Size distribution and anti-trust

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Abstract : Extensive literature notwithstanding, e¤ects of the size distribution of …rms on consumers' surplus and on social welfare leaves room for further exploration. In this paper we discover that size distribution imposes two counter-balancing e¤ects on aggregate surplus of the industry: [i] even distribution of …rm sizes typically facilitates tacit collusion compared to slightly uneven distribution, whilst [ii] very uneven distribution resembles monopoly. The trade-o¤ between these two counter-forces can make the overall welfare e¤ect of …rms' size distribution (given a …xed number of …rms) non-monotone in the degree of concentration.

Keywords : Competitive equilibrium, predatory equilibrium, collusive sustainability, concentration indeces.

JEL classi…cation : L11, D43, K22.

1 Introduction

Concentration indices are commonly used by various antitrust authorities all over the world in order to measure how anti-competitive the market is. Game-theoretic microeconomics largely supports the prediction given by these indices as far as the number of firms is concerned. That is, the more firms coexist in a market, [1] the less market power each firm can exercise in static oligopolistic equilibria, and [2] the harder it is to sustain tacit collusion as a subgame perfect equilibrium when the market is repeated, be it Cournot or Bertrand or anything in between.

In fact, however, concentration indices measure two things inseparably. They reflect not only the number of firms, but also their size distribution as well. Namely, if the number of firms is the same, the more uneven their sizes are, the higher the concentration indices are. However, from a theoretical viewpoint, it is less straightforward whether uneven size distributions necessarily make the market less competitive as opposed to more evenly sized oligopoly.

Existing studies analysing the exercise of market power by *a priori* heterogeneous firms include Harrington (1989; 1991), Lambson (1994; 1995), and Rothschild (1999), *inter alia*. Harrington (1989) considers firms with heterogeneous discount factors and relates their different degrees of myopia to the well-known effects in terms of bargaining power. Lambson (1994; 1995) characterises firms with unequally sized capacity and the effects of their size distribution on the subgame-perfect sustainability of tacit collusion with optimal punishment à la Abreu (1986; 1988) and Abreu, Pearce and Stacchetti (1986). In Harrington (1991) and Rothschild (1999), firm heterogeneity is modelled in terms of heterogeneous production costs.

Technically, our analysis in this paper is somewhere in between Lambson's capacity approach, and Harrington's and Rothschild's cost approach. More philosophically, however, we have a slightly different view from most of the existing contributions in the following sense. In our comparative statics, we fix the industry's aggregate production capacity (defined in terms of the industry-wide marginal cost function) and divide it between firms at various parametrised proportions. This contrasts with the so-far more popular view of comparing each firm's cost (or capacity as a special case thereof) parameters directly, without focusing central attention on the industry's aggregate production capacity.

The reason why we take this somewhat unconventional approach is due to our interest in associating our theoretical analysis to practical policy implications. Namely, when discussing concentration and various ways of its indexation, what interests us (or policy makers in general) the most is the distribution of production capacity across firms, no less than each firm's individual capacity or cost levels. For this purpose, we find our approach to be the most natural and also technically the most straightforward.

We show that size distribution produces two opposite effects on aggregate surplus of the industry: [i] even distribution of firm sizes typically facilitates tacit collusion compared to slightly uneven distribution, whilst [ii] very uneven distribution resembles monopoly. For a given number of firms, the trade-off between these two forces can make the overall welfare effect of firms' size distribution non-monotone in the degree of concentration of the industry.

The remainder of the paper is organised as follows. In section 2 we lay out and analyse our basic duopoly model, where we mainly concentrate on Bertrand duopoly which turns out to be the simplest although the gist of our analysis is applicable to other forms of oligopolistic markets. Then in section 3 we present a simple illustrative example to develop concrete intuition as to the non-monotonicity of the effects of size distribution on industry-aggregate welfare. Finally, section 4 concludes the paper.

2 Bertrand duopoly

2.1 Stage game

Consider an industry where the industry-wide aggregate marginal cost function is $M[Q]$, where $M[Q] > 0$ and $M'[Q] > 0$ for all $Q \geq 0$. There are two firms competing in this industry, referred to as firm 1 and firm 2 henceforth. Their capacity ratio is $k : (1 - k)$, that is, their respective marginal cost functions are

$$m_1[q_1] = M\left(\frac{q_1}{k}\right); \qquad m_2[q_2] = M\left(\frac{q_2}{1-k}\right);$$

where $0 < k < 1$. There are no fixed costs irrespective of k.

Assume for simplicity that these two firms are perfectly substitutable suppliers facing the common, industry-wide inverse demand function $D^{-1}[Q]$, such that $D^{-1}[0] > M[0]$ and $(D^{-1})'[Q] < 0$ for all $Q \geq 0$. These firms are simultaneous-move price-setters.

2.2 Full collusion in a Bertrand supergame

Now suppose that the stage game defined in 2.1 is infinitely repeated with the discount factor δ which is common between the two firms. Obviously, whenever there exists a $Q_F \in \arg\max_q \int_{Q=0}^{q} \left(D^{-1}[Q] - M[Q] \right) dQ$, if the two firms are able to sustain tacit collusion at the monopoly level

$$p_1 = p_2 = P_F$$

where $P_F = D^{-1}[Q_F]$, selling

$$q_1 = kQ_F; \qquad q_2 = (1 - k)Q_F;$$

then this is the most profitable outcome from the two firms' point of view, exercising their market power in full, earning nett profits (per stage game)

$$\pi_1 = k\Pi_F; \qquad \pi_2 = (1 - k)\Pi_F$$

where $\Pi_F = \int_{Q=0}^{Q_F} \left(D^{-1}[Q] - M[Q] \right) dQ$.

We now examine whether this monopoly pricing is sustainable by trigger strategies. As an instrument for sustaining tacit collusion, consider the one-shot Nash reversion with the following static Bertrand-Nash equilibrium as a threat point. Note that there can be multiple static Bertrand-Nash equilibria, but that there is one with zero nett payoffs for either firm.[1] This static equilibrium, denoted by $p_1 = p_2 = P_{BN}$ hereinafter, satisfies $\int_{Q=0}^{Q_{BN}} \left(D^{-1}[Q] - M[Q] \right) dQ = 0$, where $P_{BN} = D^{-1}[Q_{BN}]$.

Let $\Pi[k; P_F]$ denote firm 1's deviation profit, i.e., the maximum one-shot nett profit attainable for firm 1 given $p_2 = P_F$. Provided that $\Pi[k; P_F] \geq k\Pi_F$, firm 1 has an incentive to remain in tacit collusion if and only if

$$\delta \geq 1 - \frac{k\Pi_F}{\Pi[k; P_F]}.$$

[1] This penal code automatically ensures the "security level punishment", as defined by Lambson (1987).

Likewise, let $\pi[1-k;P_F]$ denote firm 2's deviation profit given $p_1 = P_F$. Provided that $\pi[1-k;P_F] \geq (1-k)\pi_F$ firm 2 has an incentive to remain in tacit collusion if and only if

$$\delta \geq 1 - \frac{(1-k)\pi_F}{\pi[1-k;P_F]}:$$

The minimum admissible discount factor in order to sustain tacit collusion is therefore

$$\underline{\delta} = \max\left\{1 - \frac{k\pi_F}{\pi[k;P_F]};\ 1 - \frac{(1-k)\pi_F}{\pi[1-k;P_F]}\right\}$$

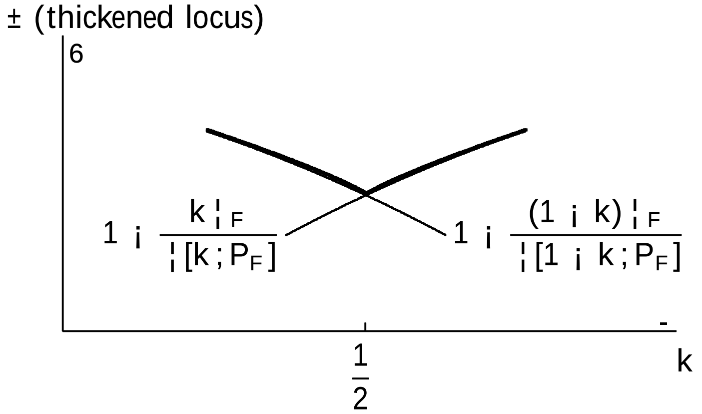
because, to sustain tacit collusion, the incentives to remain in collusion should be satisfied for both firms. Figure 1 illustrates the two functions $1 - \frac{k\pi_F}{\pi[k;P_F]}$ and $1 - \frac{(1-k)\pi_F}{\pi[1-k;P_F]}$, which obviously are mirror images of each other with respect to $k = \frac{1}{2}$.

Proposition 1 :

- If $1 - \frac{k\pi_F}{\pi[k;P_F]}$ (and thus $1 - \frac{(1-k)\pi_F}{\pi[1-k;P_F]}$ as well) happens to have a local maximum at $k = \frac{1}{2}$, then $\underline{\delta}$ also has a local maximum at $k = \frac{1}{2}$. However, insofar as these two functions are smooth, they have a zero slope at $k = \frac{1}{2}$ and so does $\underline{\delta}$, which implies that a small departure of k away from $\frac{1}{2}$ entails no first-order effect on $\underline{\delta}$.
- In all other cases, $\underline{\delta}$ has a local minimum at $k = \frac{1}{2}$. Furthermore, whenever $1 - \frac{k\pi_F}{\pi[k;P_F]}$ (and thus $1 - \frac{(1-k)\pi_F}{\pi[1-k;P_F]}$ as well) has a non-zero slope at $k = \frac{1}{2}$, the local minimum of $\underline{\delta}$ at this point is a downward-dipping kink, hence any local departure of k away from $\frac{1}{2}$ entails a first-order increase in $\underline{\delta}$.

Obviously, the latter not the former is the generic case, as drawn in Figure 1 (the local slopes of the two functions may be opposite from those in the diagram; insofar as they are non-zero our generic observation upholds).

Figure 1 : Minimum collusive discount factor near $k = \frac{1}{2}$.



In economic terms this proposition (its second half, the generic case) suggests that unequalising the size distribution of the firms away from the 50 - 50 split can help destabilise tacit collusion and thereby contribute to welfare, in spite of the fact that it increases most of the commonly used concentration indices. The intuition to this proposition can be best obtained by means of an example, as the one in section 3.

2.3 Partial collusion in a Bertrand supergame

When the discount factor δ is too low to sustain full collusion as described in 2.2, there can still be sustained some tacit price collusion that is not as profitable. This type of collusion is often referred to as partial collusion. In the spirit of "tacit collusion" where firms are not under any explicitly collusive agreement such as side payments and hence are to share profits according to their sales share, we focus on those outcomes where the firms share the market according to their capacity ratio $k : (1 - k)$.[2] In order for the two firms to collude at a price $P_T \in (P_{BN}; P_F)$ with quantities $q_1 = kQ_T$ and $q_2 = (1 - k)Q_T$

[2] Size asymmetry between the two firms in our model could generally give rise to two dimensions in which tacit price collusion can be "partial." One is the level of the collusive price, which is the dimension ordinarily considered when discussing partial collusion. The other dimension is the reshu - ing of quantities between the two firms, given any collusive price level; between equally sized firms the equal quantity shares would always be the best sustainable and the most profitable collusive configuration, whilst between unequally sized firms the quantity shares proportional to their sizes may not be the best sustainable albeit unambiguously the most profitable whenever sustainable. Fixing the quantity ratio at $k : (1 - k)$ is for us to concentrate on the former dimension not the latter.

where $P_T = D^{-1}[Q_T]$, analogously to section 2.2, the discount factor must satisfy

$$\delta \geq \max\left\{1 - \frac{k\Pi^*[P_T]}{\Pi[k;P_T]};\ 1 - \frac{(1-k)\Pi^*[P_T]}{\Pi[1-k;P_T]}\right\}$$

where $\Pi^*[P_T] = \int_{Q=0}^{Q_T} \left(D^{-1}[Q] - M[Q]\right) dQ$, and $\Pi[k;P_T]$ and $\Pi[1-k;P_T]$ denote the deviation profits for firm 1 and firm 2, respectively, given the other firm conforming to P_T.

We are interested in the most profitable partial collusion given $\delta \in (0;\underline{\delta})$ and k. Letting

$$P_T[\delta;k] = \arg_{P_T}\left\{1 - \frac{k\Pi^*[P_T]}{\Pi[k;P_T]} = \delta\right\}; \qquad P_T[\delta;1-k] = \arg_{P_T}\left\{1 - \frac{(1-k)\Pi^*[P_T]}{\Pi[1-k;P_T]} = \delta\right\};$$

the most profitable partially collusive price can be defined as

$$P^*[\delta;k] = \min\{P_T[\delta;k];\ P_T[\delta;1-k]\}:$$

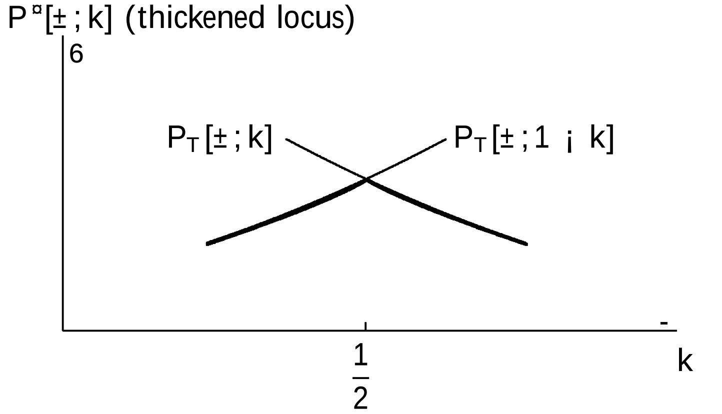
The "dual" of Proposition 1 is hereby as follows.

Proposition 2 :

- If $P_T[\delta;k]$ (and thus $P_T[\delta;1-k]$ as well) happens to have a local minimum at $k = \frac{1}{2}$, then $P^*[\delta;k]$ also has a local minimum at $k = \frac{1}{2}$. However, insofar as the two functions $P_T[\delta;k]$ and $P_T[\delta;1-k]$ are smooth, they have a zero slope at $k = \frac{1}{2}$ and so does $P^*[\delta;k]$, which implies that a small departure of k away from $\frac{1}{2}$ entails no first-order effect on $P^*[\delta;k]$.
- In all other cases, $P^*[\delta;k]$ has a local maximum at $k = \frac{1}{2}$. Furthermore, whenever $P_T[\delta;k]$ (and thus $P_T[\delta;1-k]$ as well) has a non-zero slope at $k = \frac{1}{2}$, the local maximum of $P^*[\delta;k]$ at this point is a upward-kinked ridge, hence any local departure of k away from $\frac{1}{2}$ entails a first-order decrease in $P^*[\delta;k]$.

The generic case, the second item of Proposition 2, is illustrated in Figure 2 (as in Figure 1, the local slopes of the two functions may be opposite from those in the diagram; insofar as they are non-zero our generic observation upholds).

Figure 2 : Highest partially collusive price near $k = \frac{1}{2}$.



2.4 Note on general oligopoly

Essentially, the same logic as deriving Propositions 1 and 2 could be invoked whether the market is Bertrand or Cournot or a more general game with upward-sloping supply functions (see Klemperer and Meyer, 1989). Namely, as far as collusive stability is concerned, unequalising firm sizes can help destroy tacit collusion and thereby contribute to industry-wide total surplus, defined as the sum of firms' nett profits and the consumers' surplus.

3 An illustrative example

For concrete intuition, consider the simple marginal cost function $M[Q] = \frac{Q}{2}$ (we continue to adopt the same notation as in section 2). Assume no fixed costs for production, and the industry-wide inverse demand $D^{-1}[Q] = 5 - Q$.

The static zero-profit Bertrand-Nash equilibrium is $p_1 = p_2 = P_{BN} = 1$, with quantities $q_1 = 4k$, $q_2 = 4(1 - k)$.

Full collusion is attained at the monopoly price $p_1 = p_2 = P_F = 3$, with fully collusive quantities $q_1 = 2k$, $q_2 = 2(1 - k)$, and fully collusive profits $\pi_F = 5$. The deviation profits from full collusion are $\pi[k; 3] = 6 - \frac{1}{k}$ for firm 1 and $\pi[1 - k; 3] = 6 - \frac{1}{1 - k}$ for firm 2. Since $\pi[k; 3] \gtrless k\pi_F$ iff $k \gtrless \frac{1}{5}$ and likewise $\pi[1 - k; 3] \gtrless (1 - k)\pi_F$ iff $k \lessgtr \frac{4}{5}$, the minimum admissible discount factor $\underline{\delta}$ for full collusion is

$$\underline{\delta} = \begin{cases} 1 - \dfrac{(1-k)\pi_F}{\pi[1-k; P_F]} & 0 < k < \dfrac{1}{5}; \\ \max\left\{ 1 - \dfrac{k\pi_F}{\pi[k; P_F]}; 1 - \dfrac{(1-k)\pi_F}{\pi[1-k; P_F]} \right\} & \dfrac{1}{5} \leq k \leq \dfrac{4}{5}; \\ 1 - \dfrac{k\pi_F}{\pi[k; P_F]} & \dfrac{4}{5} < k < 1; \end{cases}$$

which can be further computed as

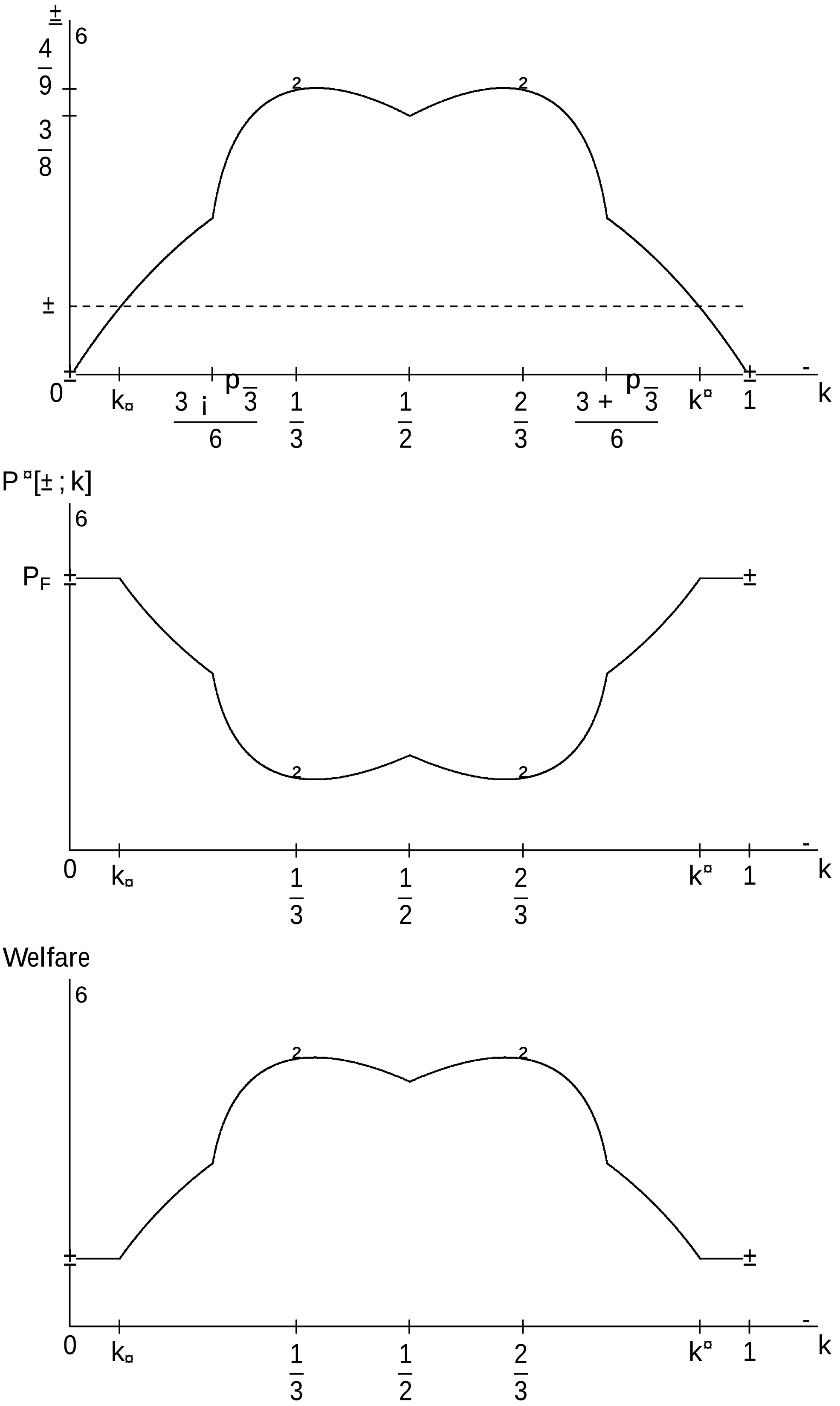
$$\underline{\delta} = \begin{cases} \dfrac{(4-5k)k}{5-6k} & 0 < k \leq \dfrac{3-\sqrt{3}}{6}; \quad \dfrac{1}{2} < k < \dfrac{3+\sqrt{3}}{6}; \\ \dfrac{(5k-1)(1-k)}{6k-1} & \dfrac{3-\sqrt{3}}{6} < k \leq \dfrac{1}{2}; \quad \dfrac{3+\sqrt{3}}{6} \leq k < 1. \end{cases}$$

The top diagram in Figure 3 plots $\underline{\delta}$ against k. Clearly from the diagram, $\underline{\delta}$ has the global infima $\underline{\delta} \searrow 0$ at $k \searrow 0$ and at $k \nearrow 1$ (the two open-ends), the local minimum $\underline{\delta} = \frac{3}{8}$ at $k = \frac{1}{2}$, and the global maxima $\underline{\delta} = \frac{4}{9}$ at $k = \frac{1}{3}$ and at $k = \frac{2}{3}$ (the filled dots in the diagram).

Hence, for any $\delta \in (0; \frac{4}{9})$, the highest sustainable collusive price $P^*[\delta; k]$ is (globally) minimised at $k = \frac{1}{3}$ and at $k = \frac{2}{3}$, locally maximised at $k = \frac{1}{2}$ $\left(\text{this local maximum becomes an interval ("plateau") when } \frac{3}{8} < \delta < \frac{4}{9}\right)$, and obviously, globally maximised at monopoly (in the neighbourhoods $k \searrow 0$ and $k \nearrow 1$). This is illustrated in the middle diagram of Figure 3, where those critical size distributions at which the most profitable partial collusion coincides with full collusion are marked with k_* and k^*.

Therefore, dually, welfare taking into account the sustainability of tacit partial collusion is (globally) maximised at $k = \frac{1}{3}$ and $k = \frac{2}{3}$, locally minimised at $k = \frac{1}{2}$, and obviously, globally minimised at monopoly (in the neighbourhoods $k \searrow 0$ and $k \nearrow 1$), as illustrated in the bottom diagram of Figure 3.

Figure 3 : Critical discount factor, collusive price and welfare plotted against productive capacity



4 Conclusion

In this paper, we have established the possibly non-monotone relation between the degree of concentration due to size distribution of oligopolistic …rms, and the resulting social welfare taking into account the (subgame perfect) sustainability of tacit price collusion in a repeated market game. When the market is highly concentrated with one …rm being overwhelmingly larger than its competitor(s) in terms of production capacity (measured by the marginal cost structure), the large …rm can exercise virtual monopoly power, whereby such extreme concentration tends to contribute negatively to welfare. However, while size distribution is not extreme, a slight perturbation away from the equally distributed capacity tends to help destabilise tacit collusion and hence contribute positively to resulting welfare. These two counterforces entail the aforesaid non-monotonicity feature.

This …nding has a very direct relevance to policy making, and more general assessment of so-called market power in a practical sense. Concentration indices, albeit extremely commonly used by various policy makers, not only mismeasure the market power in its true welfare implications, but they do so non-monotonically. An alternative “index” taking into account the aforementioned two countere¤ects is highly desired, which is indeed our future research subject.

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