

## README file

Dataset Title: “**QUANTHEM. Data for the time evolution of Rényi-2 entropy in random permutation circuits. Version 1.**”

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## Dataset Contents

The dataset consists of a .zip archive, named **QUANTHEM\_permutationcircuit.zip**, containing 18 tabular quantitative data files saved in .txt format and a README file saved in .pdf format (**README\_QUANTHEM.pdf**).

## Dataset Documentation

### Abstract:

The dataset contains data generated in the framework of Horizon Europe ERC QUANTHEM project. The data were the result of analytical and numerical computations of time evolution and long-time stationary values of the Rényi-2 entropy in random permutation circuits.

The data corresponds to numerically computed results describing the discrete time evolution of the Rényi-2 entropy under random permutation (RP) circuit dynamics and under circuit dynamics with random permutation gates with additional random

phases (RPP). The system consists of  $N$  qubits initialized in different initial states, namely: (A) a product state of the form

$$|\psi_0\rangle = (\cos \vartheta |0\rangle + \sin \vartheta |1\rangle)^{\otimes N}$$

where  $\vartheta$  is some rotation angle. We have produced data for  $\vartheta = \pi/8$  and different values of  $N$ . (B) typical (random) product states, (C) states with non-zero initial entanglement such as the Dicke-states.

### **Content of the files:**

The .zip archive **QUANTHEM\_permutationcircuit.zip** contains 18 data files.

The 5 files named as

**"QUANTHEM\_RP\_prod\_renyi2\_vs\_n\_NXX.txt"**

(with  $XX = 2, 4, 6, 8, 10$ )

contain the normalized subsystem size (column 1) and the corresponding long-time stationary value of the subsystem Rényi-2 entropy under RP circuit dynamics (column 2), for different values of the system size  $N$  and starting from the initial state

$$|\psi_0\rangle = (\cos \vartheta |0\rangle + \sin \vartheta |1\rangle)^{\otimes N}.$$

The data was produced using numerical exact diagonalization method.

The 3 files named as

**"QUANTHEM\_RP\_typ\_renyi2\_vs\_n\_NXX.txt"**

(with  $XX = 6, 12, 32$ )

contain the normalized subsystem size (column 1) and the corresponding long-time stationary value of the subsystem Rényi-2 entropy under RP circuit dynamics (column 2), for different values of the system size  $N$  and starting from random product initial states.

The data was obtained by numerical solution of the corresponding differential equation using numerical integration techniques.

The file named as

**"QUANTHEM\_RP\_typ\_renyi2\_vs\_t\_N128.txt"**

contains the discrete time steps (column 1) and the corresponding time-evolved value of the subsystem Rényi-2 entropy under RP circuit dynamics (column 2), for  $N = 128$  and starting from random product initial state.

The data was obtained by numerical solution of the corresponding differential equation using numerical integration techniques.

The file named as

**"QUANTHEM\_RP\_teq\_vs\_logN.txt"**

contains the base-2 logarithm of different systems sizes  $N$  (column 1) and the corresponding equilibration time obtained from the time evolution of the Rényi-2 entropy under RP circuit dynamics starting from random product initial state (column 2).

The data was obtained by numerical solution of the corresponding differential equation using numerical integration techniques.

The file named as

**"QUANTHEM\_RPP\_prod\_renyi2\_vs\_t\_N128.txt"**

contains the discrete time steps (column 1) and the corresponding time-evolved value of the subsystem Rényi-2 entropy under RPP circuit dynamics for  $N = 128$ , starting from random product initial state (column 2).

The data was obtained by numerical solution of the corresponding differential equation using numerical integration techniques.

The file named as

**"QUANTHEM\_RPP\_prod\_teq\_vs\_logN.txt"**

contains the base-2 logarithm of different systems sizes  $N$  (column 1) and the corresponding equilibration time obtained from the time-evolution of the Rényi-2 entropy under RPP circuit dynamics, starting from random product initial state (column 2).

The data was obtained by numerical solution of the corresponding differential equation using numerical integration techniques.

The 3 files named as

**"QUANTHEM\_RPP\_prod\_renyi2\_vs\_n\_NXX.txt"**

(with  $XX = 4, 8, 32$ )

contain the normalized subsystem size (column 1) and the corresponding long-time stationary value of the subsystem Rényi-2 entropy under RPP circuit dynamics (column 1), for different values of the system size  $N$  and starting from the initial state

$$|\psi_0\rangle = (\cos \vartheta |0\rangle + \sin \vartheta |1\rangle)^{\otimes N}.$$

The data was obtained by numerical solution of the corresponding differential equation using numerical integration techniques.

The 3 files named as

**"QUANTHEM\_RPP\_dicke\_renyi2\_vs\_n\_NXX.txt"**

(with  $XX = 16, 32, 64$ )

contain the normalized subsystem size (column 1) and the corresponding long-time stationary value of the subsystem Rényi-2 entropy under RPP circuit dynamics (column 2) for different values of the system size  $N$  and starting from a Dicke-state

$$|D\rangle = \frac{1}{\sqrt{2}}(\cos \vartheta |D_1\rangle + |D_2\rangle).$$

The data was obtained by numerical solution of the corresponding differential equation using numerical integration techniques.

## Methodology

All the data were obtained from numerical computations.

The data contained in

**"QUANTHEM\_RP\_prod\_renyi2\_vs\_n\_NXX.txt"** (with  $XX = 2, 4, 6, 8, 10$ )

were obtained using numerical exact diagonalization (state-vector representation) of random permutation circuit on the qubit Hilbert space.

The data contained in

**QUANTHEM\_RP\_typ\_renyi2\_vs\_n\_NXX.txt** (with  $XX = 6, 12, 32$ )

**QUANTHEM\_RP\_typ\_renyi2\_vs\_t\_N128.txt**

**QUANTHEM\_RP\_typ\_teq\_vs\_logN.txt**

were obtained using numerical solution of the corresponding (analytically derived) differential equation using numerical integration techniques.

The data contained in

**QUANTHEM\_RPP\_prod\_renyi2\_vs\_t\_N128.txt**

**QUANTHEM\_RPP\_prod\_teq\_vs\_logN.txt**

**QUANTHEM\_RPP\_prod\_renyi2\_vs\_n\_NXX.txt** (with  $XX = 4, 8, 32$ )

**QUANTHEM\_RPP\_dicke\_renyi2\_vs\_n\_NXX.txt** (with  $XX = 16, 32, 64$ )

were obtained using numerical solution of the corresponding (analytically derived) differential equation using numerical integration techniques.

### Notes

The data, the equations and analytical formulas used are further explained and presented in the pre-print:

D. Szász-Schagrin, M. Mazzoni, B. Bertini, K. Klobas, and L. Piroli, Entanglement dynamics and Page curves in random permutation circuits, arXiv:2505.06158 (2025)