INDUSTRY DYNAMICS AND THE DISTRIBUTION OF FIRM SIZES: A NON-PARAMETRIC APPROACH*

by

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Abstract

The aim of this paper is to analyze the evolution of the size distribution of young firms within some selected industries, trying to assess the empirical implications of different models of industry dynamics: the model of *passive learning* (Jovanovic 1982), the model of *active learning* (Ericson and Pakes, 1995), and the *evolutionary* model (Audretsch, 1995). We use a non-parametric technique, the Kernel density estimator, applied to a data set from the Italian National Institute for Social Security (INPS), consisting in 12 cohorts of new manufacturing firms followed for 6 years. Since the patterns of convergence to the limit distribution are different between industries, we conclude that the model of passive learning is consistent with some of them, the active exploration model with others, the evolutionary model with all of them.

Keywords: Cohorts; Gibrat's Law; Kernel; Industry Dynamics; Non-parametric; Shakeouts.

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1 - Introduction

Analysis of the size-growth relationship is a commonly used approach to the study of the evolution of market structure. In fact, the firm size distribution (FSD) has received considerable attention - since the seminal works of Herbert Simon and his co-authors between the late 1950s and the 1970s (cf. Simon and Bonini, 1958; and Ijiri and Simon, 1964, 1977) - in most theoretical and empirical studies dealing with the overall process of industry dynamics. The empirical evidence showed a FSD highly skewed to the right, meaning that the size distribution of firms is lognormal, both at the industry level and in the overall economy. This piece of evidence is coherent with the so-called Law of Proportionate Effect (or Gibrat's (1931) Law): as Simon and Bonini (1958) point out, if one "…incorporates the law of proportionate effect in the transition matrix of a stochastic process, […] then the resulting steady-state distribution of the process will be a highly skewed distribution".

Recent evidence based on more complete data sets, suggests that Gibrat's Law is not confirmed, either for new-born or established firms (for a survey, cf. Geroski, 1995; Lotti *et al*., 1999), since smaller firms grow more than proportionally with respect to larger ones. This finding should be consistent with a departure of the FSD from the lognormal distribution.

In this paper - using quarterly data for 12 cohorts of new manufacturing firms - we account for the evolution of the FSD over time in the case of young firms. Moreover, we try to assess the empirical implications of different models of industry dynamics. The work is organized as follows. Section 2 contains a review of the empirical evidence about Gibrat's Law and the FSD, as well as an overview of some recent models of industry dynamics. Section 3 describes the data and the methodology used, whereas Section 4 summarizes the main empirical findings. Finally, Section 5 contains some concluding remarks.

2 - Theory or Stylized Facts?

Gibrat's Law, applied to the analysis of market structure, represents the first attempt to explain in stochastic terms the systematically skewed pattern of the size distribution of firms within an industry (Sutton, 1997). In effect, the Law cannot be rejected if *a)* firm growth follows a random process and is independent from initial size, and *b)* the resulting distributions of firms' size are lognormal[1]. Although, from a theoretical viewpoint, labeled as "unrealistic" since Kalecki's (1945) study on the size distribution of factories in US manufacturing, this result was initially consistent with some empirical studies dealing with incumbent, large firms (Hart and Prais, 1956; Simon and Bonini, 1958; Hymer and Pashigian, 1962). In recent years, most studies have instead shown that these exhibit a different behavior, identifying an overall negative relationship between initial size and post-entry rate of growth (cf., among others, Mata, 1994; Hart and Oulton, 1999). However, Lotti *et al*. (2001) found that, in the case of new-born firms, the growth rates are negatively correlated with their initial size only during their infancy: Gibrat's Law fails to hold in the years immediately following start-up, when smaller firms have to rush in

[1] Of course, a FSD skewed to the right implies only that Gibrat's Law cannot be rejected. However, one cannot *a priori* exclude that the skewness is the result of *turbulence*, namely of the presence of new-born small firm in the right tail of the distribution.

order to reach a size large enough to enhance their likelihood of survival; but in the subsequent years, the patterns of growth of entrants do not differ significantly from the landscape of the industry as a whole.

One possible way to explain this phenomenon of self-selection, is to consider the firms' learning and evolution processes put forward by Jovanovic (1982), Ericson and Pakes (1995), and Audretsch (1995). By following such perspectives, entrants are uncertain about their relative level of efficiency, and only once into the market they learn about their possibilities of survival and growth. The main advantage of these models is that they allow for *a)* heterogeneity among firms, *b)* idiosyncratic sources of uncertainty and discrete possible events, *c)* entry and exit.

Boyan Jovanovic's model of *passive learning* hypothesizes that firms are initially endowed with uncertain, time-invariant characteristics (i.e. efficiency parameters), of which the firm *does not* know the distribution. But, once into the market, the firm learns *passively* about the true efficiency parameter. As a consequence, in every period the firm has to decide its strategy: whether to exit, continue with the same size, grow in size, or reduce its productive capacity. One of the consequences of this model is that, due to a particular kind of selection process, the most efficient firms survive and grow, while the others are bound to shrink or to exit from the market.

Like in the *passive learning* model, Richard Ericson and Ariel Pakes's model of *active learning* (1995) assumes that all the decisions taken by the firms are meant to maximize the expected discounted value of the future net cash, conditional on the current information set. Unlike in Jovanovic's model, a firm knows its own characteristics *and* its competitors' ones, along with the future distribution of industry structure, conditional on the current structure. Accordingly, this model can be usefully employed in explanation of 'entry mistakes' (as defined by Cabral, 1997), namely the fact that in every period and every industry more firms enter than the market can sustain. Within an *active learning* perspective, such mistakes occur due to lags in observation of rivals' entry decision or just because entry investments take time (Cabral, 1997). In a subsequent work, Pakes and Ericson (1998), using two cohorts of firms from Wisconsin, belonging to the retail and the manufacturing industries, found that the structure of the former industry is compatible with the passive learning model, while that of the latter with their model of active exploration (learning). The retail cohort, after eight years seems to have reached the size distribution of the industry as a whole, while the manufacturing one, even if showing higher growth rates, after that period is still far from the limit distribution.

David Audretsch (1995) expanded the *passive learning* approach put forward by Jovanovic (1982) into an *evolutionary* perspective, allowing for inter-industry differences in the likelihood of survival of newborn firms. Accordingly, industry-specific characteristics, such as scale economies and the endowment of innovative capabilities, exert a significant impact on entry, exit, and the likelihood of survival of newborn firms. For example, in industries characterized by higher minimum efficient scale (MES) levels of output, smaller firms face higher costs that are likely to push them out of the market within a short period after start-up. Thus, only the most efficient among newborn firms will survive and grow, whereas the other are pushed out of the market (cf. Audretsch *et al.*, 1999). In this case, the presence of more potential entrants than firms with a significant likelihood to survive in the long run can bring about a shakeout (cf. Klepper and Miller, 1995). In turn, a shakeout occurring at a certain point in the industry's history is likely to affect the long-run size distribution of firms within the same industry, depending on "how the opportunities vacated by exited firms are reallocated among surviving firms" (Sutton, 1998, p. 260; cf. also Klepper and Graddy, 1990). Conversely, in industries with a lower MES level of output the likelihood of survival will be independent of the firms' ability to grow (cf. Amaral *et al*., 1977; Brock, 1999).

With this theoretical and empirical background in mind, we look at the evolution of 12 cohorts of newborn firms in selected industries, in order to analyze the process of convergence of the firm size distribution, in terms of number of employees, with respect to the overall industry landscape. The aim of this analysis is to show *i)* whether the findings by Herbert Simon and his co-authors concerning the Skewness to the right of the FSD are confirmed also in the case of newborn, small firms and *ii)* whether the FSD resulting from application of the Kernel density estimator is consistent with models of industry dynamics - such as those surveyed above - which identify in the learning processes occurring at the firm level, and in the level of sunk costs that characterizes each industry, some possible theoretical explanations for these facts.

3 - Data and Methodology

We use a data set from the Italian National Institute for Social Security (INPS), dealing with 12 cohorts of new manufacturing firms (with at least one paid employee) born in each month of 1987, and their follow up until December 1992.

Since all private Italian firms are compelled to pay national security contributions for their employees to INPS, the registration of a new firm as "active" signals an entry into the market, while the cancellation of a firm denotes an exit (this happens when a firm finally stops paying national security contributions). For administrative reasons - delays in payment, for instance, or uncertainty about the current status of the firm - some firms are classified as "suspended". In the present work we consider these suspended firms as exiting from the market at the moment of their transition from the status of "active" to that of "suspended", while firms which have stopped their activity only temporarily were included again in the sample once they turned back active. We carry out also an accurate cleaning procedure, aimed at identifying internal inconsistencies and entry or exit due to firm transfers and acquisitions. As regards acquisitions, these are denoted as "extraordinary variations" in the INPS database, and firms involved in such activities can therefore be easily identified and cancelled from the database itself. A correct identification of firms disappeared via acquisitions permitted to avoid acquiring firms to be drawn disproportionately from the low end of the size distribution. As pointed out by Sutton (1998; cf. also Hart and Prais, 1956; Hymer and Pashigian, 1962) this would have caused a violation in the proposed bound and altered the significance of the overall analysis.

We focus our analysis on four industries - Electrical & Electronic Engineering, Instruments, Food, and Footwear & Clothing - mainly for two main reasons: the first one concerns their very different market structure in terms of cost of entry (sunk costs), and the second the fact that the latter two industries are less technologically progressive than the former two ones[2].

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[2] And this would allow to draw some conclusions on whether the FSD is or is not sensitive to technological factors.

To examine the effect of firms' age on the distribution of their sizes, we study each cohort at each quarter after start-up, and this for their first six years in the market. In Tables 1A-1D and in Table 2 some descriptive statistics are reported. In general, all industries experience a shakeout period during which the number of survivors, among new entrants, declines by 40 per cent or more. From Tables 1A, B, C, and D it turns out that, on average, the survival rate at the end of the period (i.e., after 21 quarters) is much higher within the cohorts belonging to the Electrical & Electronic Engineering and the Instruments industries, than it is the case with the Food and the Footwear & Clothing industries. Thus, consistently with Audretsch's (1995) hypothesis, industry specific characteristics, such as the commitment to innovative activities, seem to set in motion a pre-entry selection mechanism that selects only those start-ups that find in their endowment of innovative capabilities a possible competitive advantage.

Looking at Table 2 Figure 1, one immediately observes that - with the sole exception of the Food industry - the standard deviation of firm sizes is much higher at the end of the relevant period than in the first quarter. Dispersion of firm sizes tends therefore to widen as surviving firms reach the MES level of output and specialize in one of the many clusters of products which - according to John Sutton's (1998, pp. 597-605) "independent submarkets" hypothesis - characterize each industry. In turn, firm size increases along with its age for the Electrical & Electronic Engineering and the Instruments industries, but only for the first 13 and 12 quarters respectively, corresponding with a period comprised approximately between December 1989 and January 1991[3]. Afterwards, a decline in average firm size emerges, which is consistent with views of recessions (the period between 1991 and 1993 has been characterized in Italy by a significant slowdown in the GDP growth rates) as times of "cleansing" (cf. Boeri and Bellmann, 1995). In fact, the sectoral data reported in Table 3 show for both industries a significant decrease in the growth rates of value added since 1989, with a trough in 1993. This pro-cyclical pattern of the average firm size is even more marked in the Footwear & Clothing industry, in which the average size starts to decline after the eight quarter in the market (as early as 1989, that is the initial year of the recession). The data on the Footwear & Clothing industry show a substantial stability of the average firm size over time. This result is to some extent consistent with the dynamics of value added in the same industry: Table 3 points out alternate peaks and troughs in the Footwear & Clothing industry growth rates that are unlikely to affect firm size, since this needs time to adjust its patterns to variations in value added.

In a recent paper by Machado and Mata (2000) the Box-Cox quantile regression method is used to estimate the distribution of firm sizes and, accordingly, to analyze industry dynamics in Portugal. This approach consists in modeling each quantile as a function of a number of industry characteristics that are expected to affect firm size. Since our database doesn't provide any details about industry characteristics, in the present study we use instead a non-parametric approach. The basic idea is to look if, with the passing of time, the empirical distribution of firm sizes converges towards a lognormal distribution, under the hypothesis that this represents the limit distribution. To characterize the distribution, we used the Kernel density estimator (Pagan and Ullah, 1999), which can be summarized as follows.

[3] In effect, since the 12 cohorts include firms born in each month of 1987, each column in Table 2 deals with all firms and all cohorts.

Table 1A - Number of firms active at the end of each quarter – Electrical & Electronic Engineering

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21
Cohort 1	128	125	121	120	117	113	112	109	108	107	106	105	104	103	102	102	97	95	93	92	90
Cohort 2	64	61	59	56	53	51	52	51	50	50	50	50	49	47	44	43	40	38	36	37	38
Cohort 3	72	68	65	62	60	61	61	61	57	55	53	53	53	51	51	48	48	48	48	47	43
Cohort 4	49	46	47	47	47	47	45	43	43	43	42	41	40	41	41	39	38	34	33	33	33
Cohort 5	59	53	53	52	53	50	50	47	46	48	46	44	44	43	41	40	37	37	35	34	34
Cohort 6	71	68	65	64	62	62	63	59	58	55	49	49	49	48	47	45	44	42	41	37	36
Cohort 7	41	41	41	41	39	38	38	37	37	36	34	30	30	29	28	27	27	27	25	24	23
Cohort 8	18	18	18	17	17	17	17	17	16	15	15	15	15	15	14	14	14	14	14	12	12
Cohort 9	72	67	63	63	64	62	60	58	58	57	57	57	55	56	52	52	53	52	50	50	49
Cohort 10	60	58	54	50	49	50	52	49	47	47	44	44	44	41	42	42	42	42	40	39	38
Cohort 11	57	53	55	53	53	51	51	51	50	48	46	46	43	42	40	41	39	38	39	39	39
Cohort 12	29	28	26	25	25	25	25	26	25	25	24	23	23	23	23	22	22	22	21	20	19
Total	720	686	667	650	639	627	626	608	595	586	566	557	549	539	525	515	501	489	475	464	454

Table 1B - Number of firms active at the end of each quarter – Instruments

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21
Cohort 1	62	61	60	60	59	56	56	56	55	53	51	51	50	50	48	46	43	41	40	42	40
Cohort 2	38	37	35	36	35	35	34	34	34	33	32	29	28	27	27	27	26	24	24	25	25
Cohort 3	34	32	33	33	31	31	30	30	28	27	27	26	24	23	22	21	19	20	20	20	20
Cohort 4	26	26	25	24	23	23	20	19	19	18	18	17	17	17	17	16	17	17	17	17	17
Cohort 5	20	20	20	19	19	19	19	19	18	19	18	17	17	15	14	14	14	14	14	13	13
Cohort 6	33	33	32	31	28	28	28	27	27	25	24	23	21	21	21	21	21	21	21	17	19
Cohort 7	35	34	30	30	30	28	27	25	25	25	24	25	25	24	23	23	22	21	21	22	22
Cohort 8	11	11	10	10	10	10	10	10	10	10	10	10	10	10	10	10	8	7	7	7	6
Cohort 9	27	27	25	24	24	23	23	23	23	22	22	22	21	20	20	20	19	20	18	18	18
Cohort 10	32	30	28	26	26	27	25	24	23	24	22	21	21	20	19	18	18	18	18	17	17
Cohort 11	26	25	25	24	24	22	22	19	19	19	18	17	17	17	17	17	16	16	15	15	15
Cohort 12	18	18	17	16	15	14	14	14	14	14	14	13	13	12	11	11	11	11	11	11	10
Total	362	354	340	333	324	316	308	300	295	289	280	271	264	256	249	244	234	230	226	224	222

Table 1C - Number of firms active at the end of each quarter – Food

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21
Cohort 1	93	88	88	83	78	76	73	72	70	70	68	67	65	63	61	59	58	56	57	55	54
Cohort 2	47	43	40	37	34	34	33	33	29	28	28	27	24	24	24	24	22	23	23	21	21
Cohort 3	46	43	42	39	40	37	37	34	34	33	30	27	26	27	25	21	21	23	23	19	19
Cohort 4	40	35	30	29	30	29	29	29	28	28	29	27	26	25	23	19	19	20	20	19	19
Cohort 5	41	38	35	33	34	35	34	32	29	28	27	27	25	24	23	22	22	21	21	21	19
Cohort 6	44	42	37	35	32	29	29	29	28	28	25	25	25	25	25	25	24	24	24	24	22
Cohort 7	46	35	35	34	38	35	33	33	35	30	30	27	25	24	24	23	22	21	22	22	21
Cohort 8	20	16	15	15	14	13	12	8	9	8	8	8	8	8	8	8	9	7	7	7	7
Cohort 9	30	27	22	19	20	19	18	17	18	19	17	18	16	17	15	15	14	15	14	13	13
Cohort 10	51	40	34	32	32	30	30	26	29	26	23	24	26	21	19	18	23	19	18	16	19
Cohort 11	110	65	53	47	72	49	42	40	67	40	32	31	40	33	31	30	57	38	30	28	43
Cohort 12	80	42	23	23	47	29	21	18	49	19	12	12	22	10	10	9	37	25	12	11	27
Total	684	514	454	426	471	415	391	371	425	357	329	320	328	301	288	273	328	292	271	256	284

Table 1D - Number of firms active at the end of each quarter – Footwear & Clothing

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21
Cohort 1	164	159	158	156	145	143	136	132	129	126	121	120	113	112	110	110	103	100	98	95	93
Cohort 2	92	89	84	80	74	69	68	67	61	55	55	55	53	50	46	46	43	42	40	37	35
Cohort 3	85	79	76	73	71	65	62	60	59	56	51	50	48	45	45	41	40	40	38	38	37
Cohort 4	97	91	83	77	72	70	69	64	64	62	58	51	51	45	40	40	37	36	35	34	34
Cohort 5	100	93	86	83	83	79	78	74	74	70	68	66	67	65	59	55	55	48	40	49	45
Cohort 6	89	87	81	77	74	72	72	70	69	64	63	59	58	53	51	50	49	45	44	43	41
Cohort 7	88	80	73	69	69	65	63	60	57	55	54	55	53	52	48	44	43	43	42	41	41
Cohort 8	36	28	24	26	25	23	22	23	22	21	19	18	17	16	15	13	13	13	13	12	12
Cohort 9	97	95	87	84	78	75	70	68	67	63	65	63	60	59	57	56	55	55	52	51	49
Cohort 10	104	99	88	81	78	75	78	71	66	62	61	62	61	56	56	55	54	52	46	46	43
Cohort 11	96	93	86	78	75	68	63	61	61	57	54	51	49	47	43	41	40	40	38	37	34
Cohort 12	51	46	43	41	39	35	34	34	35	31	29	27	28	28	27	26	26	26	26	24	20
Total	1099	1039	969	925	883	839	815	784	764	722	698	677	658	628	597	577	558	540	522	506	484

Table 2 – Average Size and Standard Deviation, each quarter, all industries.

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21
Electrical & Electronic Eng.																					
Average Size	4.61	6.33	7.23	7.77	8.24	8.78	9.11	9.23	9.48	9.69	9.98	9.91	10.51	10.42	10.53	10.34	9.81	9.84	9.73	9.67	9.66
Standard Deviation	9.01	10.89	12.45	13.24	14.15	15.7	16.06	16.02	16.25	16.87	17.56	18.47	28.27	31.86	31.53	31.06	28.88	29.92	28.52	30.23	29.03
Instruments																					
Average Size	3.37	4.66	6.02	7.31	7.9	8.2	8.63	9.14	9.36	9.37	9.43	9.72	9.59	7.91	8.01	8.15	8.05	7.97	8.07	9.68	9.85
Standard Deviation	7.77	11.03	15.77	20.97	25.21	25.98	27.29	29.02	29.39	29.83	29.67	30.47	29.97	17.79	17.72	17.85	17.62	17.47	18.05	36.59	37.3
Food																					
Average Size	4.15	4.39	4.44	4.43	4.66	4.65	4.49	4.46	4.87	4.6	4.53	4.43	4.59	4.38	4.31	4.22	4.52	4.28	4.21	4.06	4.16
Standard Deviation	8.28	8.51	10.16	10.15	9.72	10.04	9.4	9.4	9.74	9.63	10.29	10.45	10.83	10.65	11.04	11.18	11.77	11.33	11.47	11.43	11.45
Footwear & Clothing																					
Average Size	6.31	8.67	9.36	9.76	9.78	9.81	9.76	9.88	9.81	9.64	9.39	9.28	9.16	9.14	8.91	8.76	8.68	8.43	8.08	7.74	7.17
Standard Deviation	10.26	13.95	14.85	16.03	16.29	16.98	17.34	17.7	17.7	18.02	17.84	17.83	17.81	18.32	15.57	18.62	18.87	18.92	18.59	18.32	17.5

Figure 1 – Average Size and Standard Deviation, by industry.

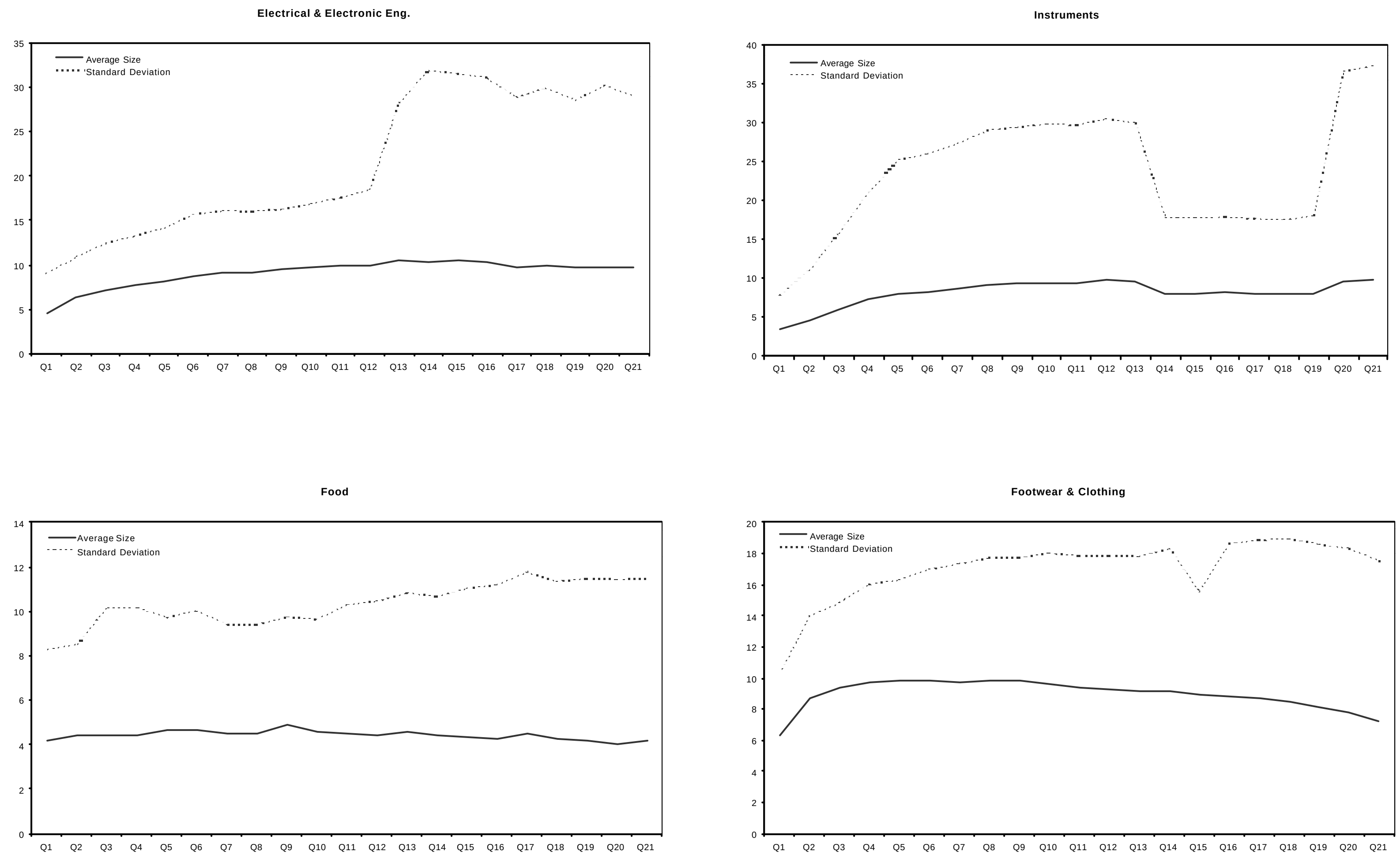

Table 3 – Growth rates (%) of Value added in constant (1995) prices

Industries	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**
Electrical & Electronic Engineer.	- 0,5	2,9	5,1	3,9	0,2	- 1,3	0,2	- 8,7	6,1
Instruments	5,9	5,3	7,8	4,9	3,7	2,6	- 1,2	- 4,2	6,5
Food	7,0	2,3	5,8	2,4	5,9	3,3	7,4	1,7	0,0
Footwear & Clothing	0,2	2,4	4,7	1,1	2,6	1,8	0,4	- 2,7	6,8

Source: ISTAT, National Statistical Office of Italy

Let *f(x)* be the unknown density to be estimated. In such a non-parametric approach, there is no need to postulate the true parametric distribution of *f*, while *f(x)* is directly estimated through the data. As a consequence, the estimates will have a stepwise nature.

The general formulation of a Kernel density estimator is:

$$\hat{f}(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x_i - x}{h}\right) = \frac{1}{nh}\sum_{i=1}^{n} K(y_i)$$

where the Kernel function *K*(•) is defined in such a way that:

$$\int_{-\infty}^{\infty} K(y_i)dy = 1, \text{ and } \qquad y_i = \frac{(x_i - x)}{h}$$

with *h* denoting the window-width (or the smoothing parameter, or band-width) and *n* the size of the sample (see the Appendix for further details). In this case, we used as Kernel function the Gaussian density.

Accordingly, for each quarter, and each industry, we estimated the distribution of the logarithm of the firms' size, and checked if a tendency towards a normal distribution does emerge. In order to test statistically the conformity of the empirical distribution to the normal, we used a simple test based on the degree of Skewness and Kurtosis, as well as a joint test (D'Agostino *et al*., 1990) for normality.

4 - Empirical Findings

The alleged structural and technological differences among the industries taken into account allow for the somewhat contradictory results obtained from the Kernel density estimates. Thus, for the Electrical & Electronic Engineering industry, the shape of the normal distribution begins to emerge after the 8^{th} quarter, as confirmed by the normality test (see Table B.1 in Appendix B). The convergence towards the normal distribution begins instead to be clear only after the 13^{th} quarter in the case of the Instruments industry (see Table B.2 in Appendix B). Thus, firm's age is a major factor affecting the FSD in these industries: as the normal distribution of sizes is reached with the passing of time, Gibrat's Law turns out to hold when firms are in their second and third year in the market, respectively for the Electric & Electronics and the Food industries.

Different is the case of the Food and the Footwear & Clothing industries (see Table B.3 and B.4 in Appendix B), for which no significant patterns of convergence do emerge. After 6 years of observation, these two industries are still far from the limit distribution and, moreover, the distributions of firm sizes are still bimodal. In the Footwear & Clothing industry, in particular, the shakeout after entry is less drastic than in the former two industries. For this reason, at the end of the relevant period, the FSD exhibits two modes: one identifies the "core" of the industry, while the second is located at the fringe of the industry, suggesting the existence of an *evolutionary* process of *active learning* that allows firms below the MES level of output to survive and grow.

A possible explanation of the contrasting results obtained for the two groups of industries is that the selection and learning processes are much slower in the traditional consumer goods industries than it is the case with two technologically progressive industries such as the Electrical & Electronic Engineering and the Instruments ones. Thus, in the Food and the Footwear & Clothing industries the process of industry dynamics should be allowed to run for more periods before a convergence to the normal distribution begins to emerge. Unfortunately, our data do not allow observing the behavior of newborn firms in these industries beyond their 21st quarter in the market.

With the aim of measuring the evolution of the FSD over time, we looked also at the moments of this distribution. In particular, we studied the patterns of evolution of the Skewness and Kurtosis indexes, to see if and how a convergence to the normal distribution does emerge. The results (summarized in Appendix C below) confirm, coherently with the Kernel estimations, the different patterns of the evolution of the size distribution of firms in the various industries. Accordingly, following Pakes and Ericson (1998), we may argue that the evolution of the FSD in the Food and the Footwear & Clothing industries is consistent with the *active learnin*g model, while in the Electrical & Electronic Engineering and the Instruments industries it turns out to be consistent with the *passive learning* model put forward by Jovanovic (1982). Nevertheless, both groups of industries display a dynamics that is to a large extent consistent with the *evolutionary* approach developed by Audretsch (1995).

5 - Conclusions

In this paper we examine the evolution of the FSD for 12 cohorts of newborn firms, to draw some conclusions about which model of industry dynamics is more consistent with the size distribution of young firms in four selected industries. In general, the process of convergence towards the limit distribution appears to be just a matter of time, although, unfortunately, our data set allows us to follow the post-entry performance of these firms only for their first 6 years in the industry.

However, we take into account four industries very different from the point of view *a)* of the productive capacity required for entering the market at the MES level of output, and *b)* of their technological content and characteristics. Differences in industry-specific characteristics concerning the levels of sunk costs and the rate of entry allow for differences in the way a convergence towards a lognormal distribution does or does not arise. This Bayesian perspective helps to explain the different speed of convergence of the FSD to a lognormal distribution. In particular, it is consistent with our empirical finding that only in the most technologically advanced industries - in which smaller entrants tend to invest in their capacity more gradually, after exploring their efficiency level with respect to their competitors - a convergence towards the lognormal distribution emerges with the passing of time. Conversely, in the most traditional industries the same tendency is less marked. Whether this is due to the fact that the selection and learning processes are much slower in the traditional consumer goods industries than it is the case with the technologically progressive ones could be detected only when and if new data will be forthcoming allowing a thorough analysis of the behavior on new-born firms in these industries beyond their 21st quarter in the market.

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Appendix A

Since the aim of this work was to look for empirical regularities and stylized facts, we employed a simple non-parametric technique of density estimation. The advantage of this methodology is that no specified functional form of the density in exam is required. In this approach the density is estimated *directly* on the data and represents the most natural way to compare, also graphically, the empirical distribution to some *a priori* known distribution.

There are several ways to estimate non-parametrically a distribution: as already pointed out, we used the Kernel method, with the Gaussian distribution as Kernel function (as in Cabral and Mata, 1996).

The band-width parameter chosen is given by the formula:

$$h = \frac{0.9m}{\sqrt[5]{n}}$$

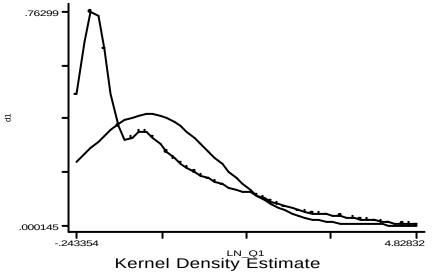
where n is the number of observations in the sample, and

$$m = \min\left(\sqrt{\mathrm{Var}(\mathrm{x})}, \frac{\textit{interquartile range}\ (\mathrm{x})}{1.349}\right)$$

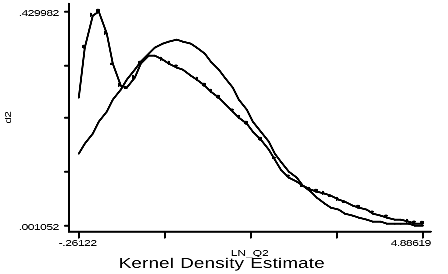
The choice of the Kernel function and of the bandwidth parameter has taken into account the trade-off between the minimization of the bias and of the variance of the estimate.

Appendix B

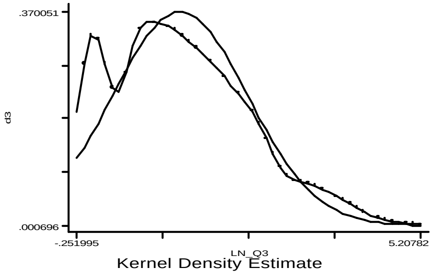
Table B.1 – Kernel Density Estimation, log(size), quarterly, Electrical & Electronic Eng. (continuous line is the Normal Distribution fitted into the data). Test of Normality below.



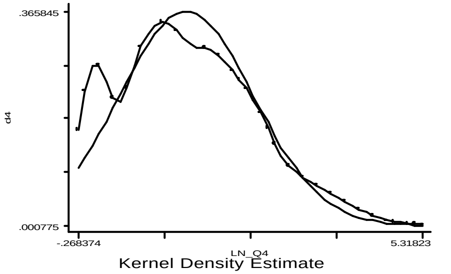
1st Q: Pr(Sk) = 0.000 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



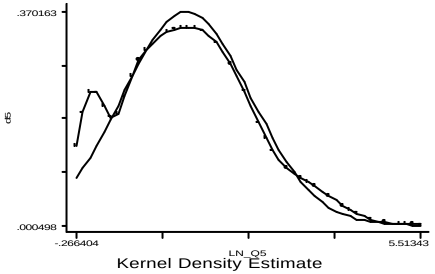
2nd Q: Pr(Sk) = 0.000 Pr(Kur) = 0.379 Pr(χ^2) = 0.0000



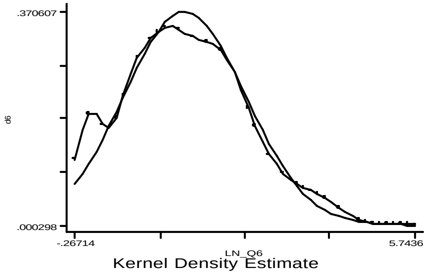
3rd Q: Pr(Sk) = 0.000 Pr(Kur) = 0.242 Pr(χ^2) = 0.0000



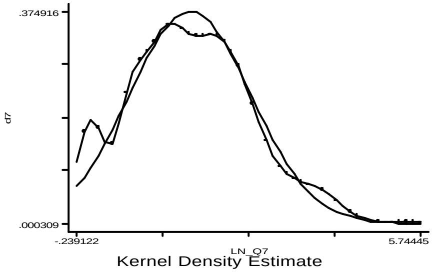
4th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.049 Pr(χ^2) = 0.0000



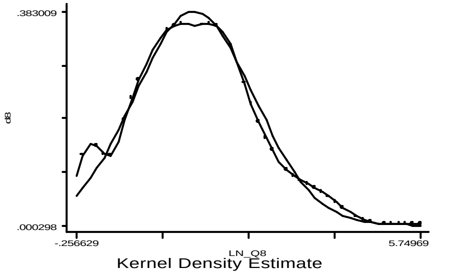
5th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.100 Pr(χ^2) = 0.0008



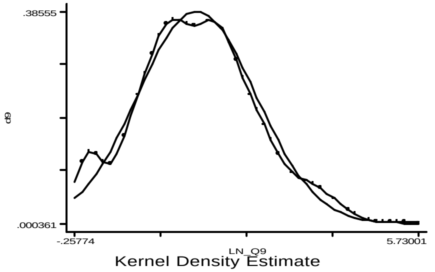
6th Q: Pr(Sk) = 0.001 Pr(Kur) = 0.198 Pr(χ^2) = 0.0023



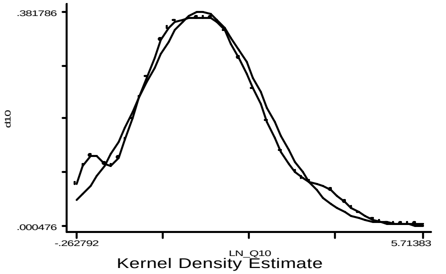
7th Q: Pr(Sk) = 0.001 Pr(Kur) = 0.229 Pr(χ^2) = 0.0055



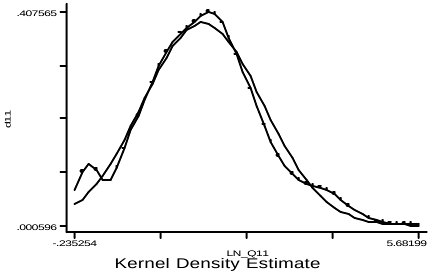
8th Q: Pr(Sk) = 0.008 Pr(Kur) = 0.565 Pr(χ^2) = 0.0274



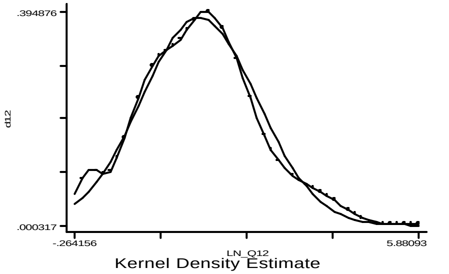
9th Q: Pr(Sk) = 0.018 Pr(Kur) = 0.716 Pr(χ^2) = 0.0574



10th Q: Pr(Sk) = 0.012 Pr(Kur) = 0.793 Pr(χ^2) = 0.0427

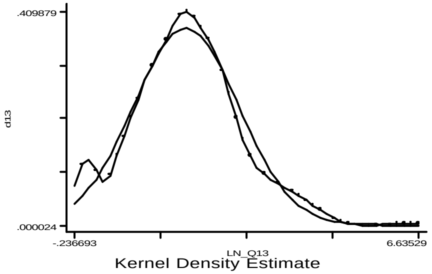


11th Q: Pr(Sk) = 0.022 Pr(Kur) = 0.745 Pr(χ^2) = 0.0679

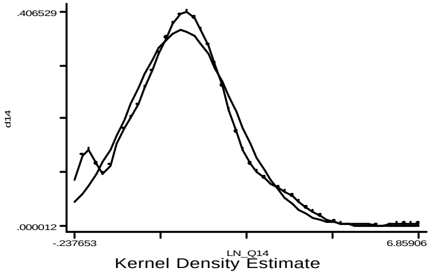


12th Q: Pr(Sk) = 0.010 Pr(Kur) = 0.800 Pr(χ^2) = 0.0372

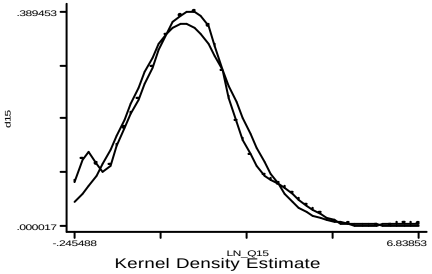
Table B.1 – following



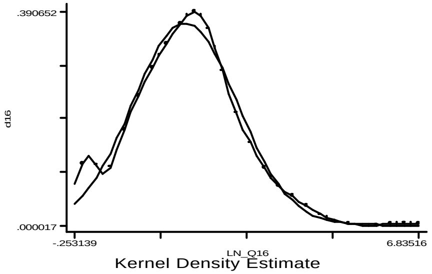
13th Q: Pr(Sk) = 0.012 Pr(Kur) = 0.155 Pr(χ^2) = 0.0180



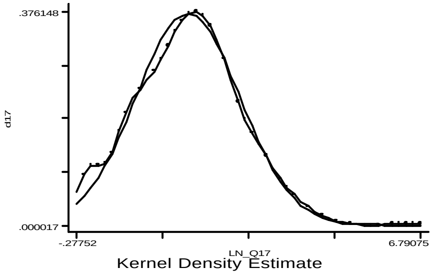
14th Q: Pr(Sk) = 0.026 Pr(Kur) = 0.150 Pr(χ^2) = 0.0320



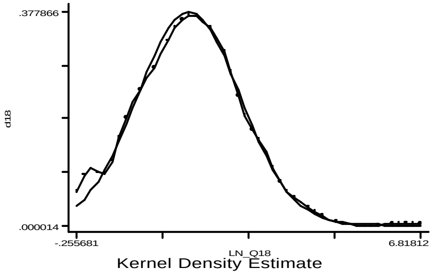
15th Q: Pr(Sk) = 0.030 Pr(Kur) = 0.173 Pr(χ^2) = 0.0388



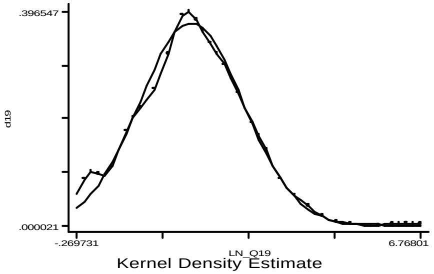
16th Q: Pr(Sk) = 0.070 Pr(Kur) = 0.243 Pr(χ^2) = 0.0980



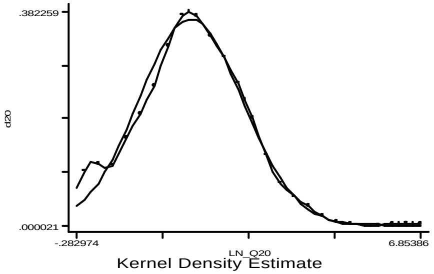
17th Q: Pr(Sk) = 0.085 Pr(Kur) = 0.588 Pr(χ^2) = 0.1909



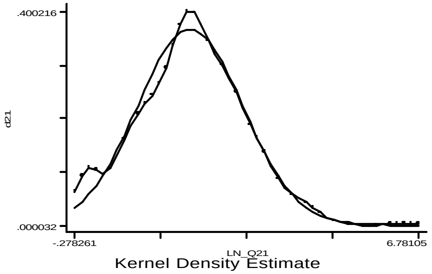
18th Q: Pr(Sk) = 0.130 Pr(Kur) = 0.405 Pr(χ^2) = 0.2249



19th Q: Pr(Sk) = 0.253 Pr(Kur) = 0.500 Pr(χ^2) = 0.4131

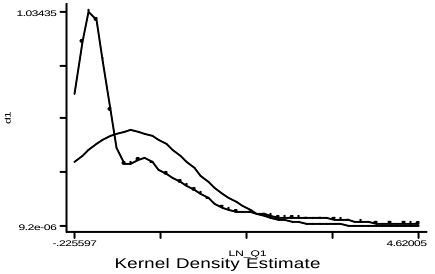


20th Q: Pr(Sk) = 0.502 Pr(Kur) = 0.434 Pr(χ^2) = 0.5859

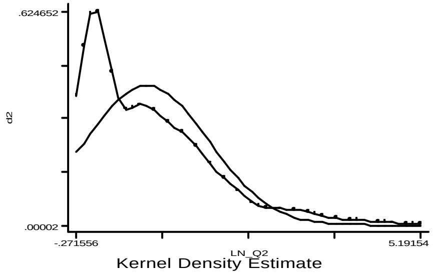


21st Q: Pr(Sk) = 0.333 Pr(Kur) = 0.488 Pr(χ^2) = 0.4904

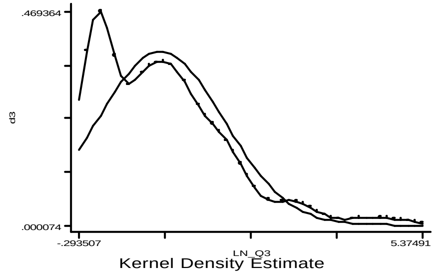
Table B.2 – Kernel Density Estimation, log(size), quarterly, Instruments. (continuous line is the Normal Distribution fitted into the data). Test of Normality below.



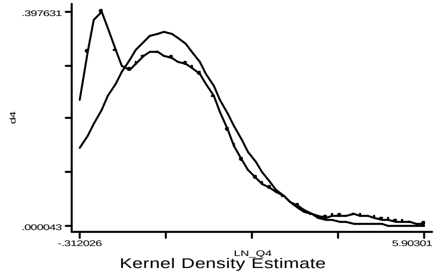
1st Q: Pr(Sk) = 0.000 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



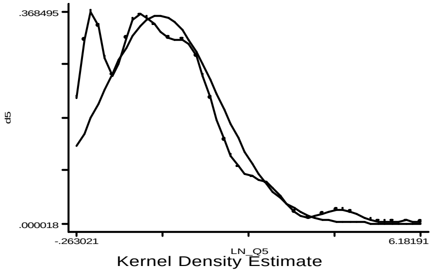
2nd Q: Pr(Sk) = 0.000 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



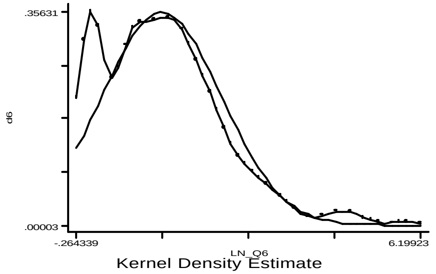
3rd Q: Pr(Sk) = 0.000 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



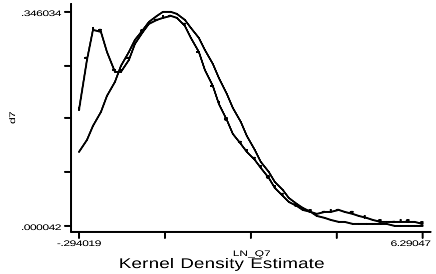
4th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.003 Pr(χ^2) = 0.0000



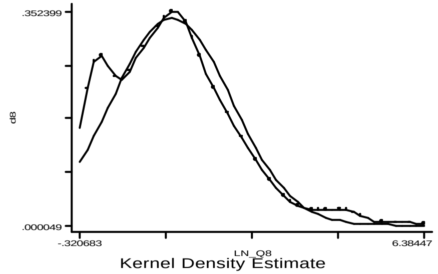
5th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.001 Pr(χ^2) = 0.0000



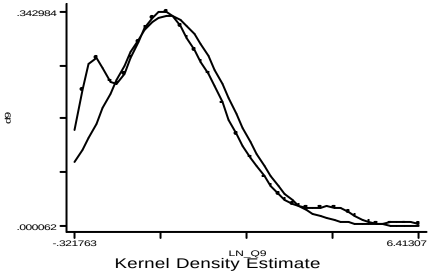
6th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.003 Pr(χ^2) = 0.0000



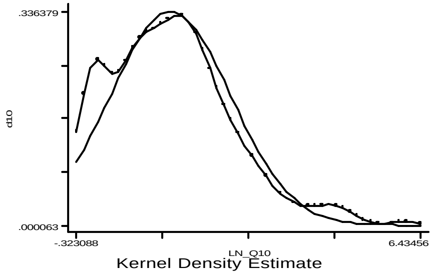
7th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.016 Pr(χ^2) = 0.0000



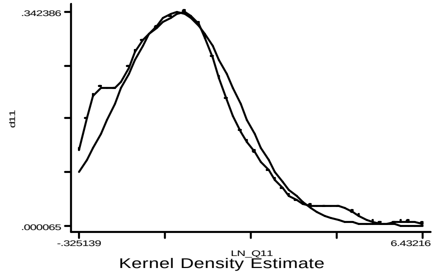
8th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.024 Pr(χ^2) = 0.0000



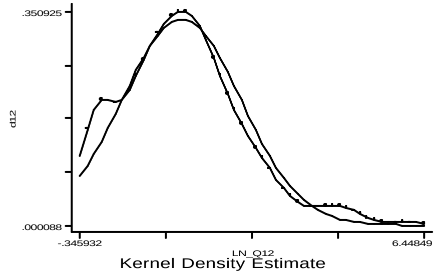
9th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.052 Pr(χ^2) = 0.0000



10th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.053 Pr(χ^2) = 0.0000

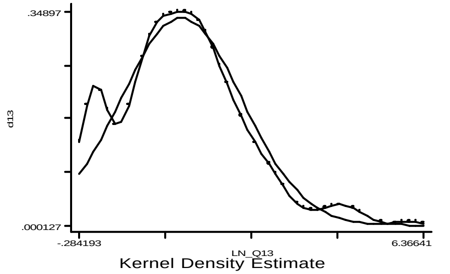


11th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.055 Pr(χ^2) = 0.0001

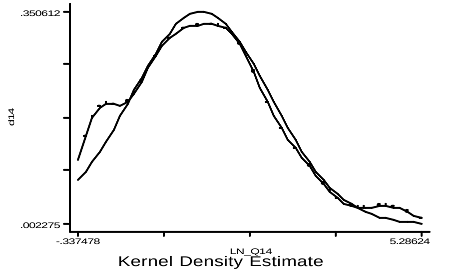


12th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.060 Pr(χ^2) = 0.0001

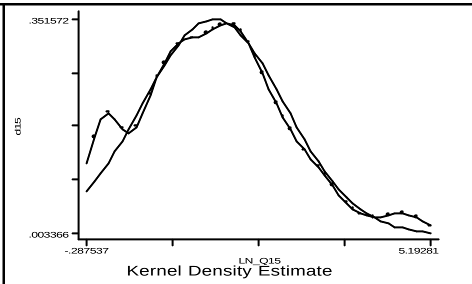
Table B.2 – following



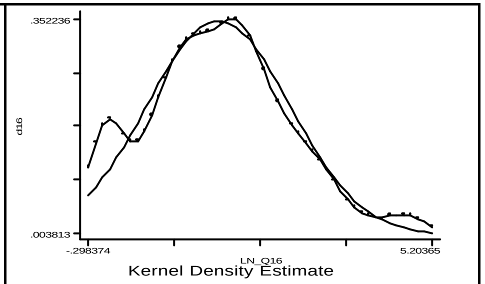
13th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.085 Pr(χ^2) = 0.0005



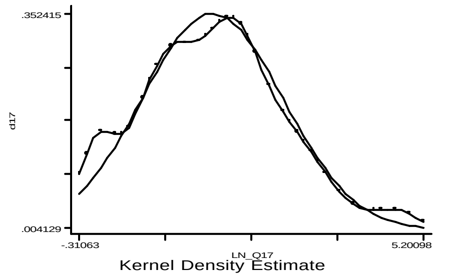
14th Q: Pr(Sk) = 0.004 Pr(Kur) = 0.883 Pr(χ^2) = 0.0219



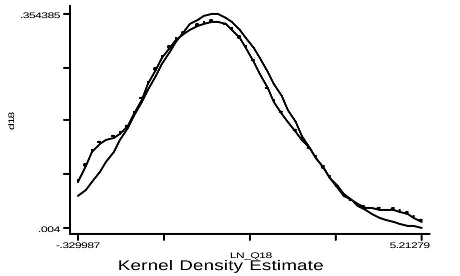
15th Q: Pr(Sk) = 0.011 Pr(Kur) = 0.996 Pr(χ^2) = 0.0432



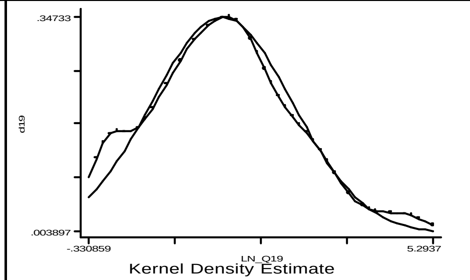
16th Q: Pr(Sk) = 0.033 Pr(Kur) = 0.884 Pr(χ^2) = 0.1009



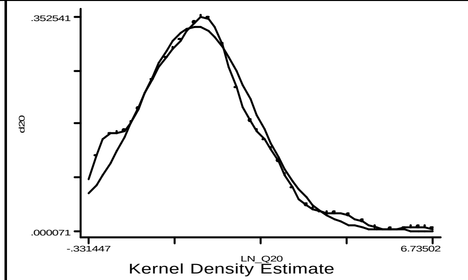
17th Q: Pr(Sk) = 0.044 Pr(Kur) = 0.747 Pr(χ^2) = 0.1218



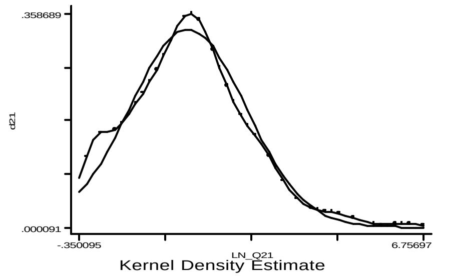
18th Q: Pr(Sk) = 0.033 Pr(Kur) = 0.766 Pr(χ^2) = 0.0961



19th Q: Pr(Sk) = 0.060 Pr(Kur) = 0.628 Pr(χ^2) = 0.1486

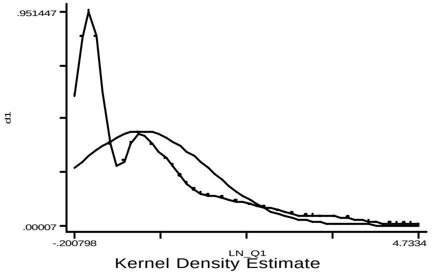


20th Q: Pr(Sk) = 0.003 Pr(Kur) = 0.126 Pr(χ^2) = 0.0066

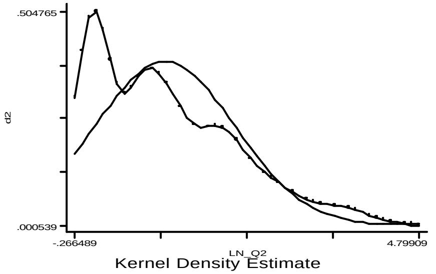


21st Q: Pr(Sk) = 0.005 Pr(Kur) = 0.162 Pr(χ^2) = 0.0118

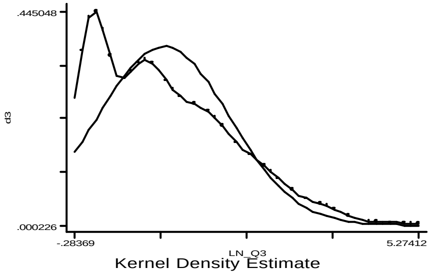
Table B.3 – Kernel Density Estimation, log(size), quarterly, Food. (continuous line is the Normal Distribution fitted into the data). Test of Normality below.



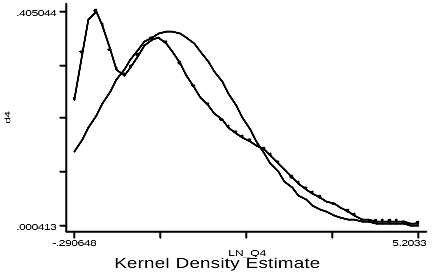
1^{st} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



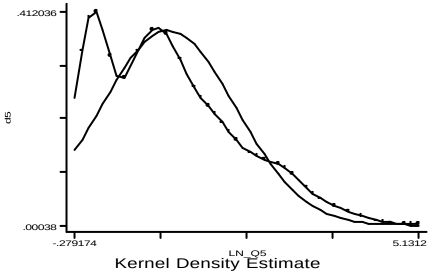
2^{nd} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.792 Pr(χ^2) = 0.0000



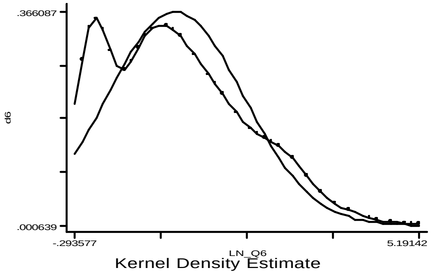
3^{rd} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.788 Pr(χ^2) = 0.0000



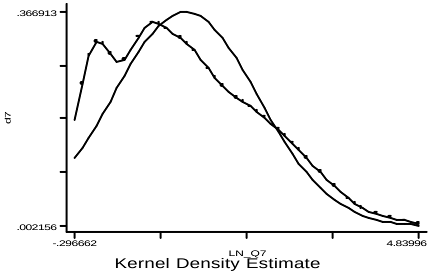
4^{th} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.487 Pr(χ^2) = 0.0000



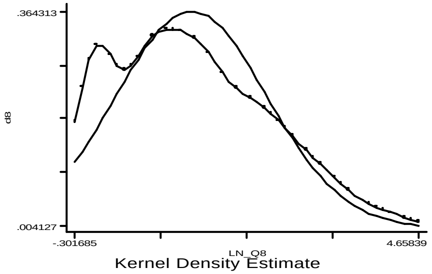
5^{th} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.671 Pr(χ^2) = 0.0000



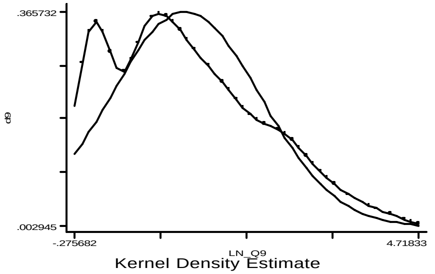
6^{th} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.063 Pr(χ^2) = 0.0001



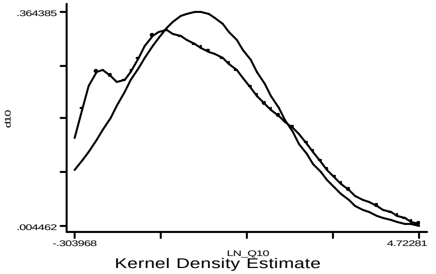
7^{th} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.010 Pr(χ^2) = 0.0001



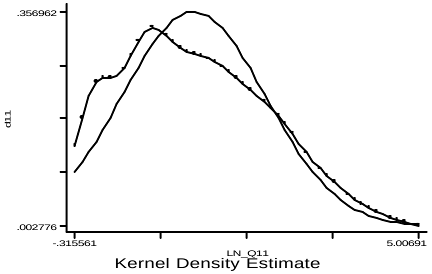
8^{th} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.009 Pr(χ^2) = 0.0001



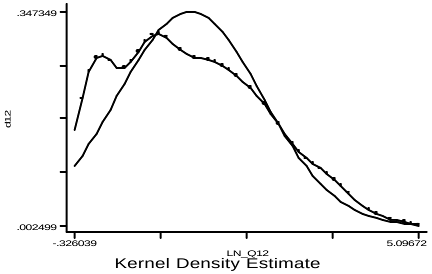
9^{th} Q: Pr(Sk) = 0.000 Pr(Kur) = 0.029 Pr(χ^2) = 0.0000



10^{th} Q: Pr(Sk) = 0.002 Pr(Kur) = 0.002 Pr(χ^2) = 0.0003

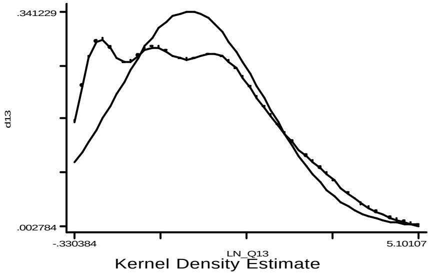


11^{th} Q: Pr(Sk) = 0.003 Pr(Kur) = 0.002 Pr(χ^2) = 0.0004

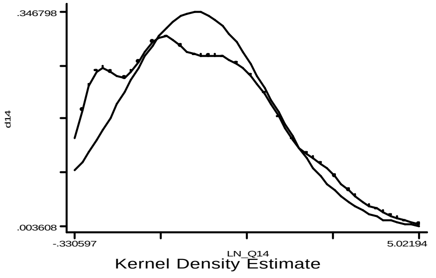


12^{th} Q: Pr(Sk) = 0.003 Pr(Kur) = 0.001 Pr(χ^2) = 0.0002

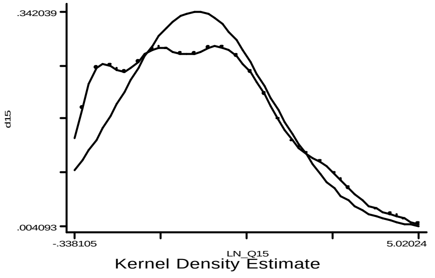
Table B.3 – following



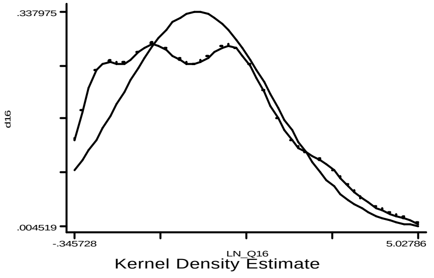
13th Q: Pr(Sk) = 0.002 Pr(Kur) = 0.049 Pr(χ^2) = 0.0001



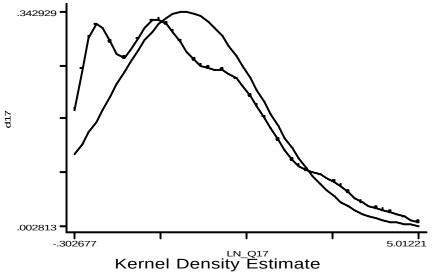
14th Q: Pr(Sk) = 0.008 Pr(Kur) = 0.002 Pr(χ^2) = 0.0008



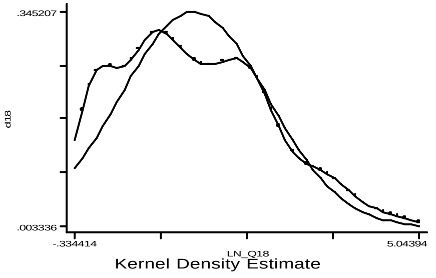
15th Q: Pr(Sk) = 0.008 Pr(Kur) = 0.004 Pr(χ^2) = 0.0014



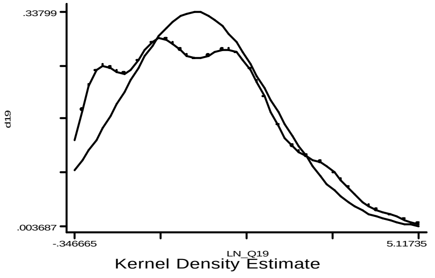
16th Q: Pr(Sk) = 0.008 Pr(Kur) = 0.004 Pr(χ^2) = 0.0013



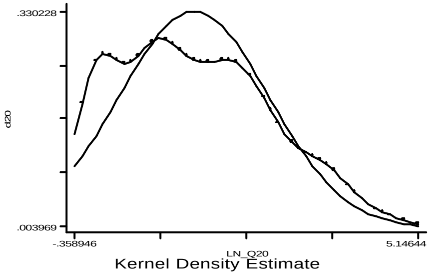
17th Q: Pr(Sk) = 0.000 Pr(Kur) = 0.218 Pr(χ^2) = 0.0002



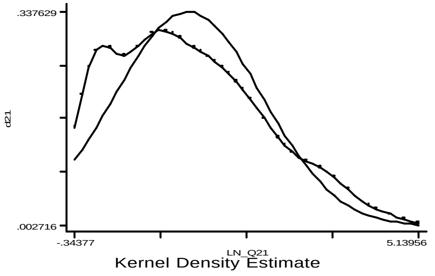
18th Q: Pr(Sk) = 0.002 Pr(Kur) = 0.042 Pr(χ^2) = 0.0023



19th Q: Pr(Sk) = 0.008 Pr(Kur) = 0.011 Pr(χ^2) = 0.0026

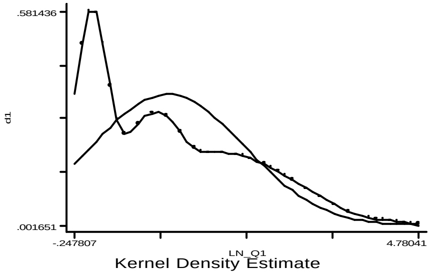


20th Q: Pr(Sk) = 0.006 Pr(Kur) = 0.003 Pr(χ^2) = 0.0011

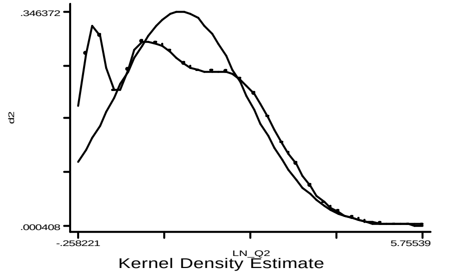


21st Q: Pr(Sk) = 0.000 Pr(Kur) = 0.071 Pr(χ^2) = 0.0010

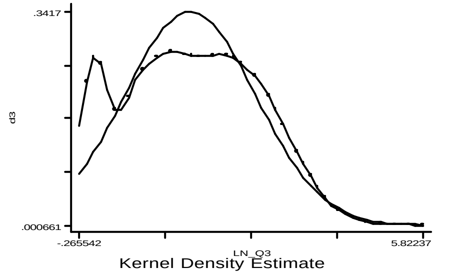
Table B.4 – Kernel Density Estimation, log(size), quarterly, Footwear & Clothing. (continuous line is the Normal Distribution fitted into the data). Test of Normality below.



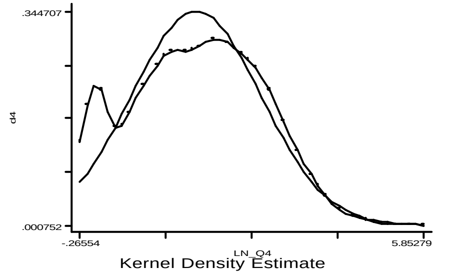
1st Q: Pr(Sk) = 0.000 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



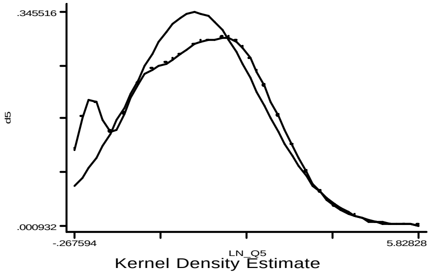
2nd Q: Pr(Sk) = 0.000 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



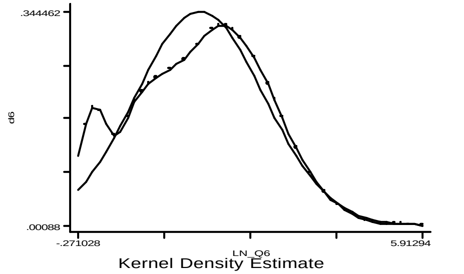
3rd Q: Pr(Sk) = 0.071 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



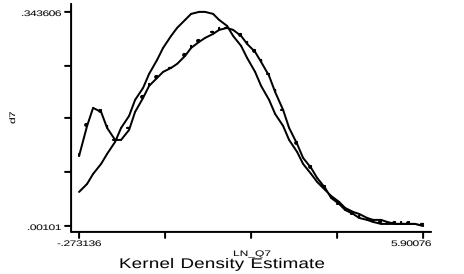
4th Q: Pr(Sk) = 0.369 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



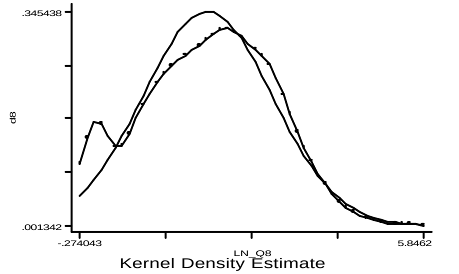
5th Q: Pr(Sk) = 0.565 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



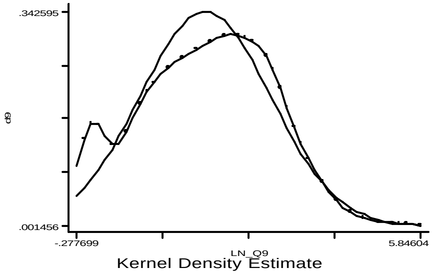
6th Q: Pr(Sk) = 0.950 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



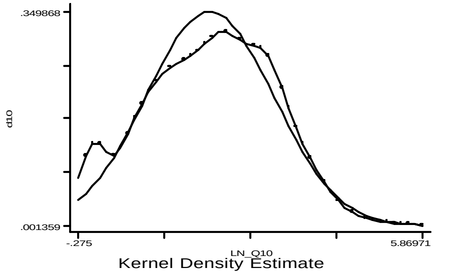
7th Q: Pr(Sk) = 0.807 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



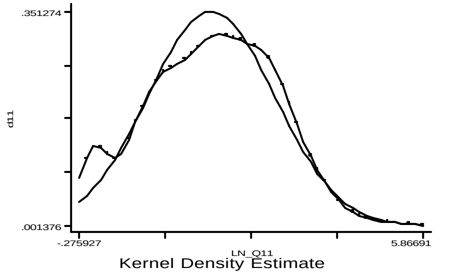
8th Q: Pr(Sk) = 0.641 Pr(Kur) = 0.049 Pr(χ^2) = 0.0000



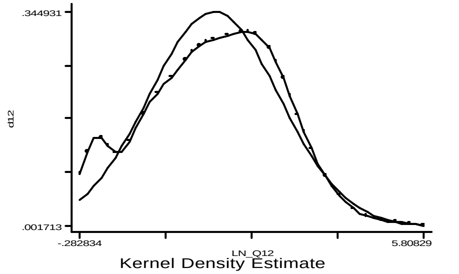
9th Q: Pr(Sk) = 0.535 Pr(Kur) = 0.000 Pr(χ^2) = 0.0000



10th Q: Pr(Sk) = 0.590 Pr(Kur) = 0.000 Pr(χ^2) = 0.0006

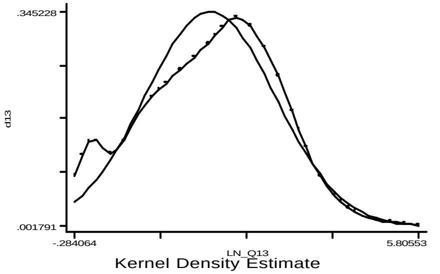


11th Q: Pr(Sk) = 0.559 Pr(Kur) = 0.000 Pr(χ^2) = 0.0007

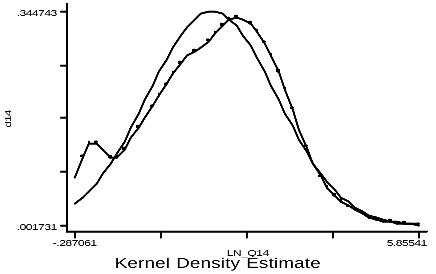


12th Q: Pr(Sk) = 0.331 Pr(Kur) = 0.000 Pr(χ^2) = 0.0003

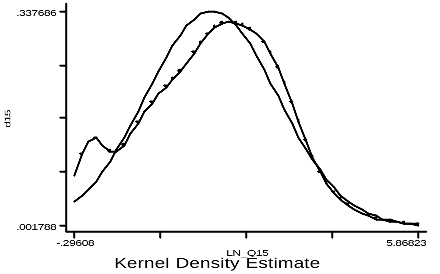
Table B.4 – following



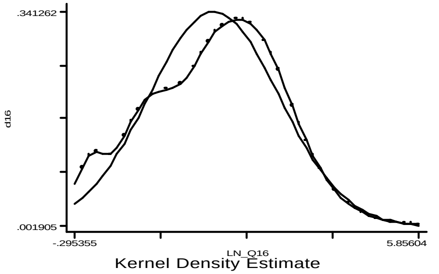
13^{th} Q: Pr(Sk) = 0.197 Pr(Kur) = 0.000 Pr(χ^2) = 0.0004



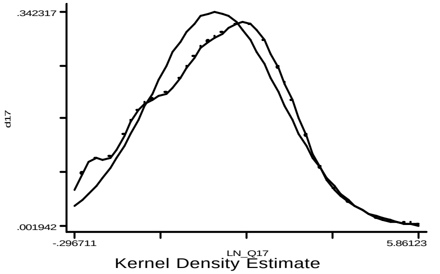
14^{th} Q: Pr(Sk) = 0.112 Pr(Kur) = 0.001 Pr(χ^2) = 0.0028



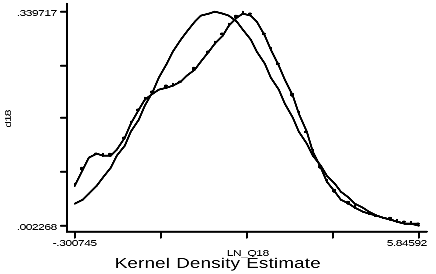
15^{th} Q: Pr(Sk) = 0.159 Pr(Kur) = 0.000 Pr(χ^2) = 0.0010



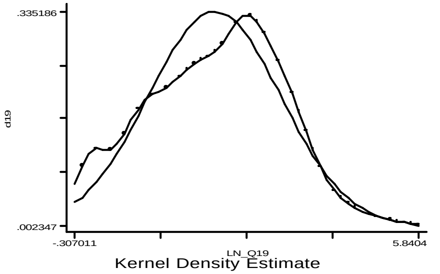
16^{th} Q: Pr(Sk) = 0.274 Pr(Kur) = 0.000 Pr(χ^2) = 0.0013



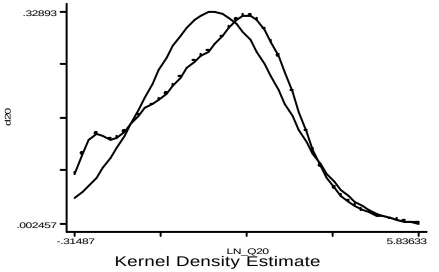
17^{th} Q: Pr(Sk) = 0.335 Pr(Kur) = 0.001 Pr(χ^2) = 0.0029



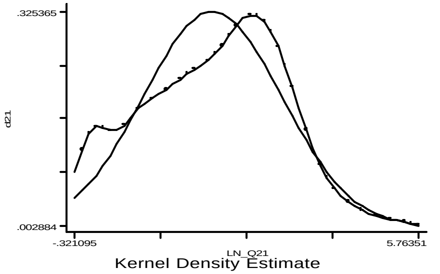
18^{th} Q: Pr(Sk) = 0.287 Pr(Kur) = 0.001 Pr(χ^2) = 0.0028



19^{th} Q: Pr(Sk) = 0.316 Pr(Kur) = 0.000 Pr(χ^2) = 0.0008



20^{th} Q: Pr(Sk) = 0.223 Pr(Kur) = 0.000 Pr(χ^2) = 0.0004



21^{st} Q: Pr(Sk) = 0.329 Pr(Kur) = 0.000 Pr(χ^2) = 0.0001

Appendix C

We started the analysis of the moments of the FSD in the various industries by standardizing the distributions, in order to obtain a more reliable comparison with the normal standard distribution. Subsequently, for the standardized distributions and for every industry, we computed, quarter by quarter, the Skewness Index and the Kurtosis Index.

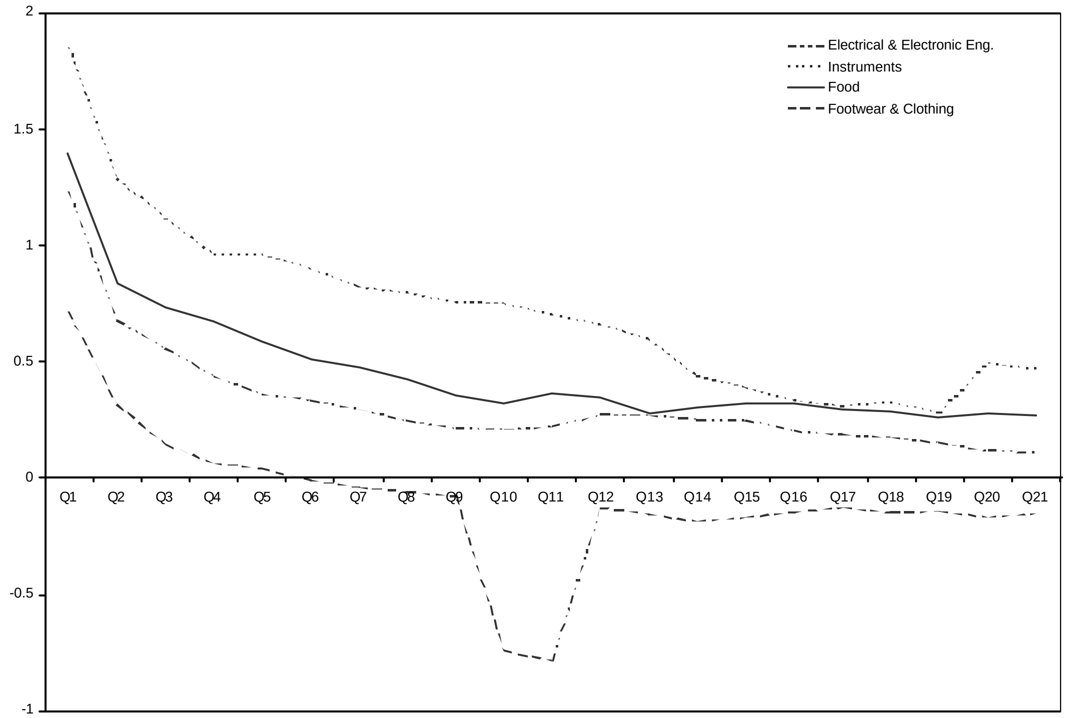
The Skewness Index, as a measure of asymmetry (or, more precisely, of the lack of symmetry), was computed as:

$$\text{Skewness} = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^3}{s^3}$$

where *s* is the standard deviation.

Since the Skewness for a normal distribution is zero - whereas it gets negative values for a distribution skewed to the left, and positive values for one skewed to the right - we expect our sequence of Skewness indexes to converge to zero.

Figure C.1 – Skewness Index, by quarter and by industry.



Looking at Figure C.1, one can note that for three out of four industries (the only exception being the Footwear & Clothing one) the FSD tends to become more symmetric over time, with different patterns of convergence. But even after 21 quarters, the FSD in the Electrical & Electronic Engineering, the Instruments and the Food industries is still skewed to the right, while in the

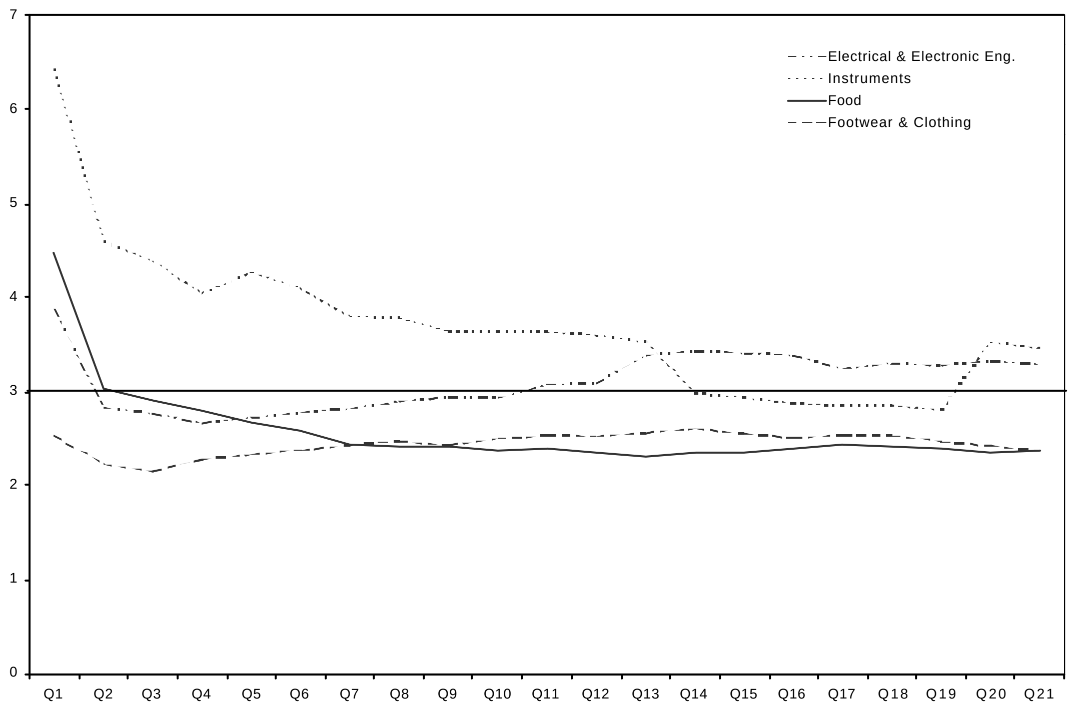
Footwear & Clothing industry, starting from a distribution skewed to the right, it turns out to be skewed to the left.

The other measure we used to characterize the evolution of the FSD is the Kurtosis index, aimed at assessing whether the data are peaked or flat relative to a normal distribution. In other words, a distribution characterized by a high Kurtosis tends to have a distinct peak near the mean, to decline rather rapidly, and to have heavy tails. On the contrary, a distribution with low Kurtosis tends to have a flat top near the mean rather than a sharp peak. We used the specification centered at 3, or *Pearson Kurtosis*:

$$\text{Kurtosis} = \frac{\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^4}{s^4}$$

If the Kurtosis index is greater than 3, the distribution is said to be *leptokurtic* (with a peak in correspondence of the mean), while if it is less than 3 the distribution is *platykurtic* (more flat and less concentrated around the mean with respect to the normal distribution). In Figure C.2 the different values of this index, by industry and for each quarter are reported.

Figure C.2 – Kurtosis Index, by quarter and by industry.



For all industries, the Kurtosis index shows a convergence towards the normal distribution, although in the case of the Electrical & Electronics and the Instruments industries, at the end of the relevant period, it appears to be more concentrated around the mean than in that of the other two industries, for which it tends to be more spread.