# Nonlinear thermal reduced model for Microwave Circuit Analysis

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***Abstract*— With the constant increase of transistor power density, electro thermal modeling is becoming a necessity for accurate prediction of device electrical performances. For this reason, this paper deals with a methodology to obtain a precise nonlinear thermal model based on Model Order Reduction of a three dimensional thermal Finite Element (FE) description. This reduced thermal model is based on the Ritz vector approach which ensure the steady state solution in every case. An equivalent SPICE subcircuit implementation for circuit simulation is proposed and discussed. The nonlinear approach of the reduced model is based on the Kirchoff transformation. The model has been successfully implemented in a circuit simulator for a PHEMT device.**

## I. Introduction

The idea of this paper consists of giving a methodology to obtain a full thermal model. The advantages of such a model rely on its ability to precisely quantify the variation of electrical performances of a device versus temperature but also to give the maximum device temperature. Thus, the electronic designer can propose an original solution to the thermal management and for the reliability of electronic equipments. This methodology is general and can be applied to any kind of active devices. It is based on the thermal Finite Element simulation of components and a reduction method we have developed to obtain a linear thermal model[1][2]. The main contribution of this paper consists of demonstrating how the linear approach can be extended to the nonlinear one. Results and implementation of the thermal model of a Triquint PHEMT in a circuit simulator have been validated.

## II. Prediction of electrical and thermal behavior

The prediction of the thermal and the electrical behavior is possible by simulating the coupled electrical and thermal systems simultaneously. In order to include the thermal effects into the circuit simulation, we use a thermal model based on the analogy between voltage and temperature as well as current and dissipated power. This thermal network has to be connected to the electrical system as shown in figure (1). The thermal model can be either linear or nonlinear depending on the thermal dependence of material conductivities.

Because reliable temperature measurements are very difficult to obtain, taken into account of the small size of the devices, the thermal model is fully based on simulation techniques. However, the simulation method has been validated in [1] by electrical measurements of temperature.

The starting point of the thermal model is the 3-dimensional thermal Finite Element simulation of the device.

The thermal behavior of the devices is governed by the heat equation (1):

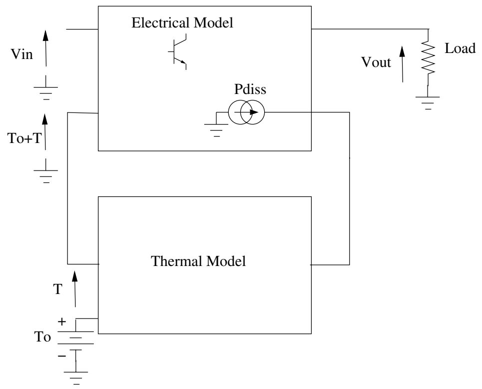


Fig. 1. Electrical model with its feedback thermal circuit

$$\nabla \cdot (\kappa(T)\nabla T) + g = \rho C_p \frac{\partial T}{\partial t} \qquad (1)$$

where $\kappa$ is the thermal conductivity, T the temperature, g the power generation density. The Finite Element formulation of equation (1) leads to the semi-discrete heat equation:

$$\mathbf{M}\dot{\mathbf{T}} + \mathbf{K}(\mathbf{T})\mathbf{T} = \mathbf{F} \qquad (2)$$

where the conductivity matrix **K(T)** and the specific heat matrix **M** are $n$-by-$n$ symmetric and positive-definite matrix, **T** the $n$-by-1 temperature vector at mesh nodes, **F** the $n$-by-1 load vector which takes into account of the power generation and the boundary conditions. The second step consists now to find an equivalent circuit to represent the FE solution. We have chosen to use a Model Order Reduction (MOR) method.

### A. *Model Order Reduction*

It has been demonstrated since few years that the Ritz vector approach [3] could be applied to the electronic domain [4]. In [1],[2], we have explained and validated the method on several structures including HBTs and PHEMTs. This approach is very powerful. to obtain reduced model when the conductivity matrix **K(T)** can be assumed to be constant. The advantages of this method rely on its ability to provide with few vectors a very good accuracy in the approximated solution of the the linear problem. A transient comparison in figure (2) of 3D linear simulation and FE model of an HBT validates this approach.

Moreover these Ritz values can be used to automatically generate a SPICE thermal subcircuit.
The thermal conductivity of substrate materials, such as GaAs, Si, GaN, .., used for RF and microwave devices exhibit a

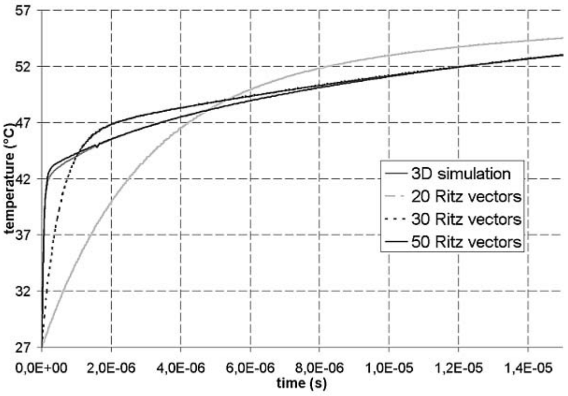


Fig. 2. Comparison between transient 3D FE simulation and Ritz vector approach

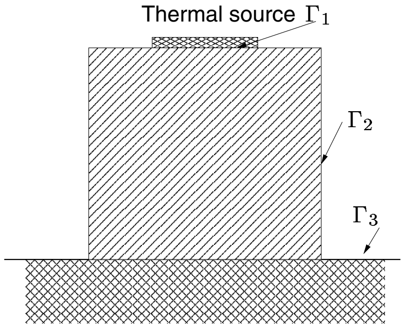


Fig. 3. Homogeneous test structure

temperature dependence of the form:

$$\kappa(T) = \kappa(T_{ref}) \left( \frac{T}{T_{ref}} \right)^{-\alpha} \quad \alpha > 0 \tag{3}$$

where $T_{ref}$ is an arbitrary reference temperature. This temperature dependence results in a decrease of the thermal conductivity from $45W/m.K$ at $293K$ to $25W/m.K$ at $473K$ for GaAs substrates. Ritz vector MOR technique can be applied to linear system only and thus must be adapted to capture the nonlinear thermal behavior. However MOR of nonlinear networks remains a difficult task [5], so we adopted as a first approximation a method based on the Kirchoff Transformation. Considering the homogeneous structure of figure (3), the 3D nonlinear thermal equation writes:

$$\begin{cases} \nabla \cdot (\kappa(T)\nabla T) + g = \rho C_p \dfrac{\partial T}{\partial t} & \\ T(x,y,z,t) = T_0 & \text{on } \Gamma_3 \\ -\kappa(T)\nabla T \cdot \hat{n}|_\Gamma = Q_\Gamma & \text{on } \Gamma_2 \text{ and } \Gamma_1 \end{cases} \tag{4}$$

where $\hat{n}$ is the outward normal vector on the boundary surface where Neumann conditions are defined. In order to linearize the equation( 4) the Kirchoff transformation [6] allows to transform the real temperature $T$ in a fictitious temperature $\theta$ as follows:

$$\theta = T_{ref} + \frac{1}{\kappa(T_{ref})} \int_{T_{ref}}^{T} \kappa(T) dT \tag{5}$$

From equation( 5) it follows that $\kappa(T_{ref})\nabla\theta = \kappa(T)\nabla T$ and the nonlinear thermal equation becomes:

$$\begin{cases} \kappa(T_{ref})\nabla^2\kappa(\theta) + g = \rho c \, \dfrac{\kappa(T_{ref})}{\kappa(T)} \dfrac{\partial \theta}{\partial t} & \\ \theta(x,y,z,t) = \theta_0 & \text{on } \Gamma_3 \\ -\kappa(T_{ref})\nabla\theta \cdot \hat{n}|_\Gamma = Q_\Gamma & \text{on } \Gamma_2 \text{ and } \Gamma_1 \end{cases} \tag{6}$$

The Neumann boundary condition is preserved by the Kirchoff transformation. Apart of the right hand side term the transformed equation is a linear one. Another transformation of the time scale can be used to fully linearize equation (4) [7]. However the experiments realized by maintaining the ratio $\dfrac{\kappa(T_{ref})}{\kappa(T)}$ constant have shown that a reasonable accuracy is preserved. Thus the treatment of the homogeneous nonlinear thermal behavior is performed by reducing the linear network using the Ritz vetor technique. This reduction provides the output of the SPICE equivalent network with a set of fictitious temperatures at the measuring nodes. To obtain the real temperature, one has to inverse the Kirchoff relation given in equation (5). Using the relation given in equation (3) the inverse relation is readily obtained as

$$T = T_{ref} \left[ \alpha + (1-\alpha) \frac{\theta}{T_{ref}} \right]^{\frac{1}{1-\alpha}} \tag{7}$$

This nonlinear relation can be easily implemented in the circuit analysis software with the aid of a nonlinear Voltage Controlled Voltage Source. However the choice of the parameter $T_{ref}$ is an important point to obtain the best accuracy of the approximation made. Indeed the Ritz vector MOR approach assumes the choice of a reference temperature of the base plate. As the model must be used for various temperatures of the base plate the $T_{ref}$ parameter is not unique. To optimize the choice of the parameter an interpolation formula is used in this approach. This interpolation is based on two reduced models corresponding to the conductivities associated with the minimum $T_{min}$ and maximum $T_{max}$ base plate temperature respectively. if $T_0$ is the actual base plate temperature, interpolation function is defined as:

$$\tilde{\theta} = (1+A)\theta_1 - A\theta_2 \quad where \quad A(T_0) = \frac{T_0 - T_{min}}{T_{min} - T_{max}} \tag{8}$$

During the simulation process two fictitious temperatures $\theta_1$ and $\theta_2$ are obtained from the reduced models derived with reference temperatures $T_{min}$ and $T_{max}$ respectively as shown in figure (4) for a particular measuring node. From these two temperatures an approximate fictitious temperature is obtained. The nonlinear voltage source is finally:

$$T = T_0 \left[ \alpha + (1-\alpha) \frac{\tilde{\theta}}{T_0} \right]^{\frac{1}{1-\alpha}} \tag{9}$$

the whole equivalent circuit is represented in figure (4).

### *B. Validation of the nonlinear approach*

The nonlinear model has been validated on two structures. First, we have applied the method on an homogeneous nonlinear structure, because Neumann boundary conditions as well as Dirichlet boundary conditions are rigorously preserved by Kirchoff transformation. The comparison between full nonlinear 3D model versus equivalent nonlinear reduced model is presented in figure (5) for severals base plate temperature

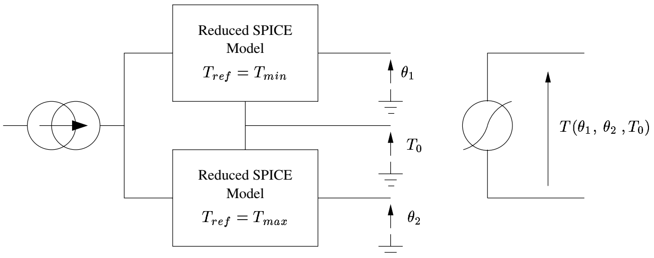


Fig. 4. Equivalent nonlinear reduced model

and two input powers (normalized by $P_0 = 86.6mW$). We can see a very good agreement. The next results concern an heterogeneous structure representing a TRIQUINT PHEMT of 12 fingers of $140\mu m$. Rigorously the Dirichlet conditions are not preserved, but [8] have shown that for some realistic structures, the error made with this assumption is reasonable. Figures (6) and (7) show the mesh and the static distribution of the temperature in half of the structure. Figure (8) show directly the comparison of $R_{th}$ between the full FE ANSYS model and the reduced model implemented in ADS. Figure (9) shows the transient simulation of the temperature ($T_{max}$) in log scale in the external finger of the PHEMT structure. This comparison exhibits a very good agreement on a large range of time.

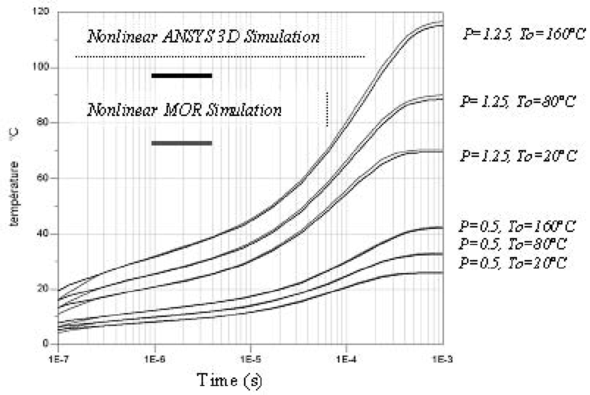


Fig. 5. Comparison full nonlinear 3D model versus equivalent reduced model

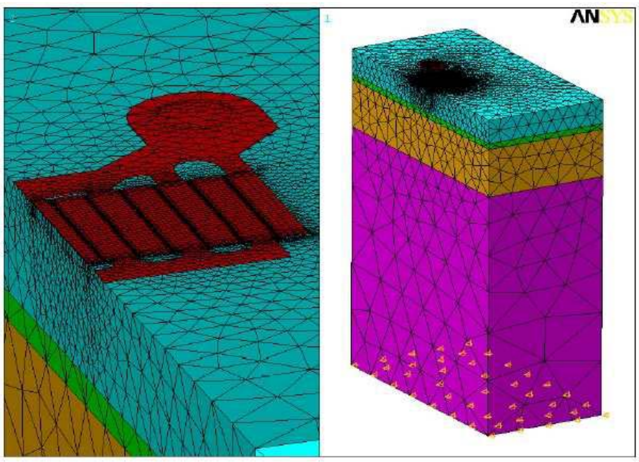

Fig. 6. Meshed TRIQUINT PHEMT

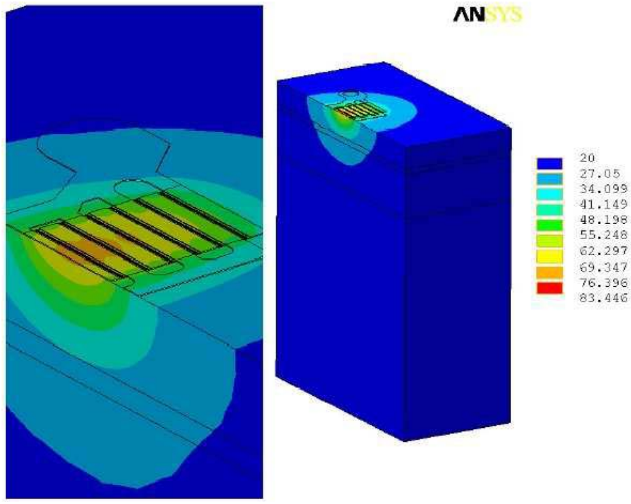


Fig. 7. Static temperature distribution in the TRIQUINT PHEMT

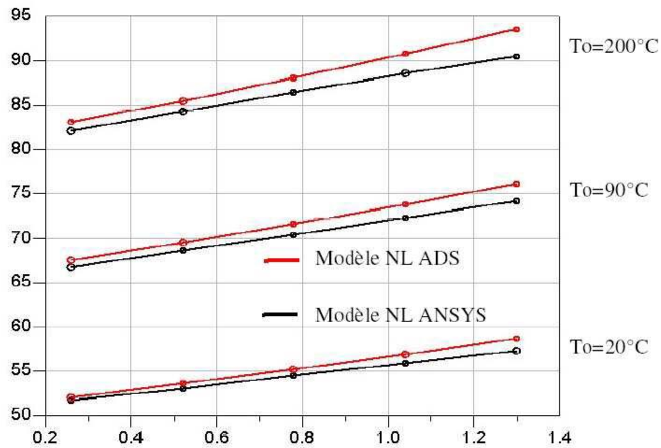


Fig. 8. Comparison between Rth (3D nonlinear simulated) and Rth (nonlinear reduced model)

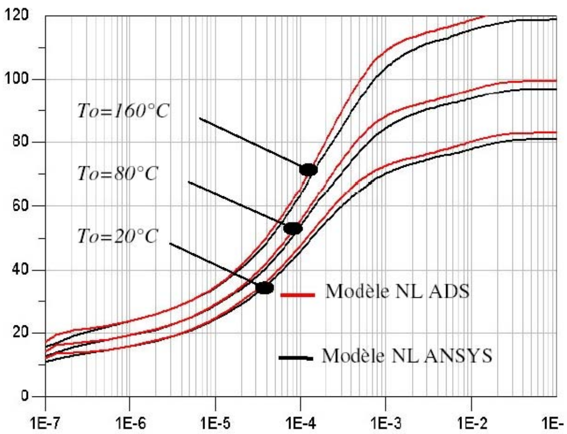


Fig. 9. Transient simulation : $T_{max}$ temperature in the external finger

## III. Conclusion

We have presented a method based on a 3D FE study to obtain a reduced thermal equivalent model for accurate prediction of electro thermal behavior of any kind of power devices. The reduction process based on Ritz vector approach leads to the generation of a SPICE format subcircuit that can be implemented in many circuit simulators. A nonlinear approach has been developed thanks to the Kirchhoff transformation. Coupled to the Ritz vector approach, the nonlinear case for a PHEMT structure has been demonstrated.

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