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Technical Progress and Structural Change in the Process of Economic Development

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1. Introduction

Economic development is, in essence, structural change. Structural change, in turn, invariably requires progress in the methods of production involving both the technical as well as the organizational and institutional aspect. Historically, evolution from agriculture-based economies, mainly producing necessities, to full fledged industrial ones has brought about steadily rising productivity and thus an increasingly larger income per head. The relationship between this economic phenomenon and broadly defined technical progress is well documented and supported by a large body of historical evidence (Landes 1969). Breadth in defining technical advance is required in order to take in due consideration institutional changes and sometimes dramatic shifts in the organizational set-up which are closely related, in the long turn, with purely technical progress. Whether this relationship runs from the former to the latter or the other way around is a highly debatable matter. The view is taken here that the two terms of this relationship are actually bound by a feed-back linkage. To grasp this point, it is convenient to turn to Adam Smith's concept of division of labour and the ensuing process of specialization. In his analysis division of labour is the consequence of a growing exchange network, of the market dimension. Both are determined by capital accumulation. The latter, however, requires growing productivity which the former engenders to provide increasing expected rates of returns. Smith's theory is, in this sense, one of the first statement of a virtuous circle linking productivity growth to investment and vice-versa. Labour division, however, and specialization thereby generated bring about organizational changes which have been historically far-reaching while deepening of market relations has demanded the putting in place of regulating institutions. Thus, economically meaningful technical progress carries with it but also presupposes institutional and organizational structural change (Ricottilli 1994).

The object of this paper is to consider the relationship between technical progress and long-term growth, within the framework of structural change dynamics. Both theory and historical observation assign a crucial role to technical progress and to the innovative capacity which lies behind it. There seems to be broad agreement that the dynamic process through which it manifests itself exhibits the following characteristics; it is (*i*) continuous, as it has become a permanent and systematic feature of economic activity (Rosenberg 1976); (*ii*) cumulative, since technological success breeds further success (Dosi 1988); (*iii*) path-dependent, for it evolves along trajectories depending on initial conditions (Arthur 1989); (*iv*) it is, for a large part, endogenous since it is embedded in and explained by the behaviour of economic activity (Nelson and Winter 1982).

The process linking capital accumulation to productivity growth and the latter to the former is clearly a process which lends itself to macroeconomic modelling; indeed, it implies a theory of effective demand placing investment at the heart of dynamic behaviour. But it also requires a suitable explanation at a microeconomic level to take into account of how single economic units generate innovations and thus promote increasing productivity. It is, in fact, the innovative activity carried out by firms, and more generally by economic agents that provide

endogeneity to the process. The characteristics mentioned above are, in fact, largely due to the searching and learning activity that firms of all kinds and sizes carry out. Both these processes can be explained in terms of a rational behaviour that learns through experience, searches alternatives or new solutions on the grounds of available and limited information, that it takes time to adjust and perceive new ways and methods. It is an innovative activity founded on bounded rationality (Egidi 1991, 1992). These principles will be used to describe a decision-making process which is shaped as a dynamical model of difference equations following recent contributions to the literature (Silverberg, Dosi, Orsenigo 1988; Warglien 1991; Wall 1993). The rationale emerging from such a model will then be used as the underlying micro-behaviour of a macroeconomic model of endogenous long-term growth sustained by technical change.

The plan of the paper is as follows. Section two sets out the microeconomic decision making process leading to innovation based on principles of bounded rationality. The analysis will be carried in the context of user-producer relationships (Freeman 1974, Pavitt 1984) at a given state of specialization. Section three is a macroeconomic model built on a different analytical plane than the micro model but it assumes behaviour discussed in that context. It is a dynamical model merging the impact of Schumpeterian technological progress with purely endogenous technical change. At the heart of its dynamics lie technological and innovative capabilities. Indeed, a taxonomy of different degrees of development can be inferred from the dynamic behaviour that different capabilities cause. Section four will draw the main conclusions.

2. INNOVATION AS A DECISION-MAKING PROCESS

The first step towards a useful description of innovative behaviour is a suitable definition of technique of production. As arqued above, this is necessary in order to obtain a meaningful definition of the state of technology. The first element conducive to such a definition is to view a productive unit, or a firm, as filling a specific slot in the existing pattern of specialization. This pattern is assumed, but it is clearly the product of a process evolving in time. Specialization reveals itself, or can in this context be taken to be, as specific knowledge applied to the production of a given good, or a bundle of goods; specific is a term which is here used to designate a type of knowledge which is relevant almost exclusively for the production process in question or, at most, to a narrow range of processes which are technically akin. In this sense, it concerns the quality and quantity of inputs to be transformed, the type and standards of equipment and tools, the organization of labour and management. A second element in this definition is clearly the means of production which crystallize the current technique in a set of capital goods which are activated to generate output. The third and final element deals with the organizational principles according to which tasks are defined, knowledge actually applied, procedures deviced to cope with production problems within a given command structure, i.e. a hierarchical set-up. Taking the three elements together a snapshot is produced at any point in time of an on-going process caught, in a manner of speaking, in a moment of a specific dynamic movement: the state just described is, in fact, the result of past innovative activity and, because of it, it is subject to further change. It can immediately be inferred from this definition that economic activity is inherently rigid, in the sense that changes can occur only by means of the specific and particular knowledge acquired and formalized in actual skills, know-how and routines strictly related to the type and kind of processes being activated. This state is the outcome of a process leading to specialization and is itself ground for further change and the seed from which further specialization stems.

As it has been mentioned, the knowledge required to set-up the process of production and actually produce an output, a good with well defined characteristics, say a numerically controlled lathe or a simple ball point pen, has been acquired and encoded in personal skills, know-how and routines in consequence of past innovations. The latter are themselves the

result of learning and search processes. Furthermore once introduced and before being mastered, they must be learnt through now well-known and well discussed processes of learning-by-using and learning-by-doing. This is a principle which applies, in a general way, to all types of economies, both to well developed and now fully industrialized ones and to as yet developing and sometimes backward ones. The difference, which can be dramatic, lies in the intensity and quality of such processes, in human capabilities involved, in resources drawn upon.

The need for routinized activity, for knowledge encoded in procedures, specialized skills and expertise lies with the fact that human behaviour and cognitive capabilities respond to principles of bounded rationality (Simon 1955, Cyert and March 1962). This concept, not normally assumed in mainstream economic analysis but borne out by empirical research both in the field of economics and in that of cognitive psychology, holds that computational ability, far from being unlimited, is in fact circumscribed, often to simple rule-of-thumb procedures and to roughly deviced linear relationships between variables (Dawes and Corrigan 1974). Thus, choice and decision making, rather than fitting into a complex pattern all relevant information, which is itself partial, asymmetric and limited to compute constrained maxima, are carried out by assimilating only very small subsets of the latter to reach tentative goals. It is a «satisficing process». People are, instead, assumed to grope towards a solution, which need not coincide with that provided by an a-priori-ly given algorithm, by learning to simplify problems by solving sub-problems given limited information and computational ability.

The innovative process is one that changes the technical state described above and its knowledge base. There is now a wide body of economic literature (Dosi 1988) showing that it is, to a large extent, endogenous: it is generated by economic activity and by its agents, firms or more generally production units. Endogenous technical progress is based upon and largely results from the state of technology. The reason of this lies with the fact that through the general process of learning, it is the main source of information, or externally acquired information and knowledge are therein effectively applied and related. Each newly achieved state of technology, each newly applied technique, is always incompletely known at first and it obviously lends itself to improvement. But, in an economic environment where there is sufficient inducement to change, focusing devices (Rosenberg 1982) chart the direction of further change. As forcefully argued by many authors, capital goods producers play a crucial role in this process: this is so since the drive to innovate their product is, if sufficient competitive pressure exists, a condition to keep their market share and the way to eventually expand it. Viewed in a long-term perspective, capital goods producers are the result of specialization dynamics, of the capital deepening which is brought about by past successful innovations. In their case, the user-producer relationship is the necessary link through which information flows and thanks to which learning occurs and search for innovation takes place. For producers of capital goods market success means being able to sell means of production, in a broad sense, to users: the equipment which is a part to be harmonized with and fitted to the other elements making up the user's production process. Users' production processes are in this sense, the producers' targets and supplying innovative capital goods provides the incentives for their purchase through the higher expected profit rate that an improved production process allows. These considerations carry the implication that while users' processes are the object of producers' innovative efforts in terms of output, they are also one of the most important source of information. Thanks to the latter, problems and opportunities exhibited by such process are learnt, all the more so the stronger is the relationship, and search for solutions occurs.

Principles of bounded rationality suggest to split up the decision making process leading to innovation within the context of user-producer relationships in three related stages: problem framing, goal setting and problem solving. Problem framing is a process through which producers or producers and users jointly learn of problems and of possibilities of improvement thanks to the capabilities hitherto developed and readyed to cope with such problems. These

capabilities are a product of a firm's evolution and they are built up as awareness and knowledge of problems develop. They can be taken to be part of or coincide with the wider spectrum of innovative capabilities, they are specific in the sense that information received from users' processes are processed through routinized learning sustained by specific skills and knowledge. The latter are, however, learnt in the process both because persons employed for the task learn by doing and because more resources are devoted to this purpose as the challenge mounts. Perception of problems is, therefore, a function of such capabilities but they, and thus the intensity of problem framing, depend on the technological complexity of users' processes.

Problem framing leads to goal-setting, i.e. to satisfacing behaviour. Thanks to perceived problems a solution procedure is framed. This second step is closely related to the third. It consists of a process through which targets of achievable improvements are set, goals within reach the attainment of which is considered to be the fulfillment of the undertaken search. Goals are, also, the consequence of an evolving process: they are continuously adjusted as discrepancies between estimates of what is feasible and guesses of what is desirable emerge. This point calls forth the third stage of this stylized decision-making process, that of problem solving. It becomes in fact necessary to make an estimate of achievable results, those which can be obtained by employing specific resources to conduct an innovative search. Such resources can be summarized under two broad categories: one is expenditure specifically devoted for the purpose, innovative investment, the other is the innovative, mainly human but organized, capability set up to support this search. Estimation is, in this case, an attempt to match means and ends, to guess the relationship to be established between the two. As mentioned above, limited computational ability and simplifying routines suggest that this relationship is ruled by a simple linear link by means of a learning parameter subject to revision as experience of the relationship develops. Given human capabilities, it is innovative investment to be established. The principle is, as in previous problems, that the quantity of means to meet ends is set by adjustment, i.e. by observing actual results from past search and measuring the distance that separates these from the set target. Innovative investment depends also on the quantity of available human resources devoted to the solution of innovation problems.

The model

The following is a simple model explaining the three tier process outlined above under very basic assumptions. A typical user-producer relationship will be considered in which only one producer is linked to only one user.

Problem framing – Let $L_j(t)$ be the quantity of organized human and physical resources, defined as an efficiency index, devoted to the understanding of user's process j and to the solution of problems likely to arise as a consequence of search «around» its characteristics by a given producer. Information on the process characteristics can be summed up by the current unit cost structure $c_j(t)$, itself the result of vector price (p), i=1,2,...,n, and of a vector of input coefficients (k_j) , inclusive of those relating to durable equipment and labour: $c_j(t) = (p_j) \cdot [k_j(t)]$. Producers scan this structure thanks to the relationship binding them to user j and assess their capability to cope with its bottlenecks, mishaps and technical opportunities through experience, by observing actual technical change as it is translated into a new cost-structure. For simplicity sake, prices are held constant. Actual technical change provides, then, an indication of innovative capabilities deemed suitable to keep abreast with it. Let such a capability be defined as $L_i^*(t)$. Its rate of change is linearly related to that of the cost structure:

$$L_{i}^{*}(t+1) \bullet L_{i}^{*}(t) / L_{i}^{*}(t) = \lambda \left[c_{i}(t) \bullet c_{i}(t+1)\right] / c_{i}(t)\lambda > 0$$

 $[c_j(t) \bullet c_j(t+1)]$ measures the cost reduction following successful technical improvement. λ is a learning parameter and it shows the degree of closeness between user and producer and their relationship strength. L_j^* is a desired target in respect to which adjustment is gradual and led by experience. A simple rule is assumed:

$$\frac{L_{j}(t+1) \bullet L_{j}(t)}{L_{j}(t)} = \frac{L_{j}^{*}(t+1) \bullet L_{j}^{*}(t)}{L_{j}^{*}(t)} + \alpha_{l} \frac{L_{j}^{*}(t) \bullet L_{j}(t)}{L_{j}(t)}; \alpha_{l} > 0$$

Goal setting – The object of searching is to bring about technical change which allows unit cost reductions. Producers' efforts, jointly with users', are rewarded when the target aimed at is reached. As argued above, the latter relates to an estimate of what end can be achieved with given means. Given such an estimate, goals are continuously reshaped to account for the cost reductions which are held as achievable. What matters here is a realistic assessment of the discrepancy between goal and estimate but also of the tendency of cost reductions to increase or decrease in intensity: a measure of the momentum of technical change: this last magnitude can be measured by an estimate of cost-reduction changes. Let the cost structure set as a goal be $c_{\rm j}^*(t)$ and that estimated $\hat{c}_{\rm j}(t)$. Then, the process of adjustment based on perceived estimates and changes is:

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and

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where $r_j(t) = [c_j(t) \bullet c_j(t \bullet 1)]$ and $\hat{r}_j(t)$ is an estimate of $r_j(t)$. In 3., the separate effects of discrepancies and of the tendency of cost reduction to accelerate (or decelerate) bear an independent impact on goal revision; 3. is, therefore, in additive form. $\hat{r}_j(t)$ is subject to revision on the basis of immediate past experience; a reflex of a very short-term perception of cost structure dynamics.

Problem solving – The final stage in this three-tier process of decision-making leading to search for technical change is the formulation of what is actually achievable; estimates of cost reduction. This implies an estimate of a function relating means, namely innovative expenditure and capabilities, to achievable ends:

$$\hat{c}_{j}(t+1) \bullet c_{j}(t) = \bullet \hat{C}[R_{j}(t), L_{j}(t)]$$

Assuming bounded rationality, perception of a functional relationship between means employed and cost reduction can be rendered as a linear relation in both independent variables:

$$\hat{C}(R_j, L_j) = \hat{\gamma}(t) R_j(t) L_j(t)$$

 $\hat{\gamma}(t)$ is a parameter which measures the extent of the impact of either L_j or R_j , or both, on $\hat{C}(\cdot)$. This impact is learnt in the sense that it is based on actual observation of γ in past periods, assigning conventional weights to each, the greater the closer is the period taken in consideration. Memory can be assumed to stretch back for an equally conventional number of periods, say m. Thus:

$$\hat{\gamma}(t) = \overline{\sigma}_1 \gamma(t \bullet 1) + ... + \overline{\sigma}_m \gamma(t \bullet m)$$

and

$$\gamma(t) = \frac{c_j(t \bullet 1) \bullet c_j(t)}{R_i(t) L_i(t)}$$

σ's are the weights. In this manner, the simple relationship postulated by 6. is subject to a learning process.

Deciding how much to spend on searching takes the form of an additive relationship based on the estimated cost reduction which is thus attainable and on the required human and physical capabilities to cope with the technological process. This procedure sets the desired levels of search expenditure which must then be matched with available resources. As evidence from empirical literature seems to indicate, the latter are often fixed as a proportion of turn-over. Thus:

$$R_i(t) = \sigma Y_i(t)$$

where Y_i is process j turn-over, and

$$R_{i}^{*}(t+1) \bullet R_{i}(t) = \gamma_{i} \left[\hat{r}_{i}(t+1) \bullet r_{i}(t) \right] + \gamma_{2} \left[L_{i}(t+1) \bullet L_{i}(t) \right] + \gamma_{3} \left[R_{i}^{*}(t) \bullet R_{i}(t) \right]$$

 γ 's are positive. 8.1 contains an adjustment term to take into account differences between actually available resources and those warranted by the innovative challenge.

The model can be closed by adding a function describing the «environment», i.e. the actual relationship tying cost reductions to the means employed for the purpose:

$$c_i(t) \bullet c_i(t \bullet 1) = C[R_i(t), L_i(t)]$$

The nature of this function is, in substance, empirical. It ties up resources addressed to innovate and cost reductions as effectively obtained.

Comments – Equations 1.-9. have some interesting features. It is quite clear, first of all, that innovators learn the shape of their estimated function $\hat{C}(\cdot)$ by observing revealed ratios between actual results $\Delta_{C_j}(t+1)$ and means activated to get them, $R_j(t)$ $L_j(t)$ as from 7.1. Their computational ability, however, restricts them to take $\hat{C}(\cdot)$ only in linear form. Thus, innovators are able to guess the actual $C(\cdot)$ only if this is, in fact, a linear function. This follows directly from 6., 7. and 7.1. To see this, substitute recursively in 7. for t-1, t-2,...,t-n, for t-1, t-

Suppose now that $c_j^* = \hat{c}_j = c_j$ i.e. innovators are able to guess exactly the cost-structure of users' processes and set consequently the goal to aim at. This, perhaps unlikely ability includes if the above stated equality is to hold for any t that $r = \hat{r}$. From 3., 4. and 9. it follows that $C(\cdot) = \hat{C}(\cdot)$. Thus, innovators can persistently hit their target without mistakes if they learn the «true» shape of the cost-decrease function. This can be so, however, if this function takes a linear form. This is quite, unlikely, however, since $C(\cdot)$ is generally non-linear.

The system dynamics are basically set by the growth rate of search expenditure which in the model sketched out above is entirely explained by an exogenous element, i.e. by turnover growth. Over and above the momentum given by market success, endogenous dynamics follow from learning and adjustment. The limit case in which innovators hit their targets, foresee exactly and learn immediately can be made. This case occurs when

 $c_j = \hat{c}_j = c_j^*$, $r_j = \hat{r}_j$, $L_j = L_j^*$ and $R_j^* = R_j$. This implies as it has been seen, that $\hat{C}(\cdot) = C(\cdot)$, which then takes a linear shape. 8.1 reduces to:

$$R_{j}(t+1) \bullet R_{j}(t) = \gamma_{2} \left[L_{j}(t+1) \bullet L_{j}(t) \right] = \sigma \left[Y_{j}(t+1) \bullet Y_{j}(t) \right]$$

from which, given $\Delta Y_j(t+1)$, the increase in L_j is obtained. Note that in this case expenditure is channeled to sustain only increasing capabilities to cope with an ever complex cost structure $c_j(t)$. None goes, in the innovators' view to support an acceleration of innovation since none is expected. Given 6. the rate of growth of $C(\cdot)$ can be approximated by the sum of the growth rates of R_j and L_j which follow directly from 10. Finally, if the turn-over growth rate is zero, both R_j and L_j remain constant implying a constant level of cost reduction and thus a falling rate of technical advancement thus measured.

The model as it stands lends itself to simulation and calibration. Such an exercise, which is not attempted here, can yield interesting results by assuming different initial conditions and different learning and search parameters for any given rate of turn-over growth.

Assumptions underlying 1.-9. are greatly simplifying in order to highlight the various steps of the decision-making process. In particular, the fact that one producer relates to one user, and vice-versa, is obviously irrealistic and binding. Indeed, the model lends itself to very interesting and useful extensions to take into account a plurality of actors. One such extension can easily be construed to take into account the possibility of many users, possibly active in different industries, for each producer. In this case, a plurality of innovative relationships would come into being providing to each one of them the benefit of externalities through feed-backs. Consider the following straightforward modifications of the above stated equations. Innovative inputs for producer i, i = 1, 2, k, from many users j = 1, 2.... n. Thus, in lieu of 1:

$$L_{i}^{*}(t+1) \bullet L_{i}^{*}(t) / L_{i}^{*}(t) = \sum_{j=1}^{n} \lambda_{j} \frac{c_{j}(t) \bullet c_{j}(t+1)}{c_{j}(t)} \bullet \frac{X_{j}(t)}{X(t)}$$

where X_j and X are output absorbed by user j and total output from producer i respectively. Different production conditions justify different learning parameters λ_j . Goal setting is now more problematic in the sense that the target of producers' efforts concerns potentially more than one user's process. For simplicity's sake, let the same procedure employed for 11. be replicated for the following goal setting procedure:

$$\sum_{j=1}^{n} \left[c_{j}^{*}(t+1) \bullet c_{j}^{*}(t) \right] X_{j}(t) / X(t) = \beta_{1} \sum_{j=1}^{n} \left[\hat{c}_{j}(t+1) \bullet c_{j}^{*}(t) \right] + \beta_{2} \sum_{j=1}^{n} \hat{r}_{j}(t+1)$$

The interesting point lies with the estimated function between means and ends which now takes the following shape for any *j* process:

$$\hat{c}_{j}(t+1) \bullet c_{j}(t) = \bullet \hat{C} \left[R_{i}(t), L_{i}(t), \sum_{j=1}^{n} \left[c_{j}(t \bullet 1) \bullet c_{j}(t) \right] \right]$$

which can be reformulated as a linear function as:

$$\hat{c}_{j}(t+1) \bullet c_{j}(t) = \hat{\gamma}_{j1}(t) R_{i}(t) L_{i}(t) + \hat{\gamma}_{j2}(t) \sum_{i=1}^{n} \left[c_{j}(t \bullet 1) \bullet c_{j}(t) \right]$$

and parameters $\hat{\gamma}_{j1}(t)$ and $\hat{\gamma}_{j2}(t)$ are learnt according to the simple rule of weighing the actual observation of results matched with means employed and with external factors, i.e. other users' (and sectors') cost reductions.

$$\gamma_{j1}(t) = \frac{c_{j}(t) \bullet c_{j}(t+1)}{R_{i} L_{i}}; \gamma_{j2} = \frac{c_{j}(t) \bullet c_{j}(t+1)}{\sum_{i=1}^{n} \left[c_{j}(t \bullet 1) \bullet c_{j}(t)\right]}$$

with the constraint $\gamma_{j1} + \gamma_{j2} = 1$ for any \emph{t} . Adjustment of expenditure would now have to be rewritten as:

$$R_{i}^{*}(t+1) \bullet R_{i}(t) = \gamma_{1} \sum_{i=1}^{n} \left[\hat{r}_{j}(t+1) \bullet r_{j}(t) \right] + \gamma_{2} \left[L_{i}(t+1) \bullet L_{i}(t) \right] + \gamma_{3} \left[R_{i}^{*}(t) \bullet R_{i}(t) \right]$$

and the «environment» function as:

$$c_{j}(t) \bullet c_{j}(t \bullet 1) = C \left[R_{i}(t), L_{i}(t), \sum_{j=1}^{n} \left[c_{j}(t \bullet 1) \bullet c_{j}(t) \right] \right]$$

All other equations can be modified in an obvious way. The main point emerging from the above reformulation is the all important externality effect. The model is now more complex and simulation necessarily more roundabout.

3. THE MACROECONOMIC MODEL

In previous pages a microeconomic model of decision-making leading to innovations has been discussed. Two of its basic features can now be used to formalize a macroeconomic model in which technical advance takes the form of an entirely endogenous process as well as one the strength of which is also due to exogenous events. In that model, technical change is the result of innovative efforts stemming from user-producer relationships. Within this linkage, capital goods producer take, in a broad sense, the protagonist' role even if still rudimentary and even if the economy is still relatively little developed or backward.

The degree of development, of course, matters greatly when considering the intensity of this relation.

The first feature to stand out is the relationship between cost reductions, a measure of technical advancement, and innovative efforts, measured by search expenditure, and innovative capabilities, the innovative equipment. A relationship which is learnt as activity evolves in time. Although, the actual empirical function may never be fully replicated due to non-linearities, it is nevertheless approximated through memory of past actual performance. In a macroeconomic context, a general definition of cost reduction capable of encompassing the whole economy is needed. A suitable measure, in this case, can be found by considering that every economy, and the one which is here assumed is closed to concentrate on purely internal processes, can be reduced to a basic system in the sense of Sraffa's (Sraffa 1960). Real wages will also be assumed as constant so that the basic system in question includes all the sectors directly or indirectly entering consumption goods on which wages are spent. No problems of consumer's choice and of distribution are addressed here in order to highlight motion due entirely to technical progress. Given this assumption, a satisfactory measure of overall, cost reduction, or increasing total productivity is the variation of the reciprocal of the system's eigenvector. Let such measure be indicated by k. The reciprocal of the eigenvector indicate the economy's potentiality in terms of its net surplus; it is in fact the maximum growth rate affordable by the system if no non-wage consumption takes place. In this model, there is no state sector and thus no taxes are levied. k is, therefore, a fair representation in economic terms of this state of technology. The macroeconomic equivalent of equation 6. which is here taken as a good approximation of 9. is:

$$\dot{k} = \theta (R, L)$$

where R and L are aggregate expenditure on innovative search processes and L is the current aggregate capability measured in efficiency units. 16. is a very general function. In a very developed and technically advanced context, R includes sophisticated forms of expenditure for what is now called research and development. In less formal contexts, however, it is expenditure to gather technical information, for technical updating, imitation and such like. Ironmongers in Sub-Saharan African villages spend very little for such purposes, if not in a very rudimentary form, machine-tool producers in India may, on the contrary, earmark an appreciable share of their financial resources to develop their product and possibly to imitate: reverse engineering, does, after all require some expense; South Korean or Indian electronic firms do now conduct their own research and development. The form which 16. takes is normally non linear. It may reasonably be conjectured that unless both R and L get over a certain threshold there are no effects on \dot{k} ; indeed, it may be supposed that below it, i.e. unless a minimum is spent on such search and minimum innovative capabilities are assured, the state of technology may deteriorate. For analytical treatment, a function of the following shape may serve the purpose to represent such characteristics:

$$\dot{k} = \bullet a_0 + a_1 R^{\eta} L^{\rho}$$
; $0 < (\rho; \eta) < 1$

first derivatives are positive while second derivatives are negative.

The second characteristic of the microeconomic model is the relationship between innovative capabilities and actual results. These again, as 1. and 2. show, are learnt. Furthermore, technical capabilities relate to technological complexity, i.e. to what is here termed the state of technology as symbolized by k. A further variable is, in a macroeconomic context, likely to play a significant role: learning-by-doing and by-using depend on the scale of economic activity. Thus, if a given state of skills, expertise and know-how organized in a routinized framework depends on cumulated activity levels, its rate of change depends also on the current one which, in a stateless, closed economy depends, via the multiplier, on the level of investment. Thus, the rate of change of L can be rendered as a function the state of technology, the current level of L and aggregate investment L:

$$\dot{L} = f_2(k, L, I)$$

As to the analytical form taken by 17. it is important that it captures the learning process. To keep it, for conveniency sake, as simple as possible but in order to reflect some of the feature seen with 1. and 2. above, call L^* the aggregate level aimed at and consider it a function of the technological state providing the information and the opportunity, the input to the learning process:

$$L^* = L^*(k); \frac{dL^*}{dk} > 0$$

Given 17.1, 17. can take a form so as to adjust L to L^* , as in a logistic function, or more simply as:

$$\dot{L} = I^{\alpha} [L^*(k) \bullet L]; \quad \alpha > 0$$

Search expenditure, as it has been seen with 8. and 8.1 depends basically on the state of technology and turn-over which, in this context, is the aggregate level of effective demand, again a function of investment.

$$R = \Phi(k, I)$$

Again, for the sake of simplicity, consider it as:

$$R = A(k) I$$
 A'>0;

Equations 16.-18.1 describe an endogenous process of technical change in the sense that the latter is explained in ultimate analysis by economic activity. What is now required to make it fully endogenous is an explanation of investment.

Investment determined by technological advance

That technical change be at the heart not only of economic development but of an endogenous virtuous circle of productivity, income and output growth is an historical fact which, as it was mentioned, was forcefully argued by A. Smith and by the classical school. Furthermore, that technical change and capital accumulation were the crucial analytical magnitudes of the process were also tenets of the theories due to, among others, Kalecki (1954), Kaldor (1961), Steindl (1979). Schumpeter (1943) constructed a view of long-term development based on the thrust given to the economy by clusters of radical innovations. This occurs when new technological principles, or indeed new paradigms, developed but not exploited during long cyclical downswings, reach roughly in the same period the stage of applicability and provide an entirely new and rich field of investment opportunities from which a long-wave of output growth ensues. Innovators who develop new goods and processes stand to reap profits above the norm. As imitators jump on the band-wagon and economic activity unfolds towards a cyclical peak, prices fall and costs rise so that quasi-rents are eroded away and the profit motive behind investment peters out. While this view may, indeed, be a cause of controversy, for instance on the empirical support to be given to clusters rather that to waves of innovation (Kleinknecht 1990), it still remains a powerful suggestion of how an economy can be propelled towards long bouts of growth thanks to technical progress. In this case, however, while corroborated by mounting and forceful historical evidence, it is an entirely exogenous factor.

The impact of exogenous technical progress can be the initial point along a technological trajectory (Dosi 1988). A set of radical innovations may constitute a break-through from which an entirely new technological paradigm stems. From that point onwards technical change becomes an endogenous process along the lines stated by equations 16.-18. The impact of this form of technical change is equally important in stimulating investment. Although less dramatic and possibly incremental in nature, given properties of continuity and cumulativeness, it still provides investment opportunities through higher expected profit rates, as argued in the previous section, due to cost reductions. This holds true, in particular, for replacement investment since, within certain circumstances (Ricottilli 1988), technical progress quickens the pace of scrapping and substitution. The opportunity that this form of investment provides, in fact, does not lie so much with greater than expected growth of demand warranting increasing capacity but with higher profit per unit of output owing to better and more advanced means of production.

Investment activity, as recent cyclical experience shows, still follows a pattern of adjustment of capacity to demand pressure and, in fact, accelerator-capital adjustment mechanisms can still account for short-term oscillations. Adjustment, moreover, is required since investment dictated by radical as well as more run of the will innovations are, to a large extent, independent of the short-term demand outlook and thus excess capacity is likely to arise.

The foregoing considerations can now be used to model investment demand seen in accordance to the three aspects reviewed. What tells them apart is the motive which is behind them, the specific investment related to different forms of technical change and to adjustment.

Exogenous technical progress

Investment led by radical and exogenous forms of technical progress is motivated, as argued above by higher than normal profit rates, by so called technological quasi rents. For analytical purposes, the normal profit rate may be defined for any new introduced technique as the one applying when its adoption is complete and the economy has fully incorporated it in its technological matrix. While this approach may be justified in a macroeconomic context, microeconomic behaviour is undoubtedly more complex, expected profit rates being compared with historical ones, both being then subject to continuous revision as adoption and imitation proceed. Quasi-rents, or the above norm profit rate is here treated as follows: given an initial condition, exogenously given, it rises as the capital stock incorporating the new technique rises. This is due to the fact that technological novelties take time to be pioneered and be brought up to efficient performance. As imitative behaviour spreads and the new capital stock rises still further a maximum is reached after which decline begins towards the above defined normal profit rate. Let the former rate be indicated by r(K) while the latter by r(k), a function of technological state as synthesized by the reciprocal of the eigenvector. The differential

$$x = r(K) \bullet \bar{r}(k) \quad ; \begin{cases} \frac{dr}{dK} < \ge 0 \text{ for } K > \le K^* \\ \frac{d\bar{r}}{dk} > 0 \end{cases}$$

 K^* indicates the critical level for which r(K) reaches a maximum. dr / dk is positive for the reasons explained above. x defines the incentive to invest to exploit the opportunities provided by exogenous innovations; let it be defined as:

$$I_1 = F_1(x); I_1(0) = 0; I_1' \ge 0$$

Note that because of the assumptions, t he time profile of 15. is such that:

$$\dot{I}_1 = \frac{dF_1}{dx} \bullet \frac{dr}{dK} I \bullet \frac{dF_1}{dx} \bullet \frac{d\overline{r}}{dk} \dot{k}$$

 \emph{I} is as usual aggregate investment and the definition contains the assumption that all investment, if it takes place, concerns innovated equipment. It follows from 15.1 that $\dot{\textbf{I}}_{\text{I}}=0$ for

$$I^* = \frac{d\bar{r}/dk}{dr/dK} \dot{k} \text{ for } dr/dK > 0.$$

Equations 20.1 and 20.2 describe a contrasting dynamic behaviour: as investment proceeds further reason to invest are provided, but decreasingly so, by a rising x; at the same time, as endogenous progress is also carried on and the underlying normal profit rate rises, fewer reasons for investment of the first type are provided. Indeed, as 20.2 shows, rising k requires an ever increasing quantity of total investment in order to keep I_1 stationary, at least as long as the new capital stock is such as to maintain a positive incentive.

Endogenous technical progress

This kind of technical change, as argued above, provides an incentive to invest in its own right. This is, in fact, investment tied to user-producer effort to improve as discussed in the previous section and thus it is related to profit opportunities created by search and learning on the part of firms, in particular capital goods producers. By their nature, innovations which are therefrom generated, do not open up new avenues of untapped demand, investment which is so

motivated is generally not new capacity readyed to meet increasing demand pressure but is only carried out to improve production efficiency. This is the main reason why it concerns replacement the actual date of which it concurs to fix. It, therefore, follows that while \dot{k} is a crucial variable in determining it, newly installed capital, stock, K, is a draw-back in further investment. In functional form:

$$I_2 = F_2(\dot{k}, K)$$
; $\partial F_2/\partial \dot{k} > 0$; $\partial F_2/\partial K < 0$

Second derivatives are both negative. By differentiating

$$\dot{\mathbf{I}}_2 = \frac{\partial \mathbf{F}_2}{\partial \dot{\mathbf{k}}} \ddot{\mathbf{k}} + \frac{\partial \mathbf{F}_2}{\partial \mathbf{K}} \mathbf{I}$$

Stationarity is reached when

$$I = \bullet \frac{\partial F_2 / \partial \dot{k}}{\partial F_2 / \partial K} \ddot{k}$$

This last relationship is positive. 21.2 indicates that the faster is technical change, as measured by \dot{k} , the greater is total investment required to keep its replacement part constant; this is, in fact, due to be breaking role played by the new capital stock being installed.

Adjustment investment

This last element can be stylized in terms of an accelerator term so that investment keeps track of demand pressure and of an adjustment term to bring capacity down to size with production requirements. As in previous equations, the former term can be rendered by aggregate investment itself while the second by the capital stock:

$$I_3 = F_3(I, K)$$
; $\partial F_3 / \partial I > 0$; $\partial F_3 / \partial K < 0$

From which

$$\dot{\mathbf{I}}_3 = \frac{\partial \mathbf{F}_3}{\partial \mathbf{I}} \dot{\mathbf{I}} + \frac{\partial \mathbf{F}_3}{\partial \mathbf{K}} \mathbf{I}$$

and the stationarity for I_3 is given by:

$$I^* = \bullet \frac{\partial F_3 / \partial I}{\partial F_3 / \partial K} \dot{I}$$

a positive relation. Finally:

$$I = \sum_{i=1}^{3} I_i$$

The dynamical system

By substituting 18. into 16., the following equation is obtained

$$k = f_1(k, L, I)$$

Furthermore, summing 20.1, 21.1, 22.1 and substituting for \ddot{k} by differentiating 16. and after some algebraic manipulations, a function of the following form is obtained:

$$I = f_3(k, L, I)$$

A system of non-linear differential equations has been obtained:

$$\dot{k} = f_1(k, L, I)$$

 $\dot{L} = f_2(k, L, I)$
 $\dot{I} = f_3(k, L, I)$

This system expresses a motion in which entirely endogenous technical progress is combined with an exogenous kind. The latter is formalized by stating a function defining x, the incentive provided for investment warranted by radical innovations from which a long-wave may ensue. Once this is given, the motion is self-sustaining. Studying the dynamics implied by 26. may be rather complex. It is, however, clearly and strongly dependent on the analytical form of the three relevant equations and therefore on the parameters used to construct them. It is quite interesting, however, to determine a stationary solution and study local stability characteristics.

To determine solutions for $(\dot{k}\;,\dot{L}\;,\dot{I})\,{=}\,0\,,$ consider 16.1, 17.2 and the following expression:

$$\dot{I} = \frac{\alpha(K) \bullet \alpha_1}{1 \bullet \sigma} I + \frac{\beta}{1 \bullet \sigma} f_1(k, L, I) + \frac{\gamma}{1 \bullet \sigma} f_2(k, L, I)$$

where α , β , γ , σ are combinations of partial derivatives and defined in the Appendix. It is then immediate that

The interpretation of the above conditions is fairly straightforward. Stationarity occurs when innovative capabilities are perfectly adapted to the targeted ones, the new capital stock is such that there is still an incentive for investment led by positive quasi-rents and finally when aggregate investment is kept at a critical threshold above which positive growth occurs. Note that $\alpha(k)$ and β are both decreasing (see Appendix) and from a certain point onwards negative. Thus, investment is certainly going to have null variations first and then to decrease even if \dot{L} and \dot{k} are positive. Although quite stringent, these condition are quite significant for an economy's development outlook. 28. *iii*) indicates, in fact, that $\dot{I}=0$ when the positive incentive given to investment because of radical innovation is entirely offset by high retrenchment due to adjustment to excess capacity as shown by α_1 . At the same time no endogenous technical change occurs, when 28 *ii*) holds, if investment does not reach past the level at which significant amounts of searching are made possible. Economies where investment is sluggish are economies unlikely to exhibit endogenous virtuous circles of productivity growth.

To assess stability properties consider the following:

$$J = [f_{ii}]i, j = 1, 2, 3$$

is the Jacobian matrix of partial derivates taken at the point of stationarity.

A necessary condition for the linearized system to possess roots with negative real

parts, i.e. for the system to converge is that the trace be negative:

In this case of a three by three matrix the following, necessary and sufficient, Routh-Hourwicz conditions can be applied:

$$a,b,c>0$$
 and $ab \bullet c>0$

where:

$$a = \bullet tr(J)$$

$$b = (f_{22} f_{33} \bullet f_{23} f_{32}) + (f_{11} f_{33} \bullet f_{13} f_{31}) + (f_{11} f_{22} \bullet f_{21} f_{12})$$

$$c = \bullet \det J$$

Signs of the Jacobian elements are as follows:

$$f_{1i} > 0$$
; $i = 1, 2, 3$; $f_{21} > 0$; $f_{22} < 0$; $f_{23} = 0$;

The signs of $f_{_{3i}}$ depend on σ appearing in 27. σ is an important parameter since it defines a combination of accelerating effects. A high σ indicates an economy which reacts strongly to demand pressures as well as to endogenous technical change, a low σ a slow-reacting one. β , on the other hand, balances the positive effect on replacement investment of endogenous change with its dissuasive impact on purely Schumpeterian investment. Stationarity is likely to happen where the latter has largely waned with respect to the former. Let it be assumed, therefore, that, for simplicity, $\beta = 0$. Two cases must now be distinguished:

i) $1-\sigma > 0$. σ is small enough so that the economy adjusts slowly to endogenous stimuli. Furthermore, being $\beta=0$ (or sufficiently small) $f_{3i}<0$. Thus the trace is negative if f_{11} , the only positive parameter, is sufficiently small. f_{11} is also a crucial parameter. It is in fact, the partial derivative of k in k: it measures therefore the impact that the state of technology bears upon its further change; it is, in other words, the strength of «focusing devices» in generating endogenous change. Because of this peculiar characteristic, it is a crucial element in determining the innovative capability of the economic system in question. Convergence to stationarity requires, therefore, as a necessary condition that it be sufficiently small.

Whether the necessary and sufficient conditions are also satisfied depends practically on the compounding effects of all parameters; it is quite easy to see that if parameters reflecting the impact of both learning processes and endogenous stimuli to investment are small, $(k\,,L\,,I)=0$ is a stable point.

ii) $1-\sigma < 0$. Acceleration is, in this case, stronger. Coefficients $f_{_{3i}}$ are all positive and it is quite unlikely that the necessary condition be, in the circumstances, satisfied. Since $f_{_{22}}$ is still negative, there remains the remote possibility that the sum of both $f_{_{11}}$ and $f_{_{33}}$ is not large enough to match it. Quite aside this very special case, the situation which is portrayed when $1-\sigma < 0$ holds is that with a strong reaction to effective demand, the system is able to avoid the stationary trap. Given positive initial conditions it is able, thanks to roots with positive real parts, to diverge. An oscillating behaviour cannot, however, be ruled out.

Bifurcation

The analysis made above can be carried further. As mentioned, the specific magnitude of

parameters is crucial. They can be viewed, in fact, as a description of structural characteristics which define a specific system especially as far as innovative behaviour, capabilities and thus capacity to sustain endogenous growth are concerned. Economies differ exactly because this set of parameters differs and not so much and not necessarily on account of their dynamic behaviour. This is especially true for those capabilities concerning search to innovate and learning.

In previous paragraphs, much importance was given, for instance, to parameter f_{11} depicting, quite broadly, the capacity of the system to learn and receive inputs from the existing technological state. Different magnitudes of f_{11} , due to the degree of development, are responsible for structurally different dynamic behaviour. Even a marginal shift of this parameter may significantly alter the system's economic destiny. To see this, consider the case in which cyclical dynamics occur. Indeed, roots of J can feature negative real parts but also be complex. This is the case in which stability prevails. In particular system 26. once linearized, exhibits one real root and two complex and conjugate ones if its discriminant $\Delta > 0$. If this condition is also satisfied, the system converges to stationarity with damped oscillations. Take now f_{11} : if small enough, as argued above, condition 29.1 and, together with appropriate values of other parameters, conditions 30. are satisfied. Let now

$$y = ab \bullet c > 0$$
; $a, b, c \neq 0$

from the definition of a, b, c it is $\partial y/\partial f_{11} < 0$: continuous positive variations of f_{11} lead to continuous but negative variations of y; there is then an f_{11}^* such that

$$y = ab \bullet c = 0$$
 for $f_{11} = f_{11}^*$ if $\lim_{f_{11} \to \infty} y = 0$

The last part of 31.1 restricts its applicability. In this case, the real part of complex and conjugate roots are zero. It is now possible to resort to Hopf's theorem using f_{11} as the bifurcation parameter (Lorenz 1989).

Theorem: if a system such as the one defined by a linearized form of 26. has an equilibrium at $(\dot{k}, \dot{L}, \dot{I}) = 0$ and f_{11}^* and (i) has two complex and conjugate roots in the real part equal to zero and (ii) the first derivative of the real part in terms of f_{11} is positive

Install Equation Editor and doubleclick here to view equation.

then there exist periodic solutions bifurcating from the stationary equilibrium point, in this case $(k\,,L\,,I)=0$.

Assumptions made insure that 31.2 is indeed positive. It is, in fact, negative for $f_{11} < f_{11}^*$ and the system is stable, it is zero for $f_{11} = f_{11}^*$ and for further positive variations it is positive, for $f_{11} > f_{11}^*$ and the system is unstable in a neighborhood of the stationary point. There are methods, moreover, the algebra of which is rather complex, which can determine the stability of a closed orbit.

The case just discussed is interesting since it clearly indicates the possibility of a closed orbit. Imagine now developing economies which are quite similar almost in every respect but differ, although slightly, for their capability to set off a technological search over their acquired, possibly still rudimentary, productive set-up. This could well be due to small differences in the capability to perceive opportunities, to see where users' problems lie, indeed in the human abilities which decisions to spend to conduct a quest for improvement in ultimate analysis owe to. Suppose now these economies to lie in the neighborhood of a stationary state: the little there is to be learnt has, in fact, been absorbed, fast radical innovations, if they ever occurred, have spent their driving thrust, effective demand given the propensity to invest for technical

change is too low to support effective levels of expenditure to improve. They have nearly similar f_{11} 's. It is now possible that if the conditions formally set out above prevail, for some positive shocks which unsettles them away from the stationary state, some will gradually but unavoidably return to equilibrium, to the trap in which they are caught, some may oscillate endlessly, while those with capabilities above the f_{11} threshold may start off on a path of sustained, even if cyclical growth.

4. CONCLUSIONS

This paper addresses the crucial issue of which forces can support a developing economy on a long-term growth path. Modern economic theory seems to be in broad agreement that growth is to a large extent the product of an endogenous virtuous circle. Neoclassical theories have contributed a view according to which human capabilities are of fundamental importance to this process; yet they consider them to be directly or indirectly, according to the particular strain a particular model belongs to, as an input in a production function with the usual solowian characteristics. The problem remains entirely that of maximizing an intertemporal stream of utility attained through consumption and thus activate a consistent saving plan which is, in neoclassical equilibrium, the same as investment. Choice of technique along the production function, in which inputs change endogenously, allows profit maximization.

A different view is taken here. The model presented in the previous pages is an evolutionary one in which behaviour, of a microeconomic kind, responds to principles of bounded rationality. In this context, firms and agents strive to innovate thanks to a given limited and highly specific routinized knowledge which is itself the product of specialization and past innovative efforts. They learn and search locally, while satisfacing rather maximizing guides their moves. Thus goals are set, problems framed and possibly solved. Actual results lend to revision and adjustment, in fact to learning. In this context the paper argues that producer-user relationships are essential to make innovation an endogenous process. Searching and learning with limited information and, clearly, limited computational abilities are the heart of innovative efforts. At any particular time, the existing stock of capital goods, know-how and skills define the technology being used which is itself the result of past innovative decisions. This technology sets both the bounds of what is technically feasible but also of what can be improved: focusing devices are at work to orient localized search. Solutions of problem and success bring about innovations which, on one hand, spread through the economic system on account of complementarities and externality effects and, on the other, establish the new set of conditions from which further progress stems. This sequence defines the main characteristics of the innovative process: seen from this viewpoint applied technical progress once set in motion by an exogenous source, say a cluster of inventions becomes (i) endogenous, (ii) continuous, (iii) cumulative and (iv) non-ergodic.

To make of this a model of long-term growth, it is necessary to shift analysis on to the macroeconomic theme, to link innovative efforts to determinants of effective demand. This is done by making investment a function of innovations through the profit opportunities that they provide. But investment is itself a crucial element in determining innovations since it is the level of activity which sustains search efforts and learning-by-using and -by-doing. This is formalized in a non-linear model of growth.

Analysis of local stability reveals that stationary solutions are likely to exist; likely not surely since this is a model which is strongly dependent on what parameters, thus history, happen to be. Parameter changes lead to bifurcation: this possibility is explored and the conclusion is drawn that small differences in capabilities to frame problems and solve them thanks to the innovative challenge that a technological state provides may cause very different economic destinies. Some economies remain trapped, others oscillate along a close orbit, others still may finally grow. Results can be quite contradictory.

Convergence is likely to occur when the accelerator is weak, in which case the system gets caught in a low level equilibrium trap with no technological search. An economy in which reaction and adjustment to demand are strong attains growth even if the initial thrust is given by technical changes. This is a very strong case for specific sector- and innovation-oriented policies.

A developing economy requires initial strong demand to set the process of structural transformation in motion. If it manages to experience endogenous technical change, growth is induced thereby but slows down with weakening technical progress. Schumpeter thought that a long wave might be generated in this process. Whether a system, once fully developed, is able to produce repeatedly the initial conditions that set it in motion is quite open to question.

By differentiating 23. given 20., 21. and 22., the following is obtained:

$$\dot{I} = \frac{\partial \ F_1}{\partial \ x} \bullet \frac{dr}{dK} \ I \bullet \frac{\partial \ F_1}{\partial \ x} \bullet \frac{\bar{dr}}{dk} \ \dot{k} + \frac{\partial \ F_2}{\partial \ \dot{k}} \ \ddot{k} + \frac{\partial \ F_2}{\partial \ K} \ I + \frac{\partial \ F_3}{\partial \ I} \ \dot{I} + \frac{\partial \ F_3}{\partial \ K} \ I$$

and by differentiating the first equation of 26. for \ddot{k} , it is

$$\dot{I} = \frac{\partial F_1 / \partial x \bullet dr / dK + \partial F_2 / \partial K + \partial F_3 / \partial K}{1 \bullet \partial F_2 / \partial \dot{k} \bullet \partial f_1 / \partial \bar{I} \bullet \partial F_3 / \partial I} +$$

$$+\frac{\partial\,F_2\,/\,\partial\,\dot{k}\bullet\partial\,f_1\,/\,\partial\,k\bullet\partial\,F_1\,/\,\partial\,x\bullet\dot{dr}\,/\,dk}{1\bullet\partial\,F_2\,/\,\partial\,\dot{k}\bullet\partial\,f_1\,/\,\partial\,I\bullet\partial\,F_3\,/\,\partial\,I}\bullet\,f_1(k\,,L\,,I)\,+$$

$$+\frac{\partial F_2/\partial \dot{k} \bullet \partial f_1/\partial L}{1 \bullet \partial F_2/\partial \dot{k} \bullet \partial f_1/\partial I \bullet \partial F_3/\partial I} \bullet f_2(k,L,I)$$

Thus:

$$\beta \equiv \frac{\partial F_2}{\partial \dot{k}} \bullet \frac{\partial f_1}{\partial k} \bullet \frac{\partial F_1}{\partial x} \bullet \frac{d \ddot{r}}{dk}$$

$$\alpha(K) \equiv \frac{\partial F_1}{\partial x} \bullet \frac{dr}{dK} ; \quad \alpha(K_0) < \ge 0 \text{ if } K < \ge K^*$$

$$\gamma \equiv \frac{\partial F_2}{\partial \dot{k}} \bullet \frac{\partial f_1}{\partial L} > 0$$

$$\bullet \alpha_1 = \frac{\partial F_2}{\partial k} + \frac{\partial F_3}{\partial k} < 0$$

$$\sigma = \frac{\partial F_2}{\partial \dot{k}} \bullet \frac{\partial f_1}{\partial I} + \frac{\partial F_3}{\partial I}$$

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