process, as follows:

$$\log\left(\xi_t^j\right) = +\left(1-\rho_\xi\right)\log\left(\xi^j\right) + \rho_\xi \log\left(\xi_{t-1}^j\right) + \varepsilon_t^\xi \qquad (5)$$

with $E\left(\varepsilon_t^\xi\right) = 0$, $Var\left(\varepsilon_t^\xi\right) = \sigma_\xi^2$.

I find this way of modelling money demand shocks more realistic than a more standard approach with money in the utility function where money demand shocks are identified as preference shocks.

In (5) and in all the equations which will follow, ξ^j without time subscript indicates the steady state values of ξ_t^j. From (3) we also see that the government levies lump-sum taxes on the representative agent in the amount of T_t^j.

The intertemporal maximization problem faced by each representative agent is a portfolio allocation problem. In fact, each agent optimizes his (her) utility function with respect to C_t^j, L_t^j, M_t^j, and B_t^j, to

[1] In the notation of the text, B_t^j, indicates the bonds held by agent j. In the same way, M_t^j is the money holding agent j.

produce the following set of first order conditions:

$$\left(C_t^j\right)^{-\frac{1}{\sigma}} = \lambda_t^j\left(1+2\xi_t^j V_t^j\right) \tag{6}$$

$$\left(L_t^j\right)^{\frac{1}{\eta}} = \lambda_t^j\frac{W_t^j}{P_t} \tag{7}$$

$$\beta E_t\frac{\lambda_{t+1}^j}{P_{t+1}} = \frac{\lambda_t^j}{P_t}\left[1-\xi_t^j\left(V_t^j\right)^2\right] \tag{8}$$

$$\beta\left(1+i_t\right)E_t\frac{\lambda_{t+1}^j}{P_{t+1}} = \frac{\lambda_t^j}{P_t} \tag{9}$$

where λ_t^j is the Lagrange Multiplier associated to the constraint of the j-th agent. Equation (6) indicates the intertemporal choice of consumption, while equation (7) represents the optimal choice of labor by equating the disutility of the work effort to the real wage weighted by the marginal utility of consumption. Equation (8) indicates the optimal choice of money and together with (9) nests a money demand function. Equation (9), finally is the results from optimal bond allocation. Attached to $(6)-(9)$ there is also a Transversality Condition on Government bond holdings in order to avoid a Ponzi Scheme with public debt.

The optimization process so far described pertains to the intertemporal dimension of the choice problem faced by each representative agent. However, given the presence of a large number of final goods, each agent chooses also the composition of a basket of differentiated goods. On this ground, the variable C_t^j represents an index of all the differentiated goods produced in this economy.

Thus, following Dixit and Stiglitz (1977) the basket C_t^j is formed by aggregating through a CES aggregator all the $i\in[0,1]$ final goods as:

$$C_t^j = \left[\int_0^1\left(C_t^j(i)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{10}$$

where $\theta>1$, indicates the elasticity of substitution between different final goods varieties. Let X_t^j be the total expenditure of agent j. In order to decide the optimal allocation among differentiated goods, each agent j maximizes the index (10) over $C_t^j(i)$ for all $i\in[0,1]$ subjected to the following constraint:

$$\int_0^1 p_t(i)C_t^j(i)di \le X_t^j \tag{11}$$

The solution to the *intra-temporal* program delivers the following expression for the aggregate price index P_t:

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}} \tag{12}$$

Also, the demand for each good i expressed by every agent j is:

$$\frac{C_t^j(i)}{C_t^j} = \left[\frac{p_t(i)}{P_t}\right]^{-\theta} \tag{13}$$

where θ is the elasticity of good i with respect to its price.

Combining the above relations, we find that the total expenditure of each agent is given by:

$$X_t^j = P_tC_t^j$$

In the present framework we need the differentiated goods assumptions in order to introduce an imperfectly competitive market for each different quality of goods and a consequent price control by each firm i producing each variety i.

2.2 Government and Fiscal Policy

The government budget constraint in nominal terms is given by:

$$B_t - (1 + i_{t-1}) B_{t-1} + M_t - M_{t-1} = P_t G_t - P_t T_t \tag{14}$$

In equation (14) the primary deficit (surplus) $G_t - T_t$ plus the interest rate proceedings paid by Government to the owner of government debt $i_{t-1}B_{t-1}$ is financed either by printing new money $M_t - M_{t-1}$ or by issuing new debt $B_t - B_{t-1}$. Despite the limited role of seignorage revenue in all the advanced economy, I have chosen to include money as a source of debt financing more for comparison matters with the existing literature than for a theoretical reason.

Of course, the equilibrium conditions on both debt and money market are such that demand meets supply at any instant t, so that the total amount of debt and money floating in the market are entirely owned by j-th households. In fact:

$$B_t = \int_0^1 B_t^j dj, \quad M_t = \int_0^1 M_t^j dj, \text{ and } \quad T_t = \int_0^1 T_t^j dj \tag{15}$$

The comparative evaluation of alternative monetary policy rules cannot be considered without constraints on the solvency of government. As pointed out by an important stream of literature known as 'Fiscal Theory of the Price Level', in order to avoid unpleasant episodes of inflation due to loose budgetary policies, we need to impose a set of restrictions on the fiscal revenue collected by the government. In this sense, by imposing a set of equilibrium fiscal policies which force the government to set the total amount of fiscal revenue (or the primary surplus, more directly) as a function of the outstanding stock of real debt, will avoid the risk of inflationary expectations.

As pointed out by Sims (1994, 1998), Cochrane (1998, 1999), and Woodford (2000), the inflation can be viewed as fiscal phenomenon: if the government is perceived of adopting a loose fiscal policy, the inflation rate will explode right from today, discounting the future increase of money supply needed to wash out the level of outstanding debt. This is an equilibrium phenomena: it will happen *in expectation* even if seignorage revenue are very small and the commitment to avoid the use of them in financing the debt is very strong from the government's side.

Thus, in order to prevent an explosive solution for price level, Leeper (1991), and Sims (1994) define an 'active' fiscal policy as one which makes taxes reacting to real debt as follows:

$$T_t = \psi_0 + \psi_1 \frac{B_{t-1}}{P_t} \tag{16}$$

with $\beta (1 + i) > \psi_1$ in order to assure stability of the debt path. It is possible to design also alternative policy rules belonging to the same family of rule (16). For example, Schmitt-Grohè and Uribe (1997) describe balanced-budget rules. In general, a set of 'passive' fiscal policies (or alternatively said 'Ricardian' fiscal policies, in the Woodford's (1996, 2000) jeargon) are such that make taxes reacting to the outstanding stock of public debt.

In what follows, I assume that in the comparative evaluation of various monetary policy rules the government adopt a rule like (16). This will allow us to concentrate on the parameter combination of monetary policy rules which will satisfy certain requirements both in terms of determinacy/indeterminacy of the solution of the Rational Expectation (henceforth RE) equilibirum model and in terms of welfare optimality.

I assume that the pure public expenditure (net of interest payment) G follows an AR(1) process, given by

$$\log (G_t) = (1 - \rho_G) \log G + \rho_G \log (G_{t-1}) + \varepsilon_{t+1}^G \tag{17}$$

with ε_{t+1}^G i.i.d. variable normally distributed with zero mean and constant variance, σ_G^2.

2.3 The equilibrium of the demand side

In what follows, I will introduce a crucial assumption: I will concentrate on a symmetric equilibrium where the choices made by each agent j is the same across all agents of the population. This will allow to drop index j from the equations of the model.

By manipulating the first Order Conditions (8) and (9), we get that money demand is given by:

$$m_t = \left[\xi_t \left(\frac{1+i_t}{i_t}\right)\right]^{1/2} C_t \tag{18}$$

where $m_t = M_t/P_t$ indicate real money balances. Equation (18) is a traditional money demand function. In fact, it is straightforward to verify that $\frac{\partial m_t}{\partial C_t} > 0$, $\frac{\partial m_t}{\partial i_t} < 0$. Additionally, shocks to money demand are transaction cost shock and hit m_t a pure multiplicative way through the function ξ_t. This is realistic description of money demand fluctuations. In fact, given the increased role of transaction cost shocks in the fluctuations in money demand recently observed in the money market of the most advanced economy.

Mixing together equations (6) and (7) we have that the labor supply curve for all agent j is given by:

$$L_t^{\frac{1}{\eta}} = \frac{C_t^{-\frac{1}{\sigma}}}{1+2\xi_t V_t}\left(\frac{W_t}{P_t}\right) \tag{19}$$

The labor supply function (19) is positively sloped with respect to wage, but negatively sloped with respect to consumption through the intertemporal elasticity of substitution of labor effort.

Given the existence of lump sum taxation, the representative agent's budget constraint can be rewritten as:

$$C_t\left(1+\xi_t\frac{C_t}{m_t}\right) = Y_t - G_t \tag{20}$$

According to equation (20) the amount of income net of public expenditure can only be consumed. In this model, I did not include investment, in order to keep the analysis as simple as possible. A fraction of income is given up in order to finance the transaction costs.

Finally, after combining (6) with (9) we get the Euler equation as follows:

$$\frac{C_t^{-\frac{1}{\sigma}}}{\left(1+2\xi_t\frac{C_t}{m_t}\right)} = \beta\left(1+i_t\right)E_t\frac{C_{t+1}^{-\frac{1}{\sigma}}}{\pi_{t+1}\left(1+2\xi_{t+1}\frac{C_{t+1}}{m_{t+1}}\right)} \tag{21}$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the inflation rate.

From equations (18)-(21), it is easy to get the steady state of the model. To solve the model I will take a lig first-order Taylor expansion around the steady state of each variable (expressed in logs).

For further reference, I will report here the loglinearized version of some of the previous relationships. From (6), (7), (9), respectively, we get:

$$\widetilde{\lambda}_t = -\eta_{c\lambda}\widetilde{c}_t + \eta_{m\lambda}\left(\widetilde{m}_t - \widetilde{\xi}_t\right) \tag{22}$$

$$\widetilde{L}_t = \eta\widetilde{\lambda}_t + \eta\left(\widetilde{w}_t - \widetilde{p}_t\right) \tag{23}$$

$$\eta_c\widetilde{c}_t + \widetilde{\pi}_t - \eta_m\left(\widetilde{m}_t - \widetilde{\xi}_t\right) = \eta_c\widetilde{c}_{t-1} - \eta_m\left(\widetilde{m}_{t-1} - \widetilde{\xi}_{t-1}\right) + \eta_i\widetilde{i}_{t-1} \tag{24}$$

Additionally, the money demand function and the representative agent's budget constraint are respectively given by:

$$\widetilde{m}_t = \frac{1}{2}\widetilde{\xi}_t + \widetilde{c}_t + \eta_i\widetilde{i}_t \tag{25}$$

$$\widetilde{c}_t = \eta_{cy}\widetilde{y}_t - \eta_{cg}\widetilde{G}_t + \eta_{cm}\left(\widetilde{m}_t - \widetilde{\xi}_t\right) \tag{26}$$

These equations expressed in log-linear terms, will be employed to get a reduced form of the aggregate supply equation curve.

3 The model based on Calvo (1983) price setting mechanism

In what follows I will introduce the reader to the profit maximization problem considered by each firm living in a world where the Calvo (1983) approach of price setting, modified along Woodford (1996) and Rotemberg and Woodford (1999) is the only source of nominal rigidity in the model.

The firm producing good i faces two types of choices about the price of good i to be charged to consumers and about the amount of work to hire.

The framework I am going to follow is based on a monopolistic competition framework close to the model of Blanchard and Kiyotaki (1987). Each firm is free to determine a price for each period in order to maximize its own profit. An additional feature of my model is represented by the inclusion of a fixed cost shock Φ in the production function , on the same lines highlighted by Hornstein (1993) and Kim (1996). This cost can be interpreted as a sort of entry barrier that each firm has to cross before entering in a given market. The advantage of this formulation is that it allows to have zero profits in the steady state.

According to Calvo (1983) model of price determination, each seller sets a new price with probability $1-\delta$ $(0<\delta<1)$ each period independently of the time passed since the last price change. Thus, δ measures the degree of nominal rigidity. I adopt this framework as a first approximation in order to make my model comparable with the existing literature. In fact, Woodford (1996) and Rotemberg and Woodford (1997) who study the role of monetary and fiscal policy rules in model like the present one, adopt the same framework with a different utility function. By following this way, in fact, I can compare more easily how the way through which money enters in the model can be crucial or not in the evaluation of fiscal and monetary policy rules. A more realistic approach for the modelling of price rigidities is considered in the next section where I will examine a model with cost of adjusting prices.

What are the advantages of a Calvo price setting formulation ? First of all, we obtain a model with one parameter which allows for arbitrary variation in average time until price changes between zero and infinity. In fact, this will help to derive an equation where the deviation of actual output from its potential level will be transferred into an higher expected inflation next period, exactly like an Aggregate Supply curve of an undergraduate-level macroeconomic textbook.

The second advantage of this formulation is that if the probability of price changes is independent of time since the last price change, we do not need to consider how existing prices vary with the age of price commitments. Although this framework cannot be considered entirely realistic, it allows an easy derivation of the AS curve, which makes the model more comparable with the existing literature.

Before we get into the model, let us consider the evolution of the price index P_t. Define with P_t the price chosen by each single firm at date t: P_t identifies the new price set by each seller at date t. Each of the previous prices appear in the new price distribution with δ times the previous frequency. Therefore, the price index can be written as:

$$P_t^{1-\theta} = \left[\int_0^1 p_t(i)^{1-\theta}di\right] = \tag{27}$$

$$= (1-\delta)\,\mathsf{P}_t^{(1-\theta)} + \delta\int_0^1 p_{t-1}(i)^{1-\theta} =$$

$$= (1-\delta)\,\mathsf{P}_t^{(1-\theta)} + \delta P_{t-1}^{1-\theta} \tag{28}$$

The choice of P_t depends only upon the current and expected fut............ure evolution of $\{P_t\}$: we do not need to keep track of the entire price distribution, across all producers.

The production function for good i is assumed to be:

$$Y_t(i) = A_t\,(L_t(j))^{(1-\alpha)} - \Gamma\Phi_t \tag{29}$$

In (29) I assume that the economy shows a constant gross growth rate of income of $\Gamma \geq 1$ and that the variables appearing in this framework are already transformed according to it. A_t is the technology process and Φ_t is the fixed cost shock. I assume that both A_t and Φ_t are common to all firms and follow the stochastic processes:

$$\log(A_t) = \rho_A \log(A_{t-1}) + (1-\rho_A)\log(A) + \varepsilon_{At} \tag{30}$$

$$\log(\Phi_t) = \rho_\Phi \log(\Phi_{t-1}) + (1-\rho_\Phi)\log(\Phi) + \varepsilon_{\Phi t} \tag{31}$$

where $A\,(\geq 0)$ and $\Phi\,(\geq 0)$ are the steady state values and ε_{At} and $\varepsilon_{\Phi t}$ are i.i.d. random variables normally distributed with $N(0,\sigma_A^2)$, $N(0,\sigma_\Phi^2)$, respectively. The fixed cost shock can be interpreted also as an additional type of technological shock which enters additively in the production function. A possible further interpretation considers Φ_t as a set up shop cost, i.e. a cost that has to be pad by each firm in order to start up with the business.

Each firm chooses its own price P_t in order to maximize the following profits function:

$$\max_{\mathsf{P}_t} \sum_{s=0}^{\infty} (\delta\beta)^s \lambda_{t+s}^j \left[\left(\frac{\mathsf{P}_t}{P_{t+s}}\right) Y_t(i) - \frac{w_t^j}{P_t} L_t^j\right] \tag{32}$$

The total demand faced by each firm i is the sum of the total demand of goods expressed both from the consumers' side and from the government's side. Therefore, according to (13) we can rewrite the demand for each single firm as follows:

$$\frac{Y_t(i)}{Y_t} = \left[\frac{p_t(i)}{P_t}\right]^{-\theta} \tag{33}$$

In the maximization process of equation (32), I exploited the demand of final goods (33), the production function (29) for L_t^j, the first order condition on C_t (6) to get λ_{t+s}^j for all j, and the first order condition on L_t^j, (7), to get $\frac{w_t^j}{P_t}$. Thus, having solved the optimization problem (32), I log-linearize the resulting expression with respect to all the variables to get the following expression:

$$E_t \sum_{i=0}^{\infty} (\delta\beta)^i \left[\widetilde{P}_{t,t+i} - \phi_{sy}\widetilde{y}_{t+i} + \phi_{s\lambda}\widetilde{\lambda}_{t+i} + \phi_{sA}\widetilde{A}_{t+i} - \phi_{s\Phi}\widetilde{\Phi}_{t+i}\right] = 0 \tag{34}$$

In (34) $\widetilde{P}_{t,t+s}$ indicates the price set at time t to hold at time $t+s$ and ϕ_{sy}, ϕ_{sA}, $\phi_{s\Phi}$, $\phi_{s\lambda}$ are coefficients function of the core parameters whose expressions are given in the appendix. Moreover, $\widetilde{X}_{t+i}$ indicates the difference $\log X_{t+i} - \log X$, with X steady state, for each variable. Equation (34) is the starting point from which we can derive an aggregate supply function. To do so, we need to log-linearize (28) around the steady state to obtain:

$$\widetilde{\pi}_t = \frac{1-\delta}{\delta}\widetilde{P}_{t,t} \tag{35}$$

where $\widetilde{\pi}_t$ is the gross inflation rate. It is also easy to get the following important relation:

$$\widetilde{P}_{t,t+s} = \widetilde{P}_{t,t} - \sum_{i=1}^{s} \widetilde{\pi}_{t+i}$$

which in virtue of (35) becomes

$$\widetilde{P}_{t,t+s} = \frac{\delta}{1-\delta}\widetilde{\pi}_t - \sum_{i=1}^{s}\widetilde{\pi}_{t+i} \tag{36}$$

after having used (35). If we define now

$$\widetilde{D}_{t+i} = \phi_{sy}\widetilde{y}_{t+i} - \phi_{s\lambda}\widetilde{\lambda}_{t+i} - \phi_{sA}\widetilde{A}_{t+i} + \phi_{s\Phi}\widetilde{\Phi}_{t+i} \tag{37}$$

After making use of (36), we can rewrite (34) as follows:

$$E_t\sum_{i=0}^{\infty}(\delta\beta_x)^i\left[\frac{\delta}{1-\delta}\widetilde{\pi}_t - \sum_{i=1}^{s}\widetilde{\pi}_{t+i}\right] = E_t\sum_{i=0}^{\infty}(\delta\beta_x)^i\,\widetilde{D}_{t+i}$$

Solving for the summation terms, we finally get:

$$\widetilde{\pi}_t = \frac{(1-\delta)(1-\delta\beta_x)}{\delta}\widetilde{D}_t + \beta E_t\widetilde{\pi}_{t+1} \tag{38}$$

A final transformation involves the elements included in $\widetilde{D}_t$. From the log-linearized version of the FOC (22), (23) and (24), solve for $\widetilde{\lambda}_t$, $\widetilde{m}_t$ and $\widetilde{c}_t$ as functions of the actual output $\widetilde{y}_t$, the public expenditure shock $\widetilde{G}_t$, the transaction cost shock $\widetilde{\xi}_t$ and the nominal interest rate $\widetilde{i}_t$. Then, if we plug the resulting expression in the definition of $\widetilde{D}_t$ given by (37) and rearrange, we get the following expression for the aggregate supply function:

$$\beta E_t\widetilde{\pi}_{t+1} = \widetilde{\pi}_t - \eta_{sy}\widetilde{y}_t + \eta_{s\xi}\widetilde{\xi}_t + \eta_{sA}\widetilde{A}_t - \eta_{s\Phi}\widetilde{\Phi}_t + \eta_{sG}\widetilde{G}_t + \eta_{si}\widetilde{i}_t \tag{39}$$

with η_{sy}, $\eta_{s\xi}$, η_{sA}, $\eta_{s\Phi}$, η_{sG} function of the core parameters of the model. From equation (39) we can also define the level of potential output as $\widetilde{y}_t^P = -\frac{\eta_{sA}}{\eta_{sy}}\widetilde{A}_t - \frac{\eta_{s\xi}}{\eta_{sy}}\widetilde{\xi}_t + \frac{\eta_{s\Phi}}{\eta_{sy}}\widetilde{\Phi}_t - \frac{\eta_{sG}}{\eta\phi_{sy}}\widetilde{G}_t$ so that we can have the following expression for the aggregate supply function:

$$\beta E_t\widetilde{\pi}_{t+1} = \widetilde{\pi}_t - \eta_{sy}\left(\widetilde{y}_t^P - \widetilde{y}_t\right) + \eta_{si}\widetilde{i}_t \tag{40}$$

which is a traditional aggregate supply curve. When $\widetilde{y}_t > \widetilde{y}_t^s$ the economy is subjected to an inflationary pressure. This an AS formulation similar to what has been proposed by Rotemberg and Woodford (1997, 1998, 1999). If we define the output gap as

$$\widetilde{x}_t = \widetilde{y}_t^P - \widetilde{y}_t \tag{41}$$

the aggregate supply curve to be employed for the analysis of the results is given by:

$$\beta E_t\widetilde{\pi}_{t+1} = \widetilde{\pi}_t - \eta_{sy}\widetilde{x}_t + \eta_{si}\widetilde{i}_t \tag{42}$$

A crucial feature of the AS curve represented by equation (42) is given by the presence of nominal interest rate. This can be explained by recalling the fact that the Lagrange multiplier from (6) and (22) is a function of real money balances, so that in solving the model for $\widetilde{\lambda}_t$, $\widetilde{m}_t$ and $\widetilde{c}_t$ as functions of $\widetilde{y}_t$, $\widetilde{G}_t$, $\widetilde{\xi}_t$ we get nominal interest rate in the AS equation.

This is a consequence of the specific way in which money enters in the model. This result is different from what has been obtained by Rotemberg and Woodford (1997, 1998, 1999) and Woodford (2000), where the AS curve was independent upon real money balances and nominal interest rate. Here, instead, Euler equation - through the definition of Lagrange multiplier λ (which includes real money balances) - directly depends upon m, so it is AS, after the proper transformations described above[2].

[2] The same result would have been obtained in a model with an utility function weakly separable in consumption and real money balances, as f.e. in Kim (2000), and Lubik (2000), within a different context.

An AS curve independent upon real money balances can only be obtained if the utility function of the representative agent is strongly separable in all its arguments: consumption, labor effort and real money balances. It is known, however, as discussed by Sims (2000), that such an utility function is highly problematic in dynamical models, for the lack of intertemporal crossed effects between all its arguments. With strong separability we loose the dimension of a true intertemporal problem. Even if the strong separable utility function (with money as argument) is widely employed in the literature, I think that the inclusion of money through transaction costs and the functional form assumed in (2) are the best compromise in terms of generality, allowing also to obtain accurate welfare calculations.

4 The model with price adjustment costs

A source of dissatisfaction in the modern monetary literature, is the way through which rigidities are modelled. The biggest shortcoming of the Calvo-Woodford approach to nominal rigidity modelling is given by the fact that the distribution of price change is exogenous, as well as it is the time between two subsequent price changes.

A remedy to this problem can be offered by a model where prices are costly to change like that described in Kim (1996). Basically the cost of adjusting prices is assumed to be:

$$AC_t^P = \frac{\phi_P}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi\right)^2 Y_t \tag{43}$$

where π is the steady state inflation rate, and ϕ_P is the adjustment cost parameter.

To explore the consequences of adopting such formulation with respect to the Calvo-Woodford approach I consider here a modelling framework similar to what has been shown before, apart from the function of price adjustment cost which is now given by (43) in place of (28).

As before, I conduct the same kind of exercise by evaluating different monetary and fiscal policy rules. Therefore, the basic structure of the model (monetary and fiscal policy rules, transaction costs, technology shocks, production function) does not change, apart from the different function of price adjustment (43).

In this case, the optimization problem of the firm becomes dynamic and not static. In (43) I set $\pi = 1$, as in the traditional menu cost approach inaugurated by Rotemberg (1982). Therefore, according to (43) a firm pays a cost in terms of output all the time she decides to change prices. In a more general case, followed for example by Kim (2000), when $\pi \neq 1$, each firm would pay a cost only when the growth rate of prices differs form the steady state level of the gross inflation rate π.

The rationale behind the formulation adopted here stays in the fact that people formulates their plans according to a stable level of inflation rate. If these plans are compatible with this inflation rate, we do not observe any deviations from rationality. This ways of modelling rigidity can be interpreted as a synthetic way to represent the complex nexus of relationships existing between each firm and her customer.

4.1 Profit Maximization

The optimal choice of productive input and prices is done by each firm through the maximization of the future stream of profit evaluated with a stochastic pricing kernel ρ_t for contingent claims, which I consider here as the firm's discount factor. Each firm maximizes the future stream of profits:

$$\max_{\{K_t(j), L_t(j)\}} E_0\left[\sum_{t=0}^{\infty} \rho_t \Omega_t(j)\right] \tag{44}$$

subjected to:

$$\Omega_t(j) = P_t(j) Y_t(j) - W_t L_t(j) - P_t AC_t^P(j) \tag{45}$$

given the demand for differentiated (33), and the cost of price adjustment function (43), after having aggregated over all $i \in [0,1]$ agents.

In taking first-order conditions for problem (44)-(45), I am going to use the relationship existing between $Y_t(j)$ and $L_t(j)$ through the production function. The first order condition with respect to the unique input of the production function $L_t(j)$ is given by:

$$\frac{W_t}{P_t} = (1-\alpha)\left(1 - \frac{1}{\varepsilon_t(j)}\right)\left(\frac{Y_t(j) + \Phi_t G^t}{L_t(j)}\right)\left(\frac{P_t(j)}{P_t}\right) \tag{46}$$

where $\varepsilon_t(j)$ is the demand elasticity, whose expression is given by:

$$\frac{1}{\varepsilon_t(j)} = \frac{1}{\theta}\left\{1 - \phi_p\left[\frac{P_t(j)}{P_{t-1}(j)} - 1\right]\frac{P_t}{P_{t-1}(j)}\frac{Y_t}{Y_t(j)} + E_t\left[\phi_p \frac{\rho_{t+1}}{\rho_t}\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right)\frac{P_{t+1}}{P_t(j)}\frac{P_{t+1}(j)}{P_t(j)}\frac{Y_{t+1}}{Y_t(j)}\right]\right\} \tag{47}$$

Equation (47) measures the gross markup of price over marginal costs. In fact, if $\phi_p = 0$ (in a perfectly flexible price framework), the markup over the costs, would be just $\frac{\theta}{\theta-1}$ over marginal cost given by equation (46). On the other hand, when $\theta \to \infty$ *and* $\phi_p = 0$ the markup will be constant and equal to one, since all goods produced by atomistic firms would be perfectly substitutable and market power would have disappeared. In fact, with an explicit design of the full intertemporal problem in the profit maximization process, both technology and the demand shocks affect the magnitude and the cyclical properties of the markup, which now becomes one of the key channels of transmission of the various policy shocks. For example, with a positive demand shock, firms will be forced to cut the markup: in fact, given the increase of the marginal revenue, output increases, but prices will increase less than in the full flexible case, leading to the cut of the markup. I consider these features as extremely important especially for the comparative evaluation of alternative monetary policy rules.

4.2 Additional Features

An important aspect for the characterization of the equilibrium of the model is given by role of the intertemporal discount factor for both firms and consumers. In the present context, I assume that each agent (consumers and firms) have access to a set of complete markets for contingent claims. This implies the existence of a unique market discount factor, as a consequence of adding the following condition:

$$\frac{\rho_{t+1}}{\rho_t} = \frac{\beta\lambda_{t+1}}{\lambda_t} \tag{48}$$

To understand condition (48) we can imagine the presence of a representative agent who can freely exchange shares of each firm operations in this economy, without paying transaction costs at all. It is clear, however, that the introduction of an additional first order condition for the optimal shares allocation will produce the same results.

Working through the first order conditions of the intertemporal choice problem of the consumers and through those by the firm, it is easy to show that even the model with cost of price adjustment nests an Aggregate Supply curve of the same type of that derived from the model with Calvo-Woodford mechanism of price adjustment. To see this, let impose the symmetrical equilibrium condition by on all the equations of the system together with (46) and (47), by setting $X_t(j) = X_t$. Then, loglinearize[3] the resulting expression around the steady state we get the following version of equations (46) and (47):

$$\widetilde{W}_t - \widetilde{P}_t = \eta_y \widetilde{y}_t + \left(1 - \eta_y\right)\widetilde{\Phi}_t + (\theta - 1)^{-1}\widetilde{\varepsilon}_t \tag{49}$$

[3] Recall that in the loglinearization of (47) I considered a steady state level of the inflation rate equal to 1.

$$\beta\phi_p E_t\widetilde{\pi}_{t+1} = \phi_p E_t\widetilde{\pi}_t - \widetilde{\varepsilon}_t \tag{50}$$

with $\eta_y \equiv (1-\alpha)\left(1-\frac{1}{\theta}\right)$. Thus, by using the same set of first order conditions, together with equations (49) and (50), we get a similar AS curve as that obtained in (42).

5 Monetary Policy Rules

Throughout the paper I will consider alternative monetary policy rules whose behavior has been carefully studied in a both theoretical and applied works. All the following rules are expressed in log-linear form.

All the rules considered in the present paper have been obtained as variants of the following general interest-rate pegging rule:

$$i_t = i + \sum_{n=1}^{T_i}\phi_{in}\left(i_{t-1}-i\right) + \sum_{n=1}^{T_\pi}\phi_{\pi n}\left(\pi_{t-1}-\pi\right) + \sum_{n=1}^{T_x}\phi_{xn}\left(Y_{t-1}-Y\right) + \eta_t^{mp} \tag{51}$$

where i_t is a measure of the nominal interest rate (the Federal Funds Rate in the empirical literature), with steady state i, and $\pi_t = P_t/P_{t-1}$ as gross inflation rate with steady state π. Additionally, Y_t indicates the level of actual output. In (51) I assumed that the target of the monetary policy authority for the nominal interest rate, inflation and output is coincident with the steady state level of these variables. The advantage of making use of rules embedded in (51) is mainly given by the restrict number of parameters to be controlled by monetary policy authority. Under rules like (51), money supply becomes endogenous, since monetary authority sets interest rate by letting the quantity of money to be determined endogenously by market clearing conditions in the money market.

In equation (53) I also included a monetary policy shock η_t^{mp} for which I assumed an AR(1) structure like the following:

$$\log\left(\eta_t^{mp}\right) = \left(1-\rho_{mp}\right)\log\left(\eta^{mp}\right) + \rho_{mp}\log\left(\eta_{t-1}^{mp}\right) + \varepsilon_t^{mp} \tag{52}$$

where ε_t^{mp} is an i.i.d. process distributed as follows $\varepsilon_t^{mp} \sim N\left(0,\sigma_{mp}^2\right)$. This assumption adds a little more persistency in the model, in order to allow a better propagation mechanism internal to the model, as suggested by Furher and Moore (1995b) and Cogley and Nason (1995a,b).

One key element of the present model is the role of various shocks: the fixed cost shock in the production function and the transaction cost shock which do not disappear in steady state. It should be noted that all these elements make the output gap - intended as the difference between actual and potential output - as time varying. Moreover, in the model based on the cost of price adjustment, the markup is not constant over the marginal cost, but is time varying, as shown in both equations (46)-(47). It has been shown that an accurate way to insert the targeting of real variables from the monetary authority is to explicitly consider the output gap variable $\widetilde{x}_t$, defined in equation (41).

Note that given the characteristics of the model at hands, the output gap derived fro mthis model is not necessarily efficient. In fact, if for efficient output gap we mean a level of output where all the frictions (both nominal and real) were eliminated, this is not the case considered here, because of the presence of the money transaction costs.

Thus, for potential output we mean the level of output associated to an economy where all nominal rigidities (here price rigidities) are completely absent from the model. It is natural, then, for monetary authority to target - together with the inflation rate - also the output gap.

Therefore, the simplest Taylor Rule assumed in the literature, which I consider here as a benchmark, is what I will call the **Taylor Rule 1**:

$$\widetilde{i}_t = \phi_\pi\widetilde{\pi}_t + \phi_x\widetilde{x}_t + \eta_t^{mp} \tag{53}$$

where η_t^{mp} has been defined in (52). According to rule (53), monetary authority sets interest rate reaction to inflationary pressure and to movements of the level of output gap. This rule is similar to what has been proposed by Taylor (1993), where nominal interest rate were assumed to react to inflation and actual output (note: note the output gap), with fixed parameters $\phi_\pi = 1.5$, $\phi_x = 0.5$.

A particular case for this rule is given by the simple interest rate pegging rule with the inflation rate assumed as unique target, by setting $\phi_x = 0$: this is the rule described by Leeper (1991) and called 'active' monetary policy.

A recent stream of literature stresses the importance of inertial behavior of the nominal interest rate. Woodford (1999), and Svensson and Woodford (1999) have pointed out that changes in nominal interest rates through open market operations or movemntes in the free reserve accounts, never happen to be a sudden and unexpected phenomena. In fact, often the changes of the discount rate have been preceded by persistent movements in Federal Funds Rate (in US, f.e.). This is done in order to avoid exaggerated reactions from financial markets and minimize the distortions on the portfolio reallocation issues.

Thus, the inclusion of a lagged nominal interest rate, in equation (53) will generate what we call **Taylor Rule 2**:

$$\widetilde{i}_t = \phi_\pi \widetilde{\pi}_t + \phi_x \widetilde{x}_t + \phi_i \widetilde{i}_{t-1} \tag{54}$$

It should be noted that in order to make both rules (53) and (54) operational, we need to specify the frequency at which data are collected by monetary authority at the decision time. This is not irrelevant especially when we consider that both (53) and (54) require the contemporaneous knowledge of current data about output gap (i.e. actual output, potential output) and inflation rate. However, in many situations, these data are collected retrospectively.

To take into account these criticism, I define a new policy rule, called **Taylor Rule 3,** given by the following equation:

$$\widetilde{i}_t = \phi_\pi \widetilde{\pi}_{t-1} + \phi_x \widetilde{x}_{t-1} + \phi_i \widetilde{i}_{t-1} \tag{55}$$

The adoption of a rule like (55) with lagged data can be justified by conisdering also a policy implementation lag, linked to the monetary authority reaction to particular kind of shocks.

Clarida, Galì and Gertler (1999) showed as all rules (53)-(55), can be derived from a general class of rules where the target of the monetary authority is given by the expected inflation, together with a contemporaneous value of the output gap and a lagged nominal interest rate. The same authors estimated a similar rule for the US economy over a predefined sample and support their choice with the good model fit provided by this type of rule. Thus, by inserting an expected inflation term into equation (53) we get the following **Taylor Rule 4**:

$$\widetilde{i}_t = \phi_\pi E_t \widetilde{\pi}_{t+1} + \phi_x \widetilde{x}_t + \phi_i \widetilde{i}_{t-1} \tag{56}$$

Monetary policy rules (53)-(56) have been also considered by McCallum and Nelson (1998) in model different from this presented here, without considering the joint design of fiscal policy rule along the lines proposed by equations (**??**) and (16). In the evaluation of monetary policy rules proposed in the literature, this aspect has been taken as given, by simply assuming that fiscal policy follows a 'Ricardian' or 'Passive' rule such as (16). Here, instead, for the first time this aspect is explicitly taken into account in the empirical evaluation of the model.

6 Calibration

Common Features

The parameter choice is realized according to the most important contributions of the existing literature on monetary RBC models, in order to make the results here presented highly comparable. In particular, for the definitions of the parameters common to both models - i.e. the 'core' parameters - I will make extensive reference to those models in the literature which have been estimated or appear to be very robust with respect to the parameter's choice. The key parameters of the model common to both approaches (Calvo-Woodford rigidity approach and Cost of Price Adjustment) are reported in the following Table 1:

Parameter	Value
β	0.9839
α	0.33
σ	0.5
Γ	1.004
θ	6
ξ	.00532
η	3.56
ψ_1	0.55

Table 1

In most cases, the choices made here indicate widely accepted parameters. To calibrate the discount rate of the representative agent in the transformed economy, recall the steady state relationship existing between nominal and real interest rates: $\frac{1+i}{\pi} = \beta^{-1}$. From data collected on US economy on a quarterly basis for the sample 1959:1-1999:4, we record a value for the gross nominal rate equal to 1.0163% (1.067% on annual basis). Thus, if we set the inflation rate equal to 1, as it has been done for example, in Rotemberg and Woodford (1999), we get a value for β as that reported in the above table. The choice of parameter α implies a share of labor's income equal to 0.67 as in Campbell (1994). Recall that Γ indicates the growth rate of stationary variables of the overall economy, and it is set equal to 1.004 as in King, Plosser and Rebelo (1988). The elasticity of substitution is always a delicate parameter to be pinned down. I have chosen a value for σ equal to 0.5, implying a degree of risk aversion equal to 2, as prescribed by a well consolidated RBC literature.

Parameter θ describes the elasticity of substitution between differentiated goods. It is set equal to 6, in order to generate a level of markup equal to 1.2.

Another crucial parameter is given by the transaction cost given by ξ: to calibrate this parameter I followed Leeper and Sims (1994). For what concerns the role of the intertemporal substitution of the labor effort, I set η equal to 3.76, in order to guarantee internal coherence of the model. This is not too far from 3.56 - a value assumed by Chari, Kehoe and McGrattan (1999) in a model with a similar utility function.

For what concerns fiscal policy parameter ψ_1 the current empirical literature does not provide any sort of help about the magnitude of this parameter. It is clear, that to make economic sense within the present context, has to be positive (strictly). Sims (1994) indicated a bound that has to be respected by this parameter given by $\psi_1 < \beta^{-1}$. Thus, I have chosen a value for ψ_1 equal to 0.55, together with a steady state level of public debt to GDP set to 60%.

Finally, the level of the technology shock in the production function has been set in order to match exactly the mean of US economy in the post-war period.

Two final parameters remain to be calibrated. The first is the parameter δ which indicates the fraction of goods supplied, whose price does not change. We know that $0 \leq \delta < 1$. I follow Woodford (1996) and I have set $\delta = 0.36$.

With these parameter, for the model based on the Calvo-Woodford approach I obtain a coefficient on the output gap in the AS function equal to 0.38, which is close to what has been taken as a fixed benchmark by Woodford (1996, 2000), based on the estimates of AS proposed by Roberts (1995).

For what concerns the model based on the Cost of Price Adjustment, I set a value for parameter ϕ_p equal to 0.806, as in Kim (2000).

To complete the calibration issue, we need now to add some considerations on the paraemters of the shocks hitting the system. I assume the following set of parameters for the Autoregressive processes: $\rho_A = 0.98$, $\rho_\Phi = 0.92$, $\rho_\xi = 0.96$, $\rho_{mp} = 0.96$, $\rho_T = 0.96$. Finally, the variance-covariance matrix of the shocks is given by:

$$\Sigma_\varepsilon = \begin{bmatrix} \sigma_A^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_\Phi^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_\xi^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{mp}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_T^2 \end{bmatrix} = \begin{bmatrix} 0.0003 & 0 & 0 & 0 & 0 \\ 0 & 0.02 & 0 & 0 & 0 \\ 0 & 0 & 0.00196 & 0 & 0 \\ 0 & 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0 & 0.0049 \end{bmatrix}$$

These parameters values were obtained by three sources: Kim (2000), Cooley and Prescott (1995) and another companion paper by Marzo (2001), with capital accumulation. I assumed a set of shocks completely orthogonal among them. This assumption represents just a simple working hypothesis.

7 Determinacy of the Equilibrium

This section is intended to extend the discussion on determinacy of the equilibrium in monetary models to the present framework. In order to focus on the determinacy issue, in what follows I will concentrate only on the analysis of the model based on the rigidity approach modelled as in the Calvo - Woodford models. The model with cost of price adjustment can be analyzed in a very similar way.

The goal of the determinacy/indeterminacy analysis is to find a set of conditions to be imposed on the policy parameters of the model in order to satisfy the requirement of determinacy of the equilibrium. In other words, the goal here is to find under which conditions on the monetary policy parameters we can rule out explosive equilibria.

To highlight the problem I will concentrate on a the simplest Taylor rule for the Calvo-Woodford model, given by Taylor Rule 1, equation (53). To properly discuss the determinacy/indeterminacy issues, we need to cast the system in a particular way, as discussed by Bullard and Mitra (2000). To do so, by using the money demand equation (25) expressed in log-linear way, together with the representative budget constraint given in equation (26) (also in log-linear form), we can directly solve for $\widetilde{c}_t$ and $\widetilde{m}_t$ as function of $\widetilde{Y}_t$, $\widetilde{G}_t$, $\widetilde{\xi}_t$ and $\widetilde{i}_t$. Thus, plugging these expressions into Euler equation (24) and rearranging, we get the following expression for the Euler equation:

$$\widetilde{Y}_{t+1} - \alpha_g \widetilde{G}_{t+1} + \alpha_i \widetilde{i}_{t+1} + \alpha_\xi \widetilde{\xi}_{t+1} - \alpha_\pi \widetilde{\pi}_{t+1} = \widetilde{Y}_t - \alpha_g \widetilde{G}_t + \alpha_\xi \widetilde{\xi}_t + \gamma_i \widetilde{i}_t \tag{57}$$

with the expression of the coefficients α_g, α_i, α_ξ, α_π, γ_i reported in the appendix together with their numerical values. Now add and subtract from equation (57) the level of potential output $\widetilde{Y}_t^P$ to get:

$$\widetilde{Y}_{t+1} - \widetilde{Y}_{t+1}^P - \alpha_g \widetilde{G}_{t+1} + \alpha_i \widetilde{i}_{t+1} + \alpha_\xi \widetilde{\xi}_{t+1} - \alpha_\pi \widetilde{\pi}_{t+1} = \widetilde{Y}_t - \widetilde{Y}_t^P - \alpha_g \widetilde{G}_t + \alpha_\xi \widetilde{\xi}_t + \gamma_i \widetilde{i}_t \tag{58}$$

Define now a composite exogenous disturbance given by:

$$\widetilde{R}_t^n \equiv \left[\left(\widetilde{Y}_t^P - \alpha_g \widetilde{G}_t + \alpha_\xi \widetilde{\xi}_t \right) - \left(\widetilde{Y}_{t+1}^P - \alpha_g \widetilde{G}_{t+1} + \alpha_\xi \widetilde{\xi}_{t+1} \right) \right] \tag{59}$$

Equation (59) defines the deviations from the so called "Wicksellian natural rate of interest" from the value consistent with a zero inflation rate, as described by Woodford (2000). Note that in the definition of $\widetilde{R}_t^n$ enter all the exogenous processes of the system. Thus, using the definition of output gap given in

(41) and using the definition of natural rate of interest in (59), we have the following expression for the IS curve:

$$\widetilde{x}_{t+1} + \alpha_i \widetilde{i}_{t+1} - \alpha_\pi \widetilde{\pi}_{t+1} = \widetilde{x}_t + \gamma_i \widetilde{i}_t - \widetilde{R}_t^n \tag{60}$$

In the subsequent analysis, however, I will concentrate only on the dynamical aspects and I will abstract form the exogenous disturbance term given by (59). Now the system to be considered is formed by the IS curve (60), the AS curve given in equation (42) and the interest rate rule 1 (from equation (53)). To reduce further the system we can plug the monetary policy rule into equation (60) and into the Aggregate Supply curve (42), to get (after rearranging) the following system of equations:

$$(1 + \alpha_i \phi_x) \widetilde{x}_{t+1} + (\alpha_i \phi_\pi - \alpha_\pi) \widetilde{\pi}_{t+1} = (1 + \gamma_i \phi_x) \widetilde{x}_t + \gamma_i \phi_\pi \widetilde{\pi}_t + \widetilde{R}_t^n + \left(\gamma_i - \alpha_i \rho_{mp}\right) \eta_t^{mp}$$

$$\beta \widetilde{\pi}_{t+1} = (1 - \eta_{si} \phi_\pi) \widetilde{\pi}_t - \left(\eta_{sy} - \eta_{si} \phi_x\right) \widetilde{x}_t - \eta_{si} \eta_t^{mp}$$

where in (42) I collected the negative sign derived from coefficient η_{si} is negative and in (60) I used the log-;linear version of the AR(1) representation of the monetary policy shock given in (52).

To study determinacy/indeterminacy it easier to cast the system in the following way:

$$\begin{bmatrix} (1 + \gamma_i \phi_x) & \gamma_i \phi_\pi \\ -\left(\eta_{sy} - \eta_{si} \phi_x\right) & (1 + \eta_{si} \phi_\pi) \end{bmatrix} \begin{bmatrix} \widetilde{x}_t \\ \widetilde{\pi}_t \end{bmatrix} = \begin{bmatrix} (1 + \alpha_i \phi_x) & (\alpha_i \phi_\pi - \alpha_\pi) \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \widetilde{x}_{t+1} \\ \widetilde{\pi}_{t+1} \end{bmatrix} + \begin{bmatrix} 1 & \left(\gamma_i - \alpha_i \rho_{mp}\right) \\ 0 & -\eta_{si} \end{bmatrix} \begin{bmatrix} \widetilde{R}_t^n \\ \eta_t^{mp} \end{bmatrix} \tag{61}$$

or:

$$A_0 \widetilde{z}_t = A_1 \widetilde{z}_{t+1} + A_2 \begin{bmatrix} \widetilde{R}_t^n \\ \eta_t^{mp} \end{bmatrix} \tag{62}$$

with $\widetilde{z}_t = [\widetilde{x}_t, \widetilde{\pi}_t]'$. To further reduce the system, let us premultiply the matrices of the RHS of (61)-(62) by the inverse of A_0. The key matrix to be employed to study determinacy/indeterminacy is given by:

$$\Psi = A_0^{-1} A_1 = \Theta^{-1} \begin{bmatrix} (1 + \alpha_i \phi_x)(1 + \eta_{si} \phi_\pi) & (1 + \eta_{si} \phi_\pi)(\alpha_i \phi_\pi - \alpha_\pi) - \beta \gamma_i \phi_\pi \\ \left(\eta_{sy} - \eta_{si} \phi_x\right)(1 + \alpha_i \phi_x) & \left(\eta_{sy} - \eta_{si} \phi_x\right)(\alpha_i \phi_\pi - \alpha_\pi) + \beta (1 + \gamma_i \phi_x) \end{bmatrix}$$

with

$$\Theta \equiv (1 + \gamma_i \phi_x)(1 + \eta_{si} \phi_\pi) + \gamma_i \phi_\pi \left(\eta_{sy} - \eta_{si} \phi_x\right) \tag{63}$$

.Since neither $\widetilde{x}_t$ nor $\widetilde{\pi}_t$ are predetermined, the condition for determinacy is that both eigenvalues of Ψ lie inside the unit circle. Thus, the sufficient conditions for determinacy are collected in the following proposition.

Proposition 1 *Given $\phi_\pi \geq 0$, $\phi_x \geq 0$, under simple contemporaneous data Taylor Rule 1, the sufficient conditions for a Rational Expectation Equilibrium to be unique are:*

$$\phi_\pi > \frac{\alpha_\pi}{\gamma_i - \alpha_i} \tag{64}$$

$$\phi_x < \frac{\eta_{sy}}{\eta_{si}} \tag{65}$$

Proof.

The necessary and sufficient conditions for uniqueness require that both eigenvalues of Γ lie inside the unit circle. Thus, the characteristic polynomial of matrix Ψ is given by $\mu^2 + q_1\mu + q_0$, where:

$$q_0 \equiv \det(\Psi) = \frac{\beta(1+\alpha_i\phi_x)}{\Theta}$$
$$q_1 \equiv -\left[\frac{(1+\alpha_i\phi_x)(1+\eta_{si}\phi_\pi) + \beta(1+\gamma_i\phi_x) + (\eta_{sy} - \eta_{si}\phi_x)(\alpha_i\phi_\pi - \alpha_\pi)}{\Theta}\right]$$

with Θ defined as in (63). Thus, by applying the Schur-Cohn criterion[4], we have that the necessary and sufficient condition for both eigenvalues of Γ lies in the unit circle is equivalent to require that the following two conditions are satisfied:

$$(i) \quad |q_0| < 1;$$
$$(ii) \quad |q_1| < 1 + q_0;$$

Condition (i) requires:

$$-1 < \frac{\beta(1+\alpha_i\phi_x)}{(1+\gamma_i\phi_x)(1+\eta_{si}\phi_\pi) + \gamma_i\phi_\pi(\eta_{sy} - \eta_{si}\phi_x)} < 1$$

which can be split up in the following two inequalities:

$$(1+\gamma_i\phi_x)(1+\eta_{si}\phi_\pi) + \gamma_i\phi_\pi(\eta_{sy} - \eta_{si}\phi_x) > \beta + \alpha_i\beta\phi_x \tag{66}$$
$$-(1+\gamma_i\phi_x)(1+\eta_{si}\phi_\pi) - \gamma_i\phi_\pi(\eta_{sy} - \eta_{si}\phi_x) < \beta + \alpha_i\beta\phi_x \tag{67}$$

From (66) we have that:

$$\phi_x(\gamma_i - \alpha_i\beta) + \eta_{si}\phi_\pi(1+\gamma_i\phi_x) + \gamma_i\phi_\pi(\eta_{sy} - \eta_{si}\phi_x) > -(1-\beta)$$

given the sign of our coefficients, the above inequality is certainly verified if $(\eta_{sy} - \eta_{si}\phi_x) > 0$, which implies (65).

The second inequality (67) implies:

$$(1+\gamma_i\phi_x)(1+\eta_{si}\phi_\pi) + \gamma_i\phi_\pi(\eta_{sy} - \eta_{si}\phi_x) + \beta(1+\alpha_i\phi_x) > 0$$

which given (65) is always verified.

Condition (ii) implies the following two inequalities:

$$\Theta + \beta(1+\alpha_i\phi_x) + (1+\alpha_i\phi_x)(1+\eta_{si}\phi_\pi) + \beta(1+\gamma_i\phi_x) + (\eta_{sy} - \eta_{si}\phi_x)(\alpha_i\phi_\pi + \alpha_\pi) > 0 \tag{68}$$

$$-\Theta - \beta(1+\alpha_i\phi_x) < -(1+\alpha_i\phi_x)(1+\eta_{si}\phi_\pi) - \beta(1+\gamma_i\phi_x) - (\eta_{sy} - \eta_{si}\phi_x)(\alpha_i\phi_\pi + \alpha_\pi) \tag{69}$$

Inequality (68) is always satisfied, given (65). From (69) we get after rearranging:

$$\phi_x(\gamma_i - \alpha_i)(1-\beta+\eta_{si}\phi_\pi) + (\eta_{sy} - \eta_{si}\phi_x)[(\gamma_i - \alpha_i)\phi_\pi - \alpha_\pi] > 0 \tag{70}$$

Thus, given that $\phi_x \geq 0$, and given that the difference $(\gamma_i - \alpha_i)$ is positive[5] (see Appendix A.2). Thus, to satisfy (70) we need:

$$1-\beta+\eta_{si}\phi_\pi > 0 \tag{71}$$
$$(\gamma_i - \alpha_i)\phi_\pi - \alpha_\pi > 0 \tag{72}$$

[4] See LaSalle (1989).

[5] With the parameter values assumed here, we have that $\alpha_i = 0.0277$, and $\gamma_i = 0.0281$, so $\gamma_i - \alpha_i = 0.0004$.

Inequality (71) implies:

$$\phi_\pi > -\frac{(1-\beta)}{\eta_{si}}$$

which is always satisfied, because $\beta < 1$, and $\eta_{si} > 0$ (after having collected the minus sign). Finally, from (72) we directly get (64). ■

Basically, the conditions here collected imply two bounds for the two parameters of the monetary policy function: an upper bound for the coefficient on output gap and a lower bound for that on the inflation rate. This means that, in order to achieve stability and determinacy for the Rational Expectation Equilibrium we need to set high values for ϕ_π together with very low values for ϕ_x. This is not in contrast with what has been considered by Woodford (2000). An analogous set of conditions can be obtained also for the model with Cost of Price Adjustment.

I considered here conditions for determinacy only for the model including Taylor Rule 1. In fact, this is because the inclusion of Taylor Rule 1 represents the only case in which we can obtain simple analytical coniditions to establish determinacy/indeterminacy. The difficulty here is represented by the presence of the nominal interest rate in the AS function and in the IS curve dated at time $t+1$. Thus if we try to analyze the determinacy issues in models with Taylor Rule 2 or 4, as monetary policy rules, we would get a set of conditionis characterizes by an hihgh level of nonlinearity in the parameter values, which makes very difficult the task of finding determinacy conditions with a sound economic economic significance.

As I discussed before, this is a typical characteristic of the model at hand, given the particular way in which money enters in the utility function. At the contrary, in a model originating from an utility function with arguments (, L, M/P) strongly separable - given its simplicity - it is not difficult to find a full set of conditions of determinacy of the REE for all the policy rules here considered, as documented by Woodford (2000) and Bullard and Mitra (2000).

8 Optimal Monetary Policy Rule

8.1 Welfare analysis in monetary models

In this section I will discuss the search for an optimal policy rule for the model with Calvo-Woodford rigidities and the model with Cost of Price adjustment rigidities. The welfare analysis has the goal of finding the combination of parameters which maximizes the welfare of the representative agent. In this case, I am going to follow the same approach highlighted in a long series of well celebrated papers, started with Rotemberg and Woodford (1997, 1998, 1999), where the welfare of the representative agent depends upon both first and second order effects. This approach is similar to that adopted by the public finance literature where it is customary to compare alternative measures of fiscal policy or welfare-oriented economy.

Second-order effects are crucial in the evaluation of alternative monetary policy rules. In fact, the volatility of the crucial variables of the system - such as output, output gap, inflation and nominal interest rate - are a critical element in the evaluation of the goodness of a particular monetary policy rule. Under this respect, as outlined since the pioneering Poole's (1970) results, a good policy rule is also that which insulates better the system from a whatsoever kind of shock, by minimizing the fluctuations of the variable of the system from their trend. In the same way, a good policy rule is the rule which induce the minimal level of volatility in the global economic system.

The approach most widely employed in the evaluation of monetary policy rules takes a second order Taylor expansion of the utility function of the representative agent. However, this methods has some subtle limitations which can be safely by-passed by using the utility function here considered.

To state the problem, consider z as a vector of endogenous variable, and let ε be the disturbances to these variables. Suppose we want to evaluate $E\left[U\left(x;\varepsilon\right)\right]$ under alternative policy rules. We can consider vector z as a set of policy rules relating the endogenous variables to the exogenous shock ε derived as solution of a dynamic macroeconomic model. Suppose now that we can express the relationship between z and the vector of shock, by using $z = f\left(\varepsilon\right)$. A log-linear approximation of the policy function z can be described as follows:

$$z = z_0 + f^{'}\varepsilon + o\left(\|\varepsilon\|^2\right) \tag{73}$$

where z_0 is at most of order $o\left(\|\varepsilon\|\right)$. This operation is legitimate because, our welfare calculations have only local validity, for small deviations of the relevant variables from their steady states induced by shocks in vector ε. A Second-order Taylor expansion of the utility function assumes the following form:

$$E\left[U\left(z;\varepsilon\right)\right] = \overline{U} + \left(U_z f_{\varepsilon}^{'} + U_{\varepsilon}\right)\widehat{\varepsilon} + \frac{1}{2}\widehat{\varepsilon}^{'} f_{\varepsilon} U_{zz}\widehat{\varepsilon} f_{\varepsilon} + \underbrace{\frac{1}{2}\widehat{\varepsilon}^{'} U_{zz} f_{\varepsilon\varepsilon}\widehat{\varepsilon}}_{\bigstar} + \frac{1}{2}\widehat{\varepsilon}^{'} f_{\varepsilon} U_{z\varepsilon} f_{\varepsilon}\widehat{\varepsilon} + \frac{1}{2}\widehat{\varepsilon}^{'} U_{\varepsilon\varepsilon}\widehat{\varepsilon} + o\left(\|\varepsilon\|^2\right) \tag{74}$$

where $\overline{U} = U\left(\overline{z};0\right)$ and $\widehat{\varepsilon}$ is the deviation from the steady state (which is assumed to be zero for what concerns ε, because I set $E\left(\varepsilon\right) = 0$, $Var\left(\varepsilon\right) = \Sigma_{\varepsilon\varepsilon}$, matrix of variance-covariance, assumed to be diagonal[6]). The problem of second-order accuracy is given by the term underbraced which depends upon $f_{\varepsilon\varepsilon}$, the term neglected in (73). The full accuracy of the second order Taylor expansion would require that the term underbraced will disappear. This is equivalent to assume that the gradient of the utility function calculated in the steady state $U_z\left(\overline{z};0\right)$ is at most of order $o\left(\|\varepsilon\|\right)$. According to Rotemberg and Woodford (1997, 1998, 1999), this is certainly true when we assume an utility function strongly separable in all its arguments. In the case considered in the present paper, with the utility function given by (2), the term indicated by $\bigstar$ in (74) can be safely neglected in the welfare calculations without endangering the accuracy of the second order approximation. Rotemberg and Woodford (1997, 1998, 1999) together with Obstfeld and Rogoff (1995, 1997) consider strongly separable utility function which differ from (2) because of the presence of real monetary balances. However, as discussed by Sims (2000b), this approach is not entirely correct in dynamic models, because it tends to exclude important complementarities between the argument of the utility function, especially the role between consumption and leisure.

Under the utility function (2) we get that a second-order Taylor expansion would result in:

$$U(C,L) = U(\overline{C},\overline{L}) + \overline{C}^{-1/\sigma}E\left(\widetilde{C}_t\right) - \frac{1}{\sigma}\overline{C}^{-1/\sigma}Var\left(\widetilde{C}_t\right) - \overline{L}^{1/\eta}E\left(\widetilde{L}_t\right) - \frac{1}{\eta}\overline{L}^{1/\eta}Var\left(\widetilde{L}_t\right) \tag{75}$$

where $\widetilde{C}_t$, and $\widetilde{L}_t$ are the solutions in loglinear forms of the dynamical system, and represent the policy function as $z = f\left(\varepsilon\right)$. The inclusion of the solution of the dynamical system into (74) or (75) can be safely done just because of the particular type of utility function here assumed.

8.2 Optimal Monetary Policy Rule for the Calvo-Woodford model

In what follows I report the welfare results for various configuration of parameters for the monetary model based on the Calvo-Woodford approach. The simulations reported in this case and for the case with price adjustment cost function, are obtained by simulating the model in quarterly frequency. Each simulated time series is obtained after having performed 1000 simulations of the model, from which I discarded the first and the last 100 simulations. All the simulations have been obtained for each values of the monetary policy parameters in the intervals $(\phi_\pi, \phi_x, \phi_i) \in [0, 20]$ with step 0.01. The volatility

[6] All the shocks are assumed to be incorrelated.

results have been obtained according to the above described method for each values of the parameter of monetary policy function. The artificial time series were passed by the Hodrik-Prescott filter.

By adopting the methodology described before, we can collect a synthesis of the results in Table 2.

Rule	ϕ_π	ϕ_x	ϕ_i	σ_π^2	σ_y^2	σ_i^2	σ_x^2	U
$\mathbf{a}_1$	1.71	0.08	0	2.78	4.21	5.23	11.43	6.5
$\mathbf{a}_2$	10	0.08	0	1.32	8.4	6.8	15.24	6.12
$\mathbf{a}_3$	1.71	10	0	1.15	3.89	8.12	10.45	6.9
$\mathbf{b}_1$	1.48	0.06	1.25	2.95	5.56	4.99	10.92	7.28
$\mathbf{b}_2$	10	0.06	1.25	2.0	7.12	5.4	14.35	6.22
$\mathbf{b}_3$	1.48	10	1.25	2.05	3.8	6.56	9.77	5.95
$\mathbf{b}_4$	1.48	0.06	10	2.12	5.14	4.85	15.22	6.88
$\mathbf{c}_1$	1.3	0.063	1.7	1.91	4.25	5.44	10.71	7.1
$\mathbf{c}_2$	10	0.063	1.7	1.5	5.3	7.23	11.52	6.97
$\mathbf{c}_3$	1.3	10	1.7	2.15	2.6	8.41	12.8	7.0
$\mathbf{c}_4$	1.3	0.063	10	1.33	2.42	4.38	7.4	7.25
$\mathbf{d}_1$	1.42	0.04	1.75	1.85	4.17	5.21	10.5	7.18
$\mathbf{d}_2$	10	0.04	1.75	1.22	4.32	5.86	11.7	7.1
$\mathbf{d}_3$	1.42	10	1.75	1.78	2.7	6.95	12.8	7.21
$\mathbf{d}_4$	1.42	0.04	10	1.81	2.6	4.1	15.21	7.05
US historical	–	–	–	2.28	4.79	7.64	12.14	7.52

Table 2: **Model with rigidities à la Calvo-Woodford**

Note that the rule indicated with "a" refer to different parameters combination for Taylor Rule 1, while those indicated by "b", "c" and "d", refer to Taylor Rule 2, 3 and 4, respectively.

From the results reported in Table 2, we find that the optimal policy rule is given by Rule b_1. In fact, in the Calvo-Woodford framework, Taylor Rule 2 with $\phi_\pi = 1.4$, $\phi_x = 0.06$, and $\phi_i = 1.25$ delivers the best results in terms of welfare of the representative agent. By going through the numbers of Table 1, we also get that the rules belonging to group d produce results very close to this optimal one, in terms of welfare. Moreover, it is possible to note a simple result: by raising the coefficient ϕ_π describing the reaction of monetary authority to the inflation rate, we get a marked reduction of the inflation rate, wihout necessarily implying a welfare increase. In fact, by considering rules belonging to group "a", we observe that if we raise ϕ_π from 1.71 to 10 (i.e. passing from rule a_1 to a_2), we get a marked reduction of the volatility of the inflation rate, but at the expenses of an increased volatility in the output gap, output and nominal interest rate. The same is true when we move from Rule b_1 to b_2, from c_1 to c_2, and from d_1 to d_2, though in this last case, the increase in the volatility is lower than what has been recorded in all previous cases. This is a consequence of the fact that the welfare criterion of the representative agent is decreasing in the volatility of both inflation and output gap: by raising the tightness on the inflation rate, we also raise the volatility of output and nominal interest rate.

The results here outlined are close to what has been obtained by the existing literature - in particular Rotemberg and Woodford (1999) - but with some additional remarks. First of all, the values of the optimal monetary policy here obtained are slightly higher than what has been obtained by Rotemberg and Woodford, who considers as benchmark values $\phi_\pi = 1.4$, $\phi_x = 0.06$, and $\phi_i = 1.25$. However, if the nominal rigidities are designed according to the Calvo-Woodford method, the optimal monetary policy rules the optimal monetary policy rule is the same as in Rotemberg and Woodford, confirming their claim that the presence of expectation in a monetary policy function is not crucial.

To have a better idea of the empirical performance of the model, let us consider the plot of some time series simulated from the model versus the historical corresponding variables for the US economy. This procedure has been considered by several authors as a complementary method of model evaluation together with the second moment calculations. Therefore, to see how the choice of monetary policy rules affects the fit of the model in Figure 1 and 2 I reported the plot of the inflation rate, nominal interest rate and output simulated from the model together the correspondent time series for the US economy (quarterly observations, sample 1974:1-1994:4). All the variables were passed with the Hodrik-Prescott filter. For all the pictures, the dark line is the actual time series (from the data of US economy), while the dashed line is the simulated time series. All the time series were obtained by simulating the model 1000 times discarding the first and the last 100 simulations.

In Figure 1, I report the simulation results for the model simulated with Taylor Rule 1, with the combination of parameters given by a_1 in Table 2. From the top panel reported in Figure 1, we observe that the time series of the inflation of the model appears to underestimate the inflation rate: the dashed curve for the inflation rate stays always below that of the inflation from the US economy. Additionally, with this monetary policy rule, we not a volatility of the nominal interest rate much higher than is displayed from the actual time series. At the same time, in the bottom panel, we observe that the simulated time series of output is too smooth, if compared with the actual time series of the model, and does not seem to capture the various peaks and throughs occurred in the sale period under exam in the US economy.

However, the adoption of an interest rate pegging rule with the lagged nominal interest rate included - like Taylor Rule 2 - show a considerable improvement in fit of the model with the actual data. This is revealed by the plots reported in Figure 2, where I reported the actual time series versus the simulated time series for inflation, nominal interest rate and output when the model is simulated with a configuration of parameters given by b_1 in Table 2. In this case, in fact, the inflation rate is not underestimated, even if does not seem to be able to capture completely the various peaks and through displayed by the actual time series. In any case, however, the time series here reported for the simulated inflation seems to capture quite well the trend of the inflation rate. Also, the pattern of the nominal interest rate seems to confirm the improved performance of the model. However the time series here reported for output seems again to appear too smoothed with respect to the actual time series. However, the simulated series captures quite well the overall trend and some fluctuations observed in real data.

In any case, it is evident that the adoption of a rule with a lagged interest rate displays a better fit of the model. This can be also taken as additional element in favor of Taylor Rule 2 for a model when nominal rigidities are modelled along the Calvo-Woodford approach.

8.3 Optimal Monetary Policy Rule for Price Adjustment Cost model

To detect the differences among the various models characterized by different monetary policy rules, I will report the same kind of simulations for the model with the Cost of Price Adjustment Rules. As in the case with the Calvo-Woodford nominal rigidity design, each monetary policy parameter varies in the interval $(\phi_\pi, \phi_x, \phi_i) \in [0, 20]$ with step equal to 0.01. For each parameter value I calculated the second moments by simulating the model 1000 times, discarding the first 100 and the second 100 simulations. The results are displayed in Table 3.

Rule	ϕ_π	ϕ_x	ϕ_i	σ^2_π	σ^2_y	σ^2_i	σ^2_x	U
$\mathbf{A}_1$	1.66	0.02	0	2.9	5.61	9.1	15.21	6.51
$\mathbf{A}_2$	10	0.02	0	1.8	4.82	10.2	13.56	6.95
$\mathbf{A}_3$	1.66	10	0	2.12	2.25	9.45	7.88	6.87
$\mathbf{B}_1$	1.75	0.03	1.5	2.33	4.95	4.71	10.23	7.22
$\mathbf{B}_2$	10	0.03	1.5	1.95	6.28	6.68	14.3	7.34
$\mathbf{B}_3$	1.75	10	1.5	2.01	4.68	5.31	10.15	6.95
$\mathbf{B}_4$	1.75	0.03	10	1.25	5.77	6.15	12.43	7.09
$\mathbf{C}_1$	1.6	0.03	1.3	3.2	4.99	7.85	13.78	6.92
$\mathbf{C}_2$	10	0.03	1.3	1.98	5.61	8.6	16.51	6.85
$\mathbf{C}_3$	1.6	10	1.3	3.31	2.1	9.77	7.21	6.9
$\mathbf{C}_4$	1.6	0.03	10	2.34	3.8	3.5	9.75	7.05
$\mathbf{D}_1$	1.88	0.06	1.65	2.25	4.85	7.22	12.21	7.85
$\mathbf{D}_2$	10	0.06	1.65	1.45	5.01	8.1	13.5	7.55
$\mathbf{D}_3$	1.88	10	1.65	2.32	3.9	9.3	11.63	7.2
$\mathbf{D}_4$	1.88	0.06	10	1.51	4.0	4.1	10.59	7.45
US historical	–	–	–	2.28	4.79	7.64	12.14	7.52

Table 3: Model with rigidities Cost of Price Adjustment

By going through the numbers reported in the Table 3 we find that the policy rules which assures the best result in terms of welfare is given by D_1, which corresponds to Taylor Rule 4 with an expectational term and a lagged nominal interest rate. This is a different result from what has been obtained in the previous case, in a model with nominal rigidities designed according to the Calvo-Woodford method of price adjustment function.

The optimality of Rule 4 in the model with price adjustment costs, is similar to what has been obtained by Clarida, Galì and Gertler (1999) in a celebrated paper. One possible explanation could probably consider the role of expectations form the private sector in the design of the optimal monetary policy. In fact, as I have shown before, the mechanics behind the method price adjustment cost function reveals a fully microfounded nominal rigidity approach, where agents make their plans in terms of optimal pricing by looking at the future path of the economy, immediately after a shock. In fact, the cost of price adjustment function (43) is included in fully perfectly optimizing model where the single agent takes into account the future path of prices and expected inflation. Thus, the inclusion of the expected inflation rate in the monetary policy function, is a way to keep up with the inflationary expectations coming from the private sector, from the monetary authority's point of view. A similar explanation for the optimality of a monetary policy rule targeting the expected inflation term has been provided by Evans and Honkapohja (2000), though in a different paper. They show, in fact that by including the expected inflation in the monetary policy function increases the region of parameters for which we get full determinacy of the Rational Expectation equilibrium.

As for the model with the Calvo-Woodford approach for modelling nominal rigidities, I consider in Figure 3 and 4 the goodness of fit for the model with cost of price adjustment. Even in this case, the dark lines represent the actual paths for the time series of inflation, nominal interest rate and output for the US economy in the sample 1974:1-1994:4, quarterly observations, while the dashed lines indicate the simulated time series, obtained after having simulated the model 1000 times and after having discarded the first and the last 100 observations. Thus, by looking at figure 3, where I reported the plot for actual vs. simulated paths when the model includes a simple contemporaneous data Taylor Rule such as Rule 1, with the parameter configuration given by rule A_1 in Table 3. In this case, it is easy to note that a model like this does not fit well US data in the sample 1974:1-1994:4. Even if the inflation rate captures quite well the underlying trend, the same cannot be said for both nominal interest rate and output. In fact, by considering the middle and the bottom panel reported in Figure 3, we observe that the dashed lines (the simulated time series) do not capture at all the pattern of the observed time series. In particular, the nominal interest rate is highly volatile and underestimated with respect to the actual time series, and output appears to be too smoothed if compared to the original time series.

Let us consider now Figure 4, where I reported the simulated vs. actual time series for the model with a monetary policy rule such as rule 4 with the parameter configuration corresponding to the optimal monetary rule, as represented in rule D_1. By looking at the panels reported in figure 4, we observe that a monetary policy rule with an expected inflation rate, like Rule 4, is shows a better fit than the previous case. In fact, both the inflation and nominal interest rate show a trend perfectly in accord with the actual time series, though not all the various peaks and throughs observed in the reality are recorded by the simulated time series. The bottom panel of Figure 4, shows a pattern of output quite volatile, though on the average trend. In any case, Rule 4 seems to offer a better fit of the model, than the case represented by Rule 1, confirming the results obtained through the welfare analysis.

9 Impulse-Response Analysis

In this section I am going to consider some policy experiments to examine the dynamic response pattern of the main variables of the system. In each case we will simulate the response of each single variables for different values of the parameters of the monetary policy function which turned out to be optimal in each type of model according to the welfare results previously discussed. This example is illustrative of how the magnitude of the parameter can affect the pattern of response of each variables. Let us start with an expansionary government expenditure shock. A contractionary monetary policy shock will follow. For all the policy experiments here considered I assume that fiscal policy is 'Ricardian', in the sense described by Woodford (1996, 2000): taxes are set to react proportionately to the stock of real debt. To accomplish this I have set the value of the parameter of the fiscal policy reaction function equal to $\psi_1 = 0.55$.

9.1 A Government Expenditure Shock

9.1.1 Calvo-Woodford model

In figure 5 the left column represents the dynamic response of y C, i, π and b (real public debt) after a one percent shock to government expenditure, for different values of the parameter ϕ_π, of Taylor Rule 2. The right column indicates dynamic response patterns for different values of the parameters describing the tightness on the output gap ϕ_x, and the last column depicts the impulse-response patterns for changes in the parameter ϕ_i. In all cases, the dark line represents what I call the 'benchmark' case, i.e. the response patterns designed for the parameters of the optimal monetary policy rules considered for this model, which, according to the results considered in the previous section, is given by Rule 2. In this case, we know that the optimal rule is given by the parameter combination given by Rule b1 reported in Table 2, so that the dark line is obtained when $\phi_\pi = 1.48$, $\phi_x = 0.06$, and $\phi_i = 1.25$.

Let us look at the first column to the left. The dotted line is the response pattern when $\phi_\pi = 10$, $\phi_x = 0.06$, $\phi_i = 1.25$ while the dashed line is when $\phi_\pi = 20$, $\phi_x = 0.06$, $\phi_i = 1.25$. After an expansionary fiscal policy shock, we observe a strong impact on aggregate output: the higher demand from the public sector stimulates the production of more goods. The increase of aggregate demand translates into an higher level of disposable income, determining a very persistent increase of the consumption level. However, these increments of aggregate demand are inflationary: monetary authority reacts by raising the nominal interest rate to contrast the inflationary pressure and the increase of actual output with respect to the potential output. Real public debt increases for two reasons: the increase of public expenditure - not accompanied by an exact equal increase in the fiscal revenue (at least in the impact period) - creates the needs for the government to finance the new expenditure by issuing new amount of debt. Secondly, the rise of nominal interest rate, due to the non-accommodating reaction from the monetary policy side, increases the burden of interest payments.

In any case, the raise of the inflation rate does not mitigate these effects on the level of real public debt b. However, it should be noted that the increase of the public debt dies out in the long run, because I assumed a Ricardian Fiscal Policy rule. This ensures a full recovering from the initial public expenditure shock, eliminating in this way the possibility of getting an inflationary equilibrium through the increase of the burden of public debt and the increase of the probability of government default.

Similar effects can be read in the middle column of Figure 5 where I reported the impulse responses for different values of the parameter ϕ_x. The dark line still represents the case with $\phi_\pi = 1.48$, $\phi_x = 0.06$, and $\phi_i = 1.25$, the dotted line indicates $\phi_\pi = 1.48$, $\phi_x = 10$, and $\phi_i = 1.25$, while the dashed line depicts the case $\phi_\pi = 1.48$, $\phi_x = 20$, and $\phi_i = 1.25$. The similarity of the response patterns indicates the fact that raising parameter ϕ_x produces similar effects on the response patterns of almost all the variables: this confirms the results obtained by Rotemberg and Woodford (1999), by which raising ϕ_π or ϕ_x produces almost the same effect on the response pattern of the various variables of the system.

If we increase the magnitude of all the parameters of the monetary policy function in turn, we increase the ability from the monetary authority in controlling the fluctuations of all variables. In particular, when ϕ_π is very high (say 10 or 20), we need a small change in the nominal interest rate to produce a persistent deflationary phenomena. Thus, the adoption of monetary policy rules with a strong commitment on the inflation rat target, will produce a good stabilizing effect on the variables of the system around their steady state. Therefore, a very high value for ϕ_π and ϕ_x will imply a reduction of the volatility of all variables after a destabilizing shock hitting the economy.

Note also that all the Impulse-Response functions were drawn by raising the value of various coefficients in the monetary policy function, each by taking the value of all the other coefficients equal to their optimal value. This is done in order to preserve the determinacy property of the Rational Expectation equilibrium.

9.1.2 Cost of Price Adjustment

In Figure 6 I reported the impulse response functions after an expansionary policy shock in the model with cost of price adjustment. Recall that in this case the welfare analysis revealed that the optimal policy rule is given by Rule 4, i.e., the rule with a target on the expected inflation rate. As before, the dark lines in all the pictures belonging to Figure 6 represent the benchmark case, i.e. the parameter configuration for monetary policy rule 4 with expected inflation corresponding to the welfare maximizing case, indicated by the parameter configuration D_1 in Table 3. Thus, in the left column of Figure 6 I reported the pictures for changes in the parameter ϕ_π: the dark line is for $\phi_\pi = 1.88$, $\phi_x = 0.06$, $\phi_i = 1.65$, the dotted line is for $\phi_\pi = 10$, $\phi_x = 0.06$, $\phi_i = 1.65$ and the dashed line is for $\phi_\pi = 20$, $\phi_x = 0.06$, $\phi_i = 1.65$. The middle column considers the variations in parameter ϕ_x: the dark line is again for $\phi_\pi = 1.88$, $\phi_x = 0.06$, $\phi_i = 1.65$, the dotted line is for $\phi_\pi = 1.88$, $\phi_x = 10$, $\phi_i = 1.65$ and the dashed line is for $\phi_\pi = 1.88$, $\phi_x = 20$, $\phi_i = 1.65$. Finally, the right column indicates the impulse responses for changes in the parameter values of ϕ_i: the dark line is again for $\phi_\pi = 1.88$, $\phi_x = 0.06$, $\phi_i = 1.65$, the dotted line is for $\phi_\pi = 1.88$, $\phi_x = 0.06$, $\phi_i = 10$ and the dashed line is for $\phi_\pi = 1.88$, $\phi_x = 0.06$, $\phi_i = 20$.

By looking at the pictures reported in Figure 6 we note several similarities in the dynamic response of each variable to the responses derived from the model with Calvo-Woodford nominal rigidities. It is not difficult to note a better persistence of the inflation rate after the expansionary fiscal policy shock. The most important similarities can be found in the behavior of the variables following changes in the parameters of the monetary policy function: a more aggressive monetary policy towards the inflation (higher ϕ_π) will minimize the fluctuations of all the variables around their steady states, improving the ability of monetary authority in stabilizing the system. The same is true also for the increase of the values of parameters ϕ_x, ϕ_i. Remarkably, the signs of impulse-response functions are the same as those considered for the model with the Calvo-Woodford mechanism of price adjustment.

9.2 A Contractionary Monetary Shock

In what follows I consider another policy exercise represented by a contractionary monetary policy shock realized through an unexpected increase of nominal interest rate. The results are collected in Figures 7 and 8. As for the case with an expansionary public expenditure shock, the impulse-response functions are considered by changing the parameter values one per time, in order to preserve stability of the Rational Expectation Equilibrium, along the lines proposed by Woodford (2000) and Rotemberg and Woodford (1999).

9.2.1 Calvo-Woodford model

The impulse response functions for such type of exercise are reported in Figure 7, for the Calvo-Woodford model with Taylor Rule 2. The meaning of the various lines denomination are exactly the same as those we have discussed before. The impact of a monetary policy contraction through the increase of the nominal interest rate, generates a recessionary effect observed in the strong negative impact on all the variables of the system (both nominal and real). Like for the expansionary fiscal policy shock, the various effects determined by monetary policy shocks can be interpreted along the lines of a simple IS-LM model textbook based level. Thus, if we concentrate on the dark line, which still represents the model with the parameters for the optimal monetary policy rule, we observe that after the contractionary monetary shock (through the increase of the nominal interest rate), we observe a strong negative impact on output, consumption, and a marked reduction of the inflation rate. Together with this, we observe a sharp increase in the level of the real debt, caused by the increase in the burden of interest payment due to the increase of the interest rate.

Compared with the increase of public debt observed in Figures 5 and 6, here we note a much lower persistence for the recovering of public debt towards the equilibrium.

If we look at the effects of changing the parameters of the Taylor Rule 2 here adopted, we do not observe significant differences with the previous case. Higher are the parameters values, slower will be the fluctuations of the variables. Thus, even in this case we observe the stabilization property assigned to he monetary policy function. Similar results are obtained also for changes in the persistence parameter ϕ_i, whose results are reported in the right column of Figure 7.

9.2.2 Cost of Price Adjustment

Figure 8 collects the impulse response functions for this type of model, where the optimal monetary policy rule is given by Taylor Rule 4 (see equation (56)), with a target on the expected inflation and a lagged interest rate. For the model where nominal rigidities are modelled through the price adjustment costs à la Rotemberg (1982), we find a set of results not too dissimilar from those reported in Figure 7 for the model with Calvo-Woodford price rigidities.

Even in this case, the contractionary monetary policy shock substantiates into a marked and persistent increase of the nominal interest rate, which causes the recessionary effect on the economy: output, consumption and inflation decline. Real debt increases because of the increased cost of paying the burden of interest payment, and because of the reduced inflaiton rate. It is worth to note that this case shows a much higher persistence of the various responses of the variables, after the monetary policy shock. In particular, inflation shows an higher level of persistence, together with output and real debt (as a consequence of the persistence in the reduction of the inflation rate). The persistence showed by these variables is an argument in favor of modelling nominal rigidities by using the cost of price adjustment approach, given the lack of internal propagation mechanism deriving from the simple monetary business cycle with Calvo-Woodford approach. Another element which is worth to note is the strong effects generated by monetary policy: after a contractionary monetary shock all the variables show significant fluctuations.

The changes of the parameter values produce similar effects as those displayed by the model with Calvo-Woodford approach. Thus, even in this case, by increasing the magnitude of the parameters of the monetary policy function helps to minimize the fluctuations of the variables of he system around their trend.

10 Concluding Remarks

In this paper I compared the search for an optimal policy rules for two distinct models, in order to detect if the way by which nominal rigidities enter in the model makes a significant difference. The models under comparison where two simple monetary RBC models without capital accumulation: the first models nominal rigidities through the Calvo (1983) mechanism of price adjustment, where the revision of the final good price is exogenously given by a fixed fraction of firms populating the economy. In other words, in each period prices are changed through fixed intervals, exogenously given. The second model consider the nominal rigidities through the presence of quadratic price adjustment costs, along the lines proposed by Rotemberg (1982). Both models share a similar structure of the economy: money enters via transaction costs in the Representative agent's budget constraint, the production function includes a fixed cost component interpreted as a business set up cost. Both models contain several features which makes the natural level of output inefficient. Additionally, according to the recent contribution by Fiscal Theory of the Price Level, I assume a fiscal policy configuration defined 'Ricardian' or 'Passive'.

Withe these features at hand, the results from model simulations show that the choice of the modelling nominal rigidities is not indifferent to the determination of the optimal monetary policy rule: with rigidities modelled à la Calvo (1983), the optimal monetary policy rule turns out to be an interest rate pegging rule with contemporaneous inflation and output gap target together with a lagged interest rate to account for the persistency on nominal interest rate. However, in the model with cost of price adjustment, the optimal monetary policy rule is identified with an interest rate pegging rule whose argument are the expected inflation a contemporaneous level of the output gap and a lagged nominal interest rate. The optimality result is obtained through a grid search for the configuration of parameters which maximizes the utility of the representative agent, along the lines recently discussed by Woodford (2000) and Sims (2000). Various impulse responses of the main variables of the system are showed for different values of the parameters of the monetary policy function when the economy is hit by two types of shocks: and expansionary fiscal policy shock ad a contractionary monetary policy shock.

Overall, the method based on the price adjustment cost seems to guarantee better results also in terms of a better internal propagation mechanism of the model. These results for the first time show a critical comparison of the two method of modelling rigidities, and represent a step ahead towards the definition of the elements of a well established microfounded model to be used as a benchmark in comparative policy evaluation.

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Appendix

Coefficients for equation (57)

$$
\begin{aligned}
\alpha_g &= \frac{\eta_{cg}}{\eta_{cy}} \\
\alpha_i &= \frac{\eta_{mi}\left(\eta_{c\lambda}\eta_{cm} - \eta_{m\lambda}\right)}{\eta_{cy}\left(\eta_{m\lambda} - \eta_{c\lambda}\right)} \\
\alpha_\xi &= \frac{\eta_{c\lambda}\eta_{cm} - \eta_{m\lambda}\left[2\left(1 - \eta_{cm}\right) - 1\right]}{\eta_{cy}\left(\eta_{m\lambda} - \eta_{c\lambda}\right)} \\
\alpha_\pi &= \frac{\left(1 - \eta_{cm}\right)}{\eta_{cy}\left(\eta_{m\lambda} - \eta_{c\lambda}\right)} \\
\gamma_i &= \frac{\eta_{mi}\left[\eta_{c\lambda}\eta_{cm} - \eta_{m\lambda} - 2\eta_{mi}\left(1 - \eta_{cm}\right)\right]}{\eta_{cy}\left(\eta_{m\lambda} - \eta_{c\lambda}\right)}
\end{aligned}
$$

Fig.1: Actual vs. Simulated Paths – Rule 1, Calvo Approach

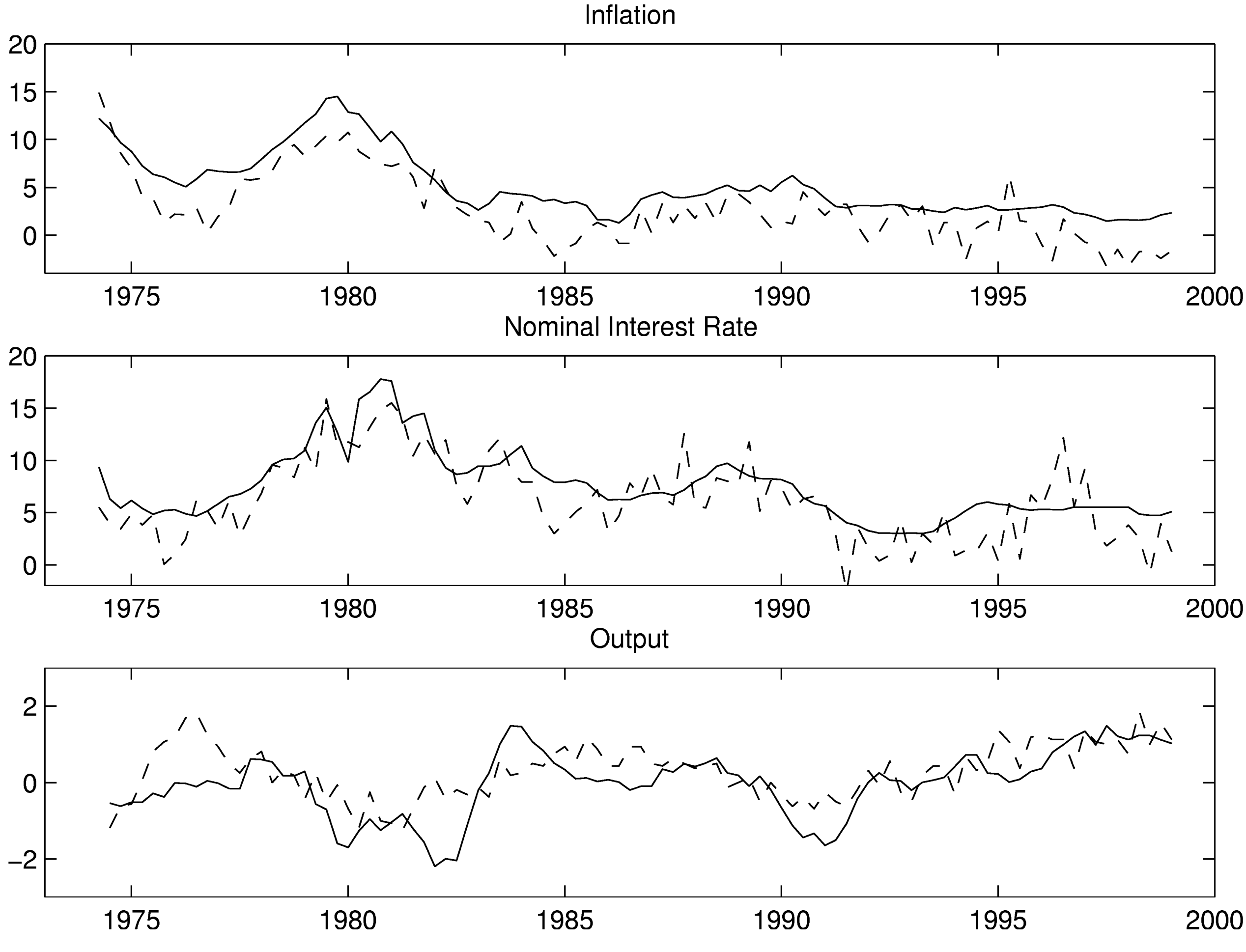

Fig.2: Actual vs. Simulated Paths – Rule 2, Calvo Approach

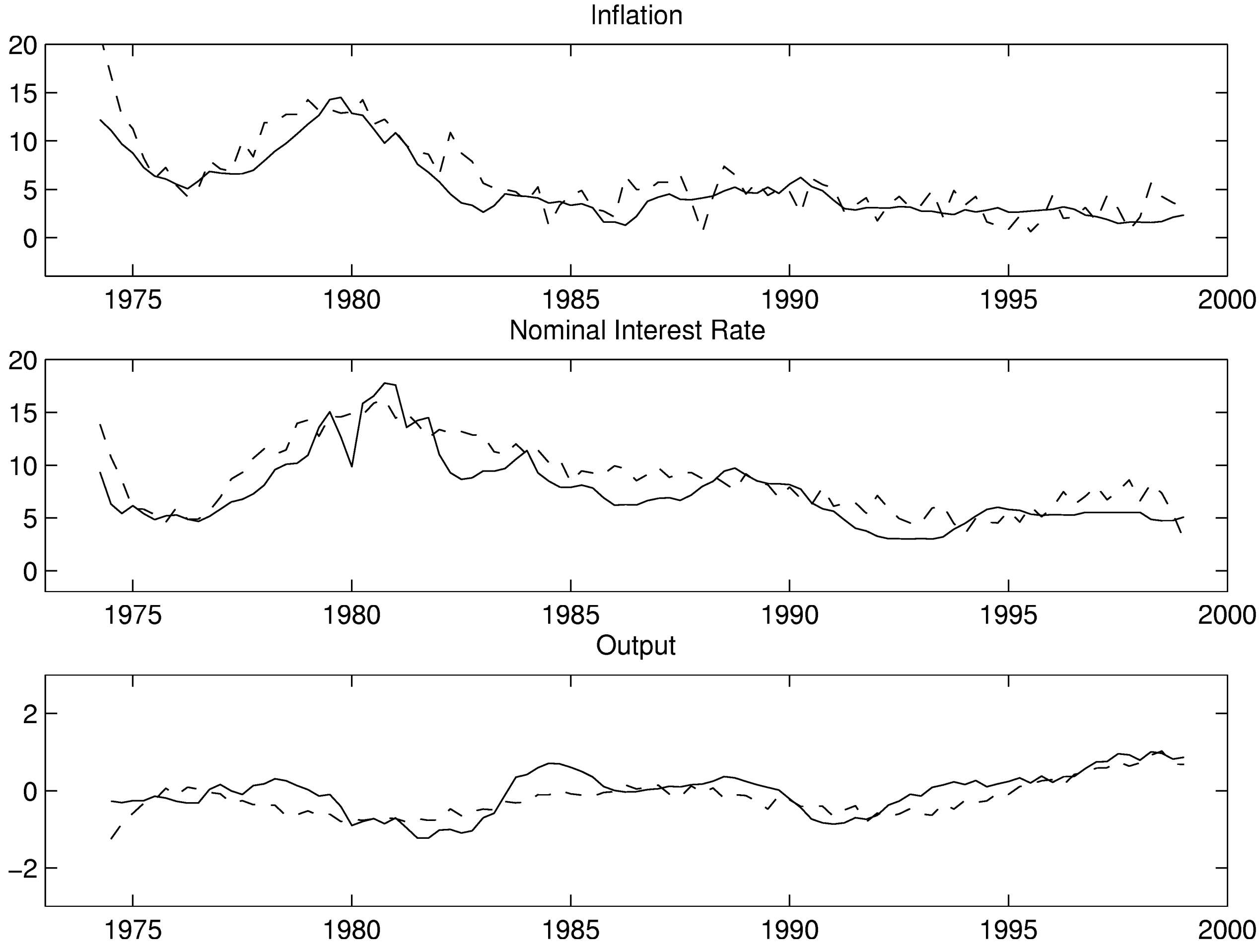

Fig.3: Actual vs. Simulated Paths −Rule 1, CAP

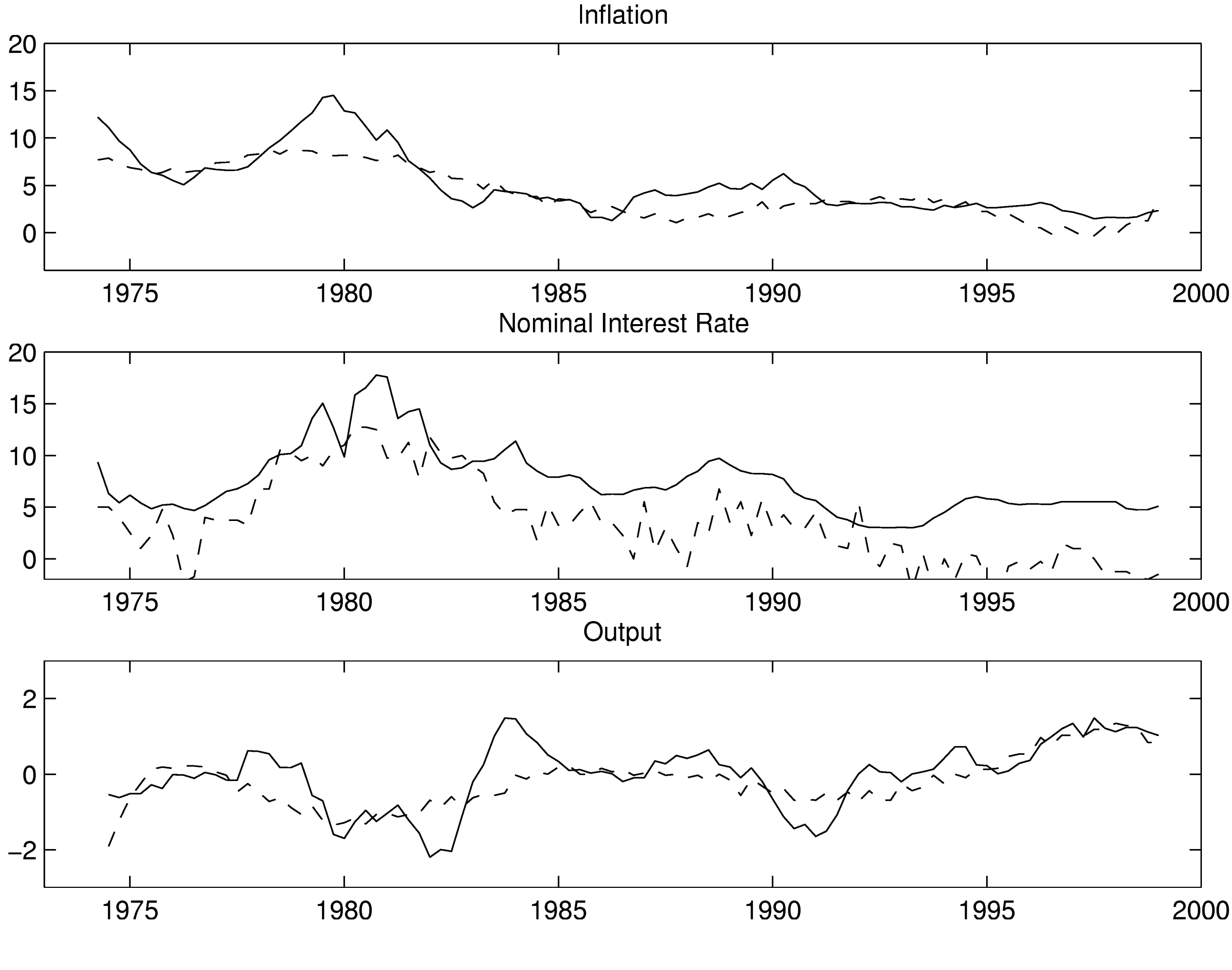

Fig.4: Actual vs. Simulated Paths – Rule 4, CAP

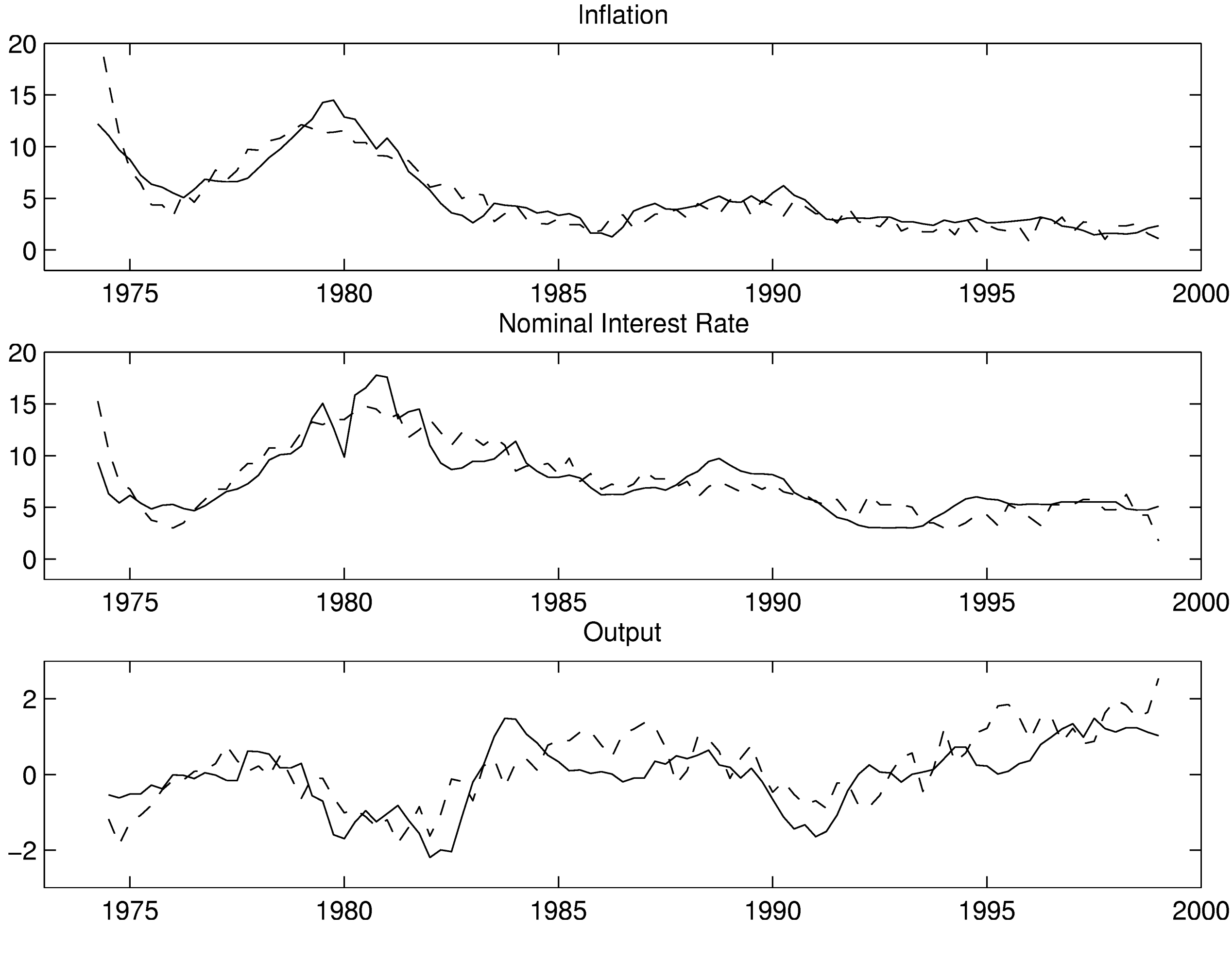

Figure 5. Fiscal Policy Shock: Rule 2

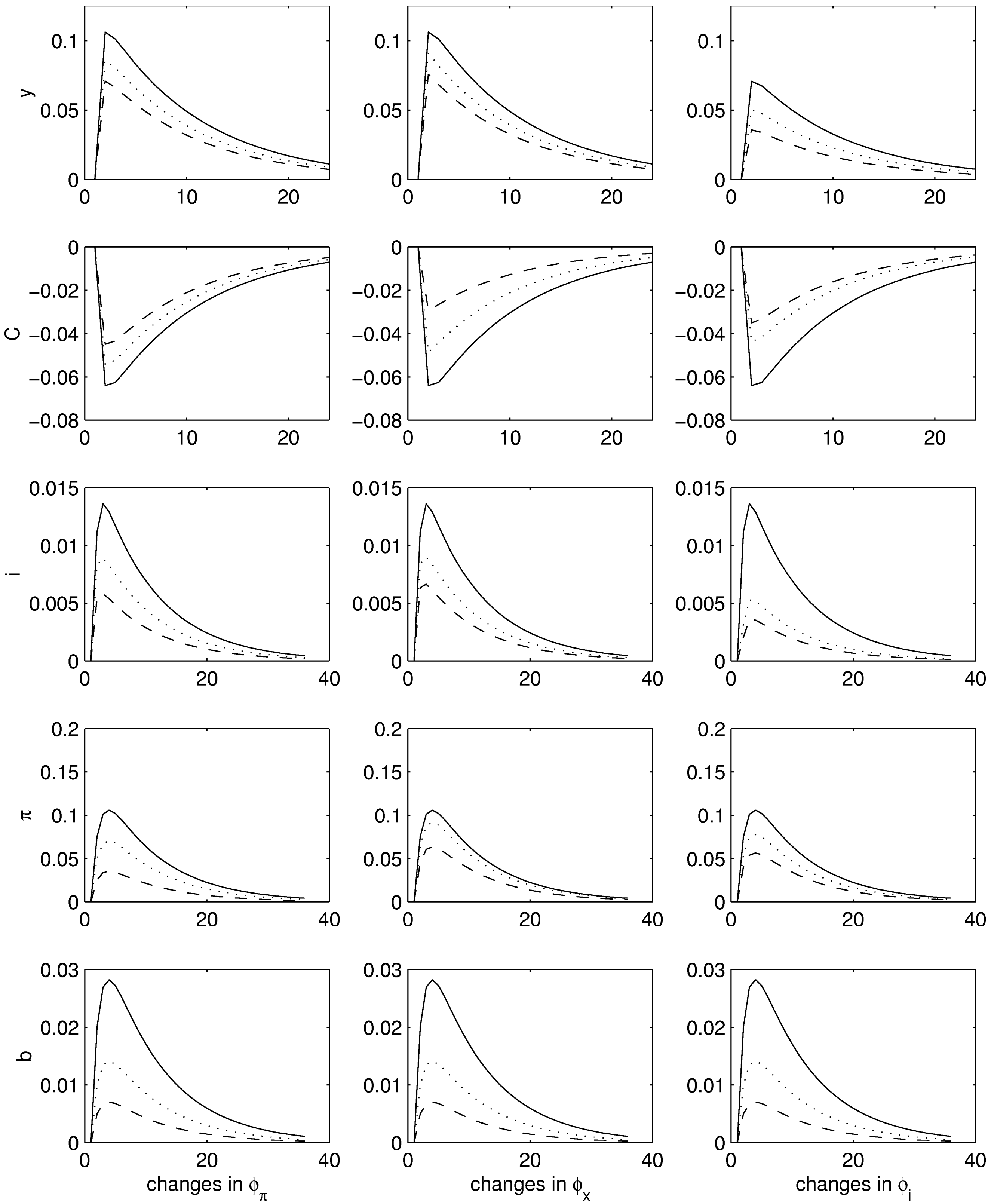

Figure 6. Fiscal Policy Shock. Rule 4 − CAP

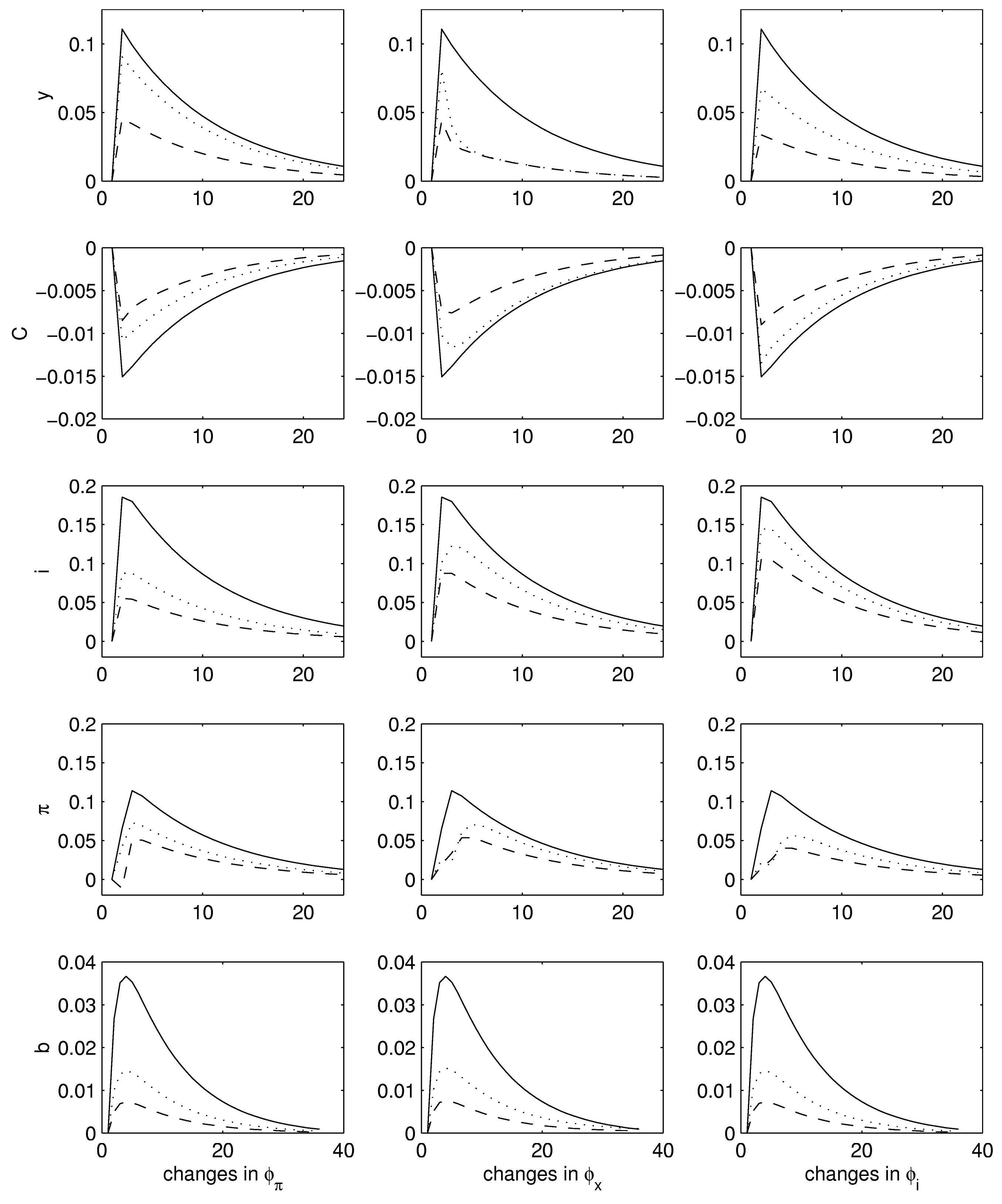

Figure 7. Monetary Policy Shock: Rule 2. C−W model

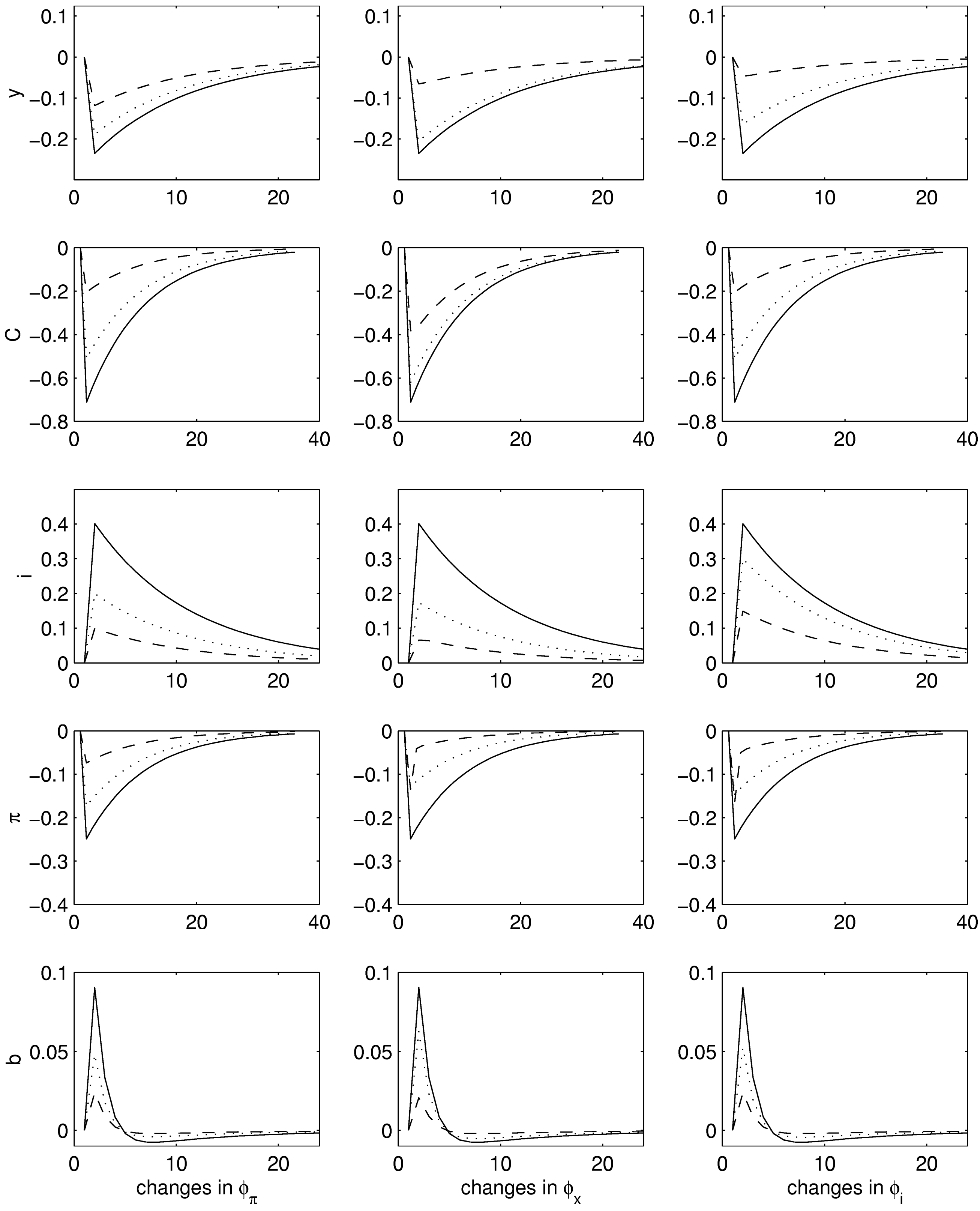

Figure 8. Monetary Policy Shock – Rule 4, CAP